

TURING MACHINES

$$(Q, \Sigma, \Gamma, \delta, q_0, t, \gamma)$$

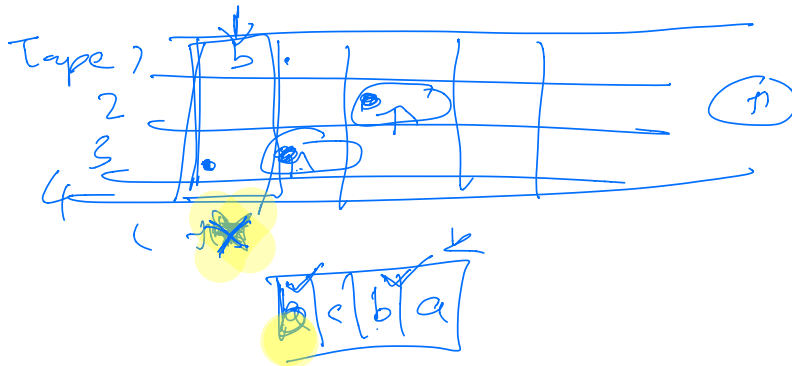
$$\delta \subseteq (Q \times \Gamma) \times (Q \times \Gamma \times \{L, R\})$$

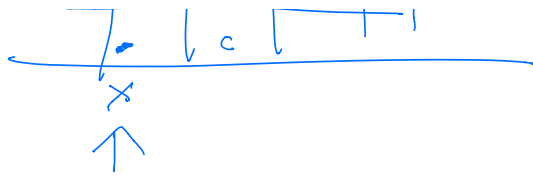
Any NDTM can be simulated by a DTM

Variants: - bi-infinite tape



- k tapes





↳ tape.

$$\delta \subseteq (Q \times \Gamma^k) \times (Q \times \Gamma^k \times \{L, R\}^k)$$

$$q (a_1 b_1 c) \rightarrow q_1$$

$$q \quad a \quad b \quad b \quad \rightarrow \quad q_2$$

- k heads.

$$\begin{array}{l} \text{TM} \quad M \\ L(M) = \{ w \mid \vdash w \vdash \rightsquigarrow u \oplus v w \} \\ \uparrow \\ \text{language} \\ \text{recognized by } M \end{array}$$

\uparrow
 accepting state

"Turing Recognizable Languages"
 L is " if $\exists M$ st
 $L = L(M)$

"recursively enumerable languages"

$$x \in L = \{w_1, w_2, \dots\}$$

Enumerator:

$$w_1 \# w_2 \# w_3 \# \dots$$

w

$$\begin{array}{ccccccc} \underline{a_1} & \underline{a_1 b_1} & \underline{a_1 b_1 b_2} & \dots & \dots & \dots & \dots \\ w_1 & w_2 & w_3 & \dots & \dots & \dots & \dots \end{array}$$

enumerating strings recognized by a TM

let w_1, w_2, \dots be an enumeration
of Σ^*

in the i th round

simulate i steps of computation
on words w_1, \dots, w_i

if any of these words is accepted
within i steps, output it.

"Recursively enumerable languages" } Countable
= "Turing Recog"

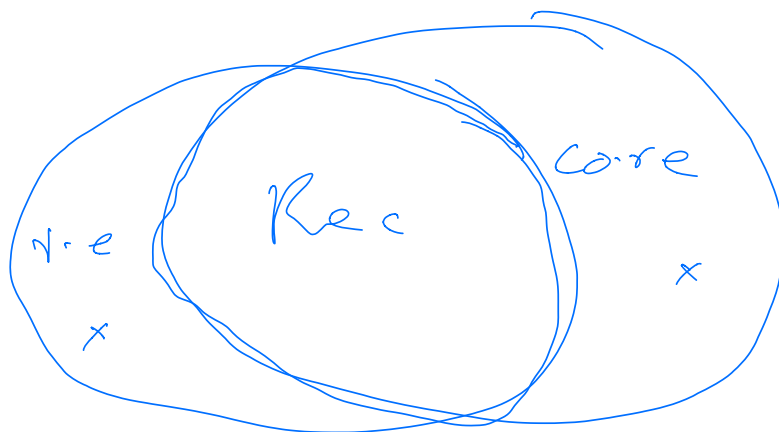
\exists languages that are not r.e.

"co-rec enumerable"

L is co-r.e if L^c is r.e

"Total Turing Machines" = ~~\mathbb{R}~~
"Recursive"

Recursive iff both r.e & co-r.e



"Decidable"
= is \exists total TM
 \Rightarrow comp. Lang is "recursive"

iff: w
 \exists : is $w \in L$

"semi-decidable"
= is \exists a TM

\Rightarrow const. lang is rec. enumⁿ

Universal Turing Machines

$$U = \{ M \# w \mid w \in L(M) \}$$