Problem Set 4

Theory of Computation

September 3, 2017

Do. Or do not. There is no try

Problem 4. *For a fixed* $L \subset \Sigma^*$, *consider a function*

$$f : \Sigma^* \to \Sigma^*$$

$$f : X \mapsto L \cdot X \cup \{\epsilon\}$$

Let $L_0 = \{\epsilon\}$ *and we define* $L_{i+1} = f(L_i)$. *We call* $\tilde{L}$ *a minimal fixed point of* f *if* $f(\tilde{L}) = \tilde{L}$, *and for all* L'' *such that* $f(L'') = L''$, $\tilde{L} \subset L''$. *Let* L' *be the minimal fixed point of* f. *Show the following:*

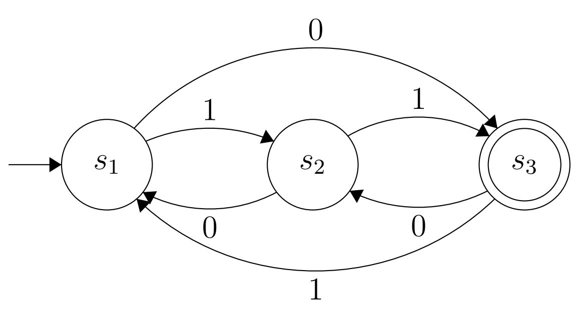
1. *There exists a unique minimal fixed-point of* f *for all* $L \subset \Sigma^*$
2. *Give an example of a language* L *such that* $L_k = L'$.
3. *Give an example of a language* L *such that* $L_k \neq L'$ *for all* $k \in \mathbb{N}$.
4. *If* L *is regular,* L' *is regular*
5. *If* L *is not regular, still* L' *can be regular by giving an example*

Problem 5. *Let* $\Sigma = \{a, b\}$. *Suppose* h *is a homomorphism from the alphabet* $\Sigma \to \Sigma^*$, *and* L *is some language in* Σ^*, *and* $h(L)$ *is regular. Prove or disprove that* L *is always regular.*

Problem 6. *Give an example of a language whose NFA requires* n *states, whereas any DFA accepting the same language would require* $\Omega(2^n)$ *states. Conclude that NFAs are exponentially succinct than DFAs.*

Problem 1. *Construct the Nerode Automata for* $L = (a^* + b)ab^*$.

Problem 2. *Write a regular expression for the language accepted by the DFA given:*



Problem 3. *We call* $k \in \mathbb{N}$ *a period of a language* $L \subset \{a\}^*$ *(language over a unary alphabet) if* $a^n \in L$ *implies* $a^{n+k} \in L$ *for large enough* n*. For* $L \subset \{a\}^*$*, prove that the following are equivalent:*

1. L *is regular*
2. L *can be represented as a finite union of languages of the form*

$$L_{a,d} = \{a^n : n = a + kd, k \in \mathbb{N}\}$$

3. $\exists! t \in \mathbb{N}$ *such that* k *is a period of* L *if and only if it is an integral multiple of* t.

Problem 7. *Prove or disprove the regularity of the following languages:*

1. $BB\text{-}8 = \{0^k u 0^k \mid k \in \mathbb{N}, u \in \{0,1\}^*\}$
2. $K\text{-}2SO = \{a^i b^j c^k \mid \text{if } i = 1 \text{ then } j = k\}$
3. $R2\text{-}D2 = \{a^{n^2} \mid n \in \mathbb{N}\}$
4. $C\text{-}3PO = \{a^p \mid p \text{ is prime }\}$

Rogue one. *Suppose* L *is a regular language, and a function* $f : \mathbb{N} \to \mathbb{N}$*, we define the language* $L_f = \{x \mid |y| = f(|x|), x \cdot y \in L\}$*. Let* $g(x) = 2^{|x|}$*, and* $h(x) = |x|^2$ *be two functions. Prove or disprove the regularity of the languages* L_g *and* L_h*.*

May the force be with you.