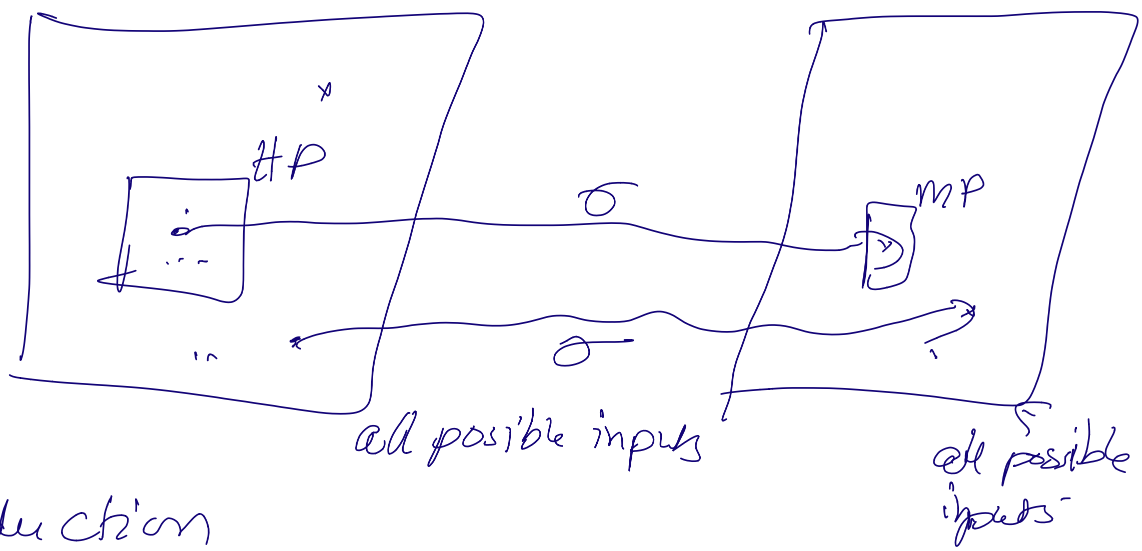
Reductions

$HP = \{ \underline{M\#x} \mid M \text{ halts on } x \}$

$MP = \{ M\#x \mid M \text{ accepts } x \}$



Reduction
from HP to MP

$$\underline{M\#x} \xrightarrow{\sigma} M'\#x'$$

$$\in HP \iff \in MP$$

On input $\underline{M\#x}$

compute $\sigma(M\#x) = M'\#x'$

Run the Total TM for MP on $M'\#x'$

$HP \leq_m MP$

[hard] $\Rightarrow$ [at least as hard]

$A \subseteq \Sigma^*$ $\qquad$ $B \subseteq \Sigma^*$

$A \leq_m B$

$x \xrightarrow{\sigma} \sigma(x)$

$x \in A \iff \sigma(x) \in B$

$HP \leq_m MP.$

$M\#x \xrightarrow{\sigma} \underline{M'\#x}$

M': new machine which simulates M

M' on input y.
Simulate M on y.
if M halts on y

then accept

Give $M'\#x$ as i/p to MP

Turing Machines

Algorithmic Questions

1) HP
2) MP
3) non ~~emptiness~~.

I/p : TM M

Qn : Is $L(M) \neq \phi$?

$MP \leq_m NE$

Reduction from MP to NE

$M\#x \xrightarrow{\sigma} M'$

M': on input y

1 - ignore y

2 - Simulate x on M

3 - if M accepts x (this simulation)

then "accept"

$$L(M') = \begin{cases} \Sigma^* & \text{if } M \text{ accepts } x \\ \phi & \text{o/w} \end{cases}$$

Check if M' is NE

$$MP \leq_m NE$$

not co.r.e $\Rightarrow$ not co.r.e.

<u>Inclusion</u>

I/p: M_1, M_2

Qn Is $L(M_1) \subseteq L(M_2)$?

$M_1 \# M_2$

<u>Emptiness</u> $\leq$ Inclusion.

$$M \xrightarrow{\sigma} M_1 \# M_2$$

$M_2 = M_\phi$ is a ~~fixed~~ TM with $L(M_\phi) = \phi$

$M_1 = M$

$L(M) \subseteq \phi$

$L(M) = \phi$?

$L(M) \subseteq L(M_\phi)$

<u>Universality</u>

i/p : M

Qn : Is $L(M) = \Sigma^*$?

HP $\leq$ Univ.

$M \# x \rightsquigarrow N$

$L(N) = \begin{cases} \Sigma^* & \text{if } M\#x \in HP \\ \phi & \text{o/w} \end{cases}$ or.

N : on i/p y.
1) ignores y.
2) simulates M on x
3) if the sim ~~accepts~~ halts

"Accept"

$A \leq B$

if B is easy then
A is also easy

if A is hard then B is hard.

Rice's theorem

Any non-trivial property of r.e languages is undecidable.

Property: Languages $\rightarrow$ {yes, no}

trivial property:

Input: TM M
Qn: Does $L(M)$ has property P?

$HP \leq P$

$P(\phi) = \text{false}$. Assume.

$P(L) = \text{true}$

$\uparrow$ a r.e. language : TM $\underline{\underline{K}}$

$M \# x \rightsquigarrow M'$

M': on input y
1) ignore y
2) simulate M on x
3) if simulation halts then simulate K only accept if K does.

$$L(M') = \begin{cases} L, & \text{if } M\#x \in HP \\ \phi, & \text{o/w} \end{cases}$$