Rice's theorem

Every non-trivial property of r.e languages is undecidable

$L_1 \subseteq L_2$ r.e

Property holds for L_1
not for L_2

then this property not r.e

$L_1 \subseteq L_2$ re

property doesn't hold for L_1
holds for L_2

then not co r.e

M_1 recognizes L_1
M_2 recognizes L_2 $L_1 \subseteq L_2$

Input to the HP $M\#x$:

Construct M':

On input 'y'

do in parallel
- (simulate M_1 on y
  accept if M_1 accepts y)
- Simulate M on x and check if it halts
  - if yes
    simulate M_2 on y
    and accept if M_2 accepts y.

$$L(M') = \begin{cases} L_1 & \text{if } M \text{ does not halt on } x \\ L_2 & \text{if } M \text{ halts on } x. \end{cases}$$

L_2 has the property (does not have the property)

L_1 does not (has)

property (doesn't) holds for $L(M')$

$\Leftrightarrow$ M halts on x

$M\#x \in HP$

$L_1 \subseteq L_2$

false true

$HP^{(c)} \leq_m$ propety-testing.

therefore property-testing is not co-re.

or $HP \leq_m$ property^c-testing.

therefore property-testing not r.e.

Consider this property of being a regular language.

$P = \{ \langle M \rangle \mid L(M) \text{ is regular} \}$

input: A TM M

Question is $L(M)$ regular?

$L_1 \subseteq L_2$

$\emptyset$ regular

palindromes not regular.

Hence it is not r.e. //

$L_1 \subseteq L_2$

palindromes $\quad \Sigma^*$

not regular. $\quad$ regular

it is not co-re.

Quiz 3 On Tuesday 17/10/2017

End Sem On Saturday 21/10/2017

To cover

more undecidable problems

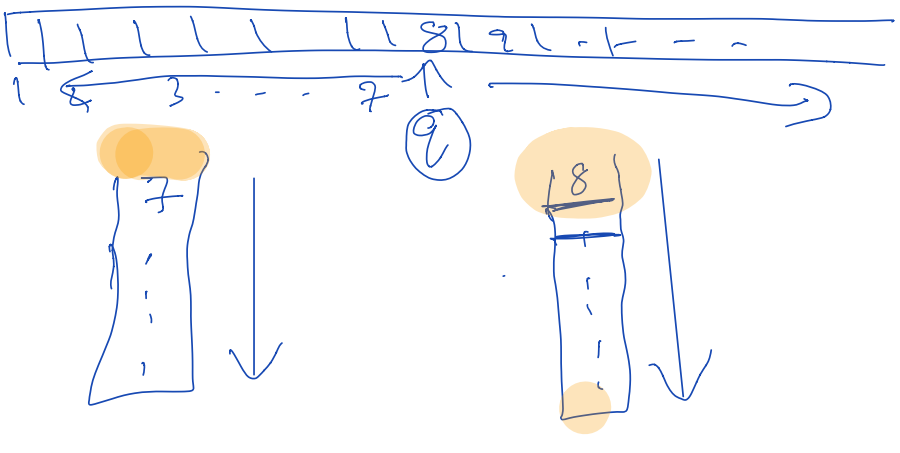
- PCP, tiling?

more Turing powerful machine models

- k-stack automata, counter machines, queue machines.

2-stack automata.

$$\delta \subseteq ((Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times \Gamma) \times (Q \times \Gamma^* \times \Gamma^*))$$



$q_1\ q_2\ \ldots\ q_n$

Counter machines

Operations on counter

- increment
- decrement
- if z

Simulate 2-stack machines
using 3-counter machines

Γ is the stack alphabet

$|\Gamma| = \gamma$

pushing a symbol d

$(\times \gamma) + d$

popping a symbol d

$(-d) / \gamma$

use the 3rd counter for storing intermediate values.

$(n \quad m \quad l)$

$2^n \; 3^m \; 5^l$

Simulate 3 counter machines with 2 counter machines

- one counter representation of a 3-counter configuration.
- simulate operations on 3-counter machines by using the 2nd counter as a temporary place holder (ex. decr ct-2 corresponds to dividing the number by 3)