Problem Set 6

Theory of Computation

October 8, 2017

Problem 1. *Identify if the following languages are recursively enumerable or co-recursively enumerable.*

1. $L_1 = \{M : M \textit{ is the encoding of a Turing Machine}\}$
2. $L_2 = \{M : M \textit{ is the encoding of a Turing Machine that accepts the string } \epsilon\}$
3. $L_3 = \{M : M \textit{ is the encoding of a Turing Machine that recognizes the empty language}\}$
4. $L_4 = \{M : M \textit{ is the encoding of a Turing Machine with three states}\}$
5. $L_5 = \{M : M \textit{ is the encoding of a Turing Machine that accepts every input}\}$
6. $L_6 = \{M : M \textit{ is the encoding of a Turing Machine that halts on every input}\}$

$\dashv$

Problem 2. *For a binary alphabet Σ, formally describe a Turing Machine that recognizes the language $L = \{ww : w \in \Sigma^*\}$*

$\dashv$

Problem 3. *Consider the language $PRIMES$ over the unary alpahabet $\{1\}$, where $PRIMES = \{1^p : p \textit{ is a prime number }\}$. Prove that membership in $PRIMES$ is decidable.*

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Problem 4. *Define a stay-put Turing Machine to be one whose tape head can stay at the same cell it reads, in addition to moving either left or right. If $M = (Q, \Sigma, \Gamma, \delta, s, t, r, \vdash, \dashv)$ is a stay-put T.M., the transition function δ is as follows:*

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$$

If $\delta(q, a) = (p, b, S)$, it means that the T.M. in state q on reading an a moves to state p, overwrites a with b and the tape head stays at the same cell. Show that for every stay-put T.M. M, there is a T.M. N s.t.$L(M) = L(N)$.

$\dashv$

Problem 5. *A right-only stay-put Turing Machine is similar to a stay-put Turing machine, except for the transition function which will be of the form,*

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$$

(i.e) at each step, the machine can either move right or stay in the same tape cell. Construct a language L which can be recognized by a Turing machine, but not by a right-only stay-put Turing machine. Identify the set of all languages that these machines can recognize.

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Problem 6. *Consider the following model of computation called k-Tape Turing machine. This model contains k tapes, with each tape having its own corresponding head to read and write. The input is given on the first of the k tapes it contains. More formally, the transition function of this Turing Machine is as follows:*

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R\}^k$$

Prove that this model of computation is exactly as powerful as Turing machines.

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Problem 7. *Are recursive (decidable) languages closed under the following properties:*

- *Union*
- *Intersection*
- *Complementation*
- *Homomorphism*
- *Inverse Homomorphisms*

What can you say about the closure properties of recursively enumerable languages for the about list of properties?

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Problem 8. *We know that the set of languages recognized by push-down automata is strictly smaller than the set of languages recognized by a Turing machine. We extend PDAs to a model with two stacks such that on reading a word we have access to the top of both the stacks and can also push elements into both the stacks at each step. Which is to say that the transition function looks like:*

$$\delta : Q \times \Sigma \times \Gamma \times \Gamma \to Q \times \Gamma^* \times \Gamma^*$$

Show that the language $L = \{w \cdot w \mid w \in \Sigma^*\}$ is recognized by this model. What can you say about the set of languages recognized by this model of computation? Are there languages which are not recognized by this model but by PDAs with more than two stacks?

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A day may come when we can tell if a Turing machine fails on a given input;
when we are able to determine if a Turing machine will accept all words.
But it is not this day.