

Post's Correspondence Problem

Input: $(u_1, v_1), (u_2, v_2), (u_3, v_3) \dots (u_k, v_k)$
Qn: $\exists i_1 i_2 \dots i_n \in \{1, \dots, k\}^+$
 $u_{i_1} u_{i_2} u_{i_3} \dots u_{i_n} = v_{i_1} v_{i_2} \dots v_{i_n}$

MPCP

Input: $(u_1, v_1), (u_2, v_2), (u_3, v_3) \dots (u_k, v_k)$
Qn: $\exists i_1 i_2 \dots i_n \in \{1, \dots, k\}^*$
 $u_1 u_{i_1} u_{i_2} u_{i_3} \dots u_{i_n} = v_{i_1} v_{i_2} \dots v_{i_n}$

$$MPCP \leq_m PCP$$

To prove that MPCP is undecidable.

$$MP \leq_m MPCP$$

\uparrow

$M \# w$

$\Rightarrow (\#, \#q_0w\#)$

$\cdot (A, A)$ for all tape symbols

$\cdot (\#, \#)$

(q, A, B, p)	$\delta(q, A) = (p, B, R)$
(xqA, pXB)	$\delta(q, A) = (p, B, L)$
$(q\#, A, p\#)$	$\delta(q, \sqcup) = (p, A, R)$
$(xq\#, pXA\#)$	$\delta(q, \sqcup) = (p, A, L)$

$(x\sqcup, \sqcup)$
 (\sqcup, \sqcup)
 $(x\sqcup, \sqcup)$
 $(\sqcup\#\#, \#)$

$\delta(q_0, \sqcup) = (p, \sqcup, R)$

$\# \dots \# \sqcup \sqcup \sqcup \# \sqcup \sqcup \sqcup \# \sqcup \sqcup \sqcup \# \sqcup \sqcup \sqcup \#$
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Undecidable Problems of CFL

1) Ambiguity checking.

PCP \leq_m ambiguity checking.

$(u_1, v_1), (u_2, v_2), \dots, (u_k, v_k)$

$$u_i, v_i \in \{a, b\}^*$$

$$\in \{1 \dots k\}$$

$$- (u_1 + u_2 + u_3 + \dots + u_k)^+$$

$$- (v_1 + v_2 + v_3 + \dots + v_k)^+$$

$$\overline{(u_1 u_3 u_2)} = v_3 v_3$$

$$A \rightarrow u_1 A_1 \mid u_2 A_2 \mid u_3 A_3 \mid \dots \mid u_k A_k$$

$$\mid u_1 1 \mid u_2 2 \mid u_3 3 \mid \dots \mid u_k k$$

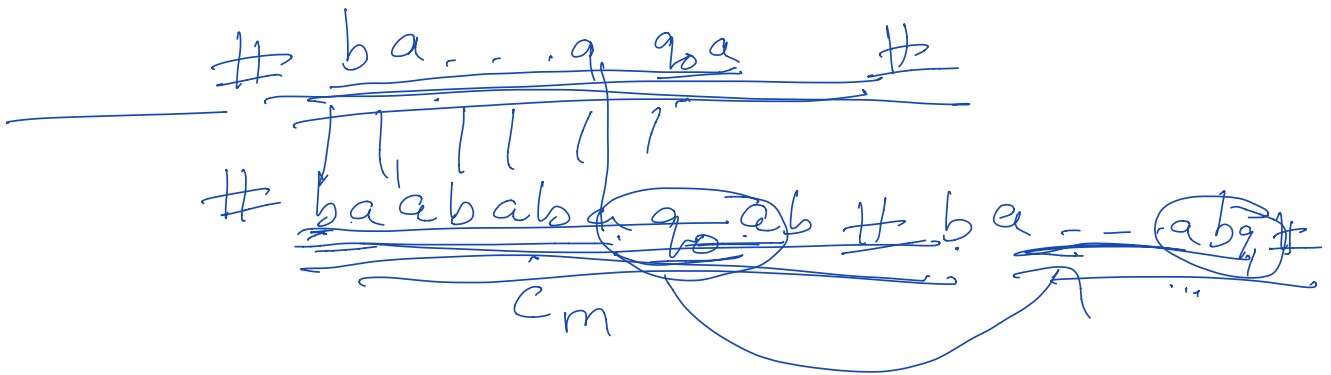
$$B \rightarrow v_1 B_1 \mid v_2 B_2 \mid \dots \mid v_k B_k$$

$$v_1 \mid v_2 \mid \dots \mid v_k$$

$$S \rightarrow A \mid B$$

$$\underline{u_2 u_1 u_3 u_2 (3112)}$$

$$\underline{v_2 v_1 v_1 v_3 v_2 (23112)}$$



$$\begin{pmatrix} b \\ b \end{pmatrix} \begin{matrix} a \\ a \end{matrix}$$

$$C_m \xrightarrow{\delta} C_m^Z$$

$$\begin{pmatrix} a_0 a \\ b a_1 \end{pmatrix}$$

Universality of CFL

$$L(A) = \Sigma^4 \text{ ?}$$

$\{ww \mid w \in \Sigma^*$ ^c is a CFL

M#w

Valid computation histories

~~$\#c_0\#c_1\#c_2\#c_3\cdots\#c_m\#$~~

~~$(w\#w)$~~

Invalid computation histories