Defining relations on graphs: how hard is it in the presence of node partitions?

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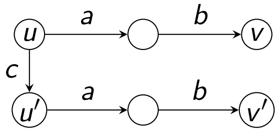
CMI

9 February 2015

Regular Path Queries on Graphs



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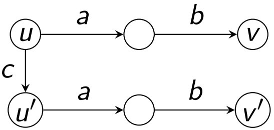
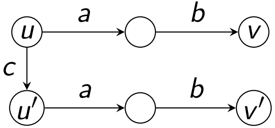


Regular path query $Q : x \xrightarrow{ab} y$

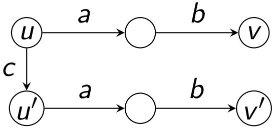
Regular Path Queries on Graphs



Regular path query $Q : x \xrightarrow{ab} y$

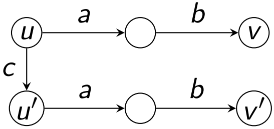
$$Q(G) = \{\langle u, v\rangle, \langle u', v'\rangle\}$$

RPQ-definability

- 

- $\{\langle u, v\rangle\}$

RPQ-definability

- 

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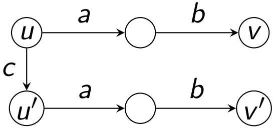
- Is there a RPQ Q such that $Q(G) = \{\langle u, v\rangle\}$?

RPQ-definability

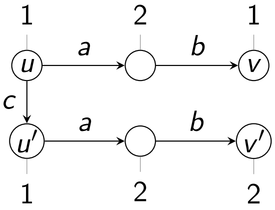
- 

- $\{\langle u, v \rangle\}$

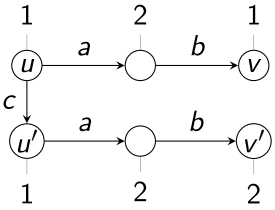
- Is there a RPQ Q such that $Q(G) = \{\langle u, v \rangle\}$?

- [Antonopoulos, Neven, Servais 2013] RPQ-definability is PSPACE-complete.

Node partitions

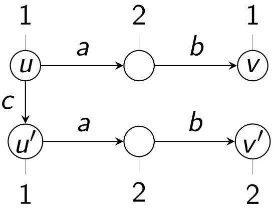


Node partitions



Regular data path query $Q : x \xrightarrow{\downarrow r_1 . ab[r_1^=]} y$

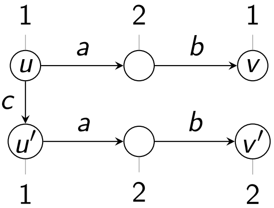
Node partitions



Regular data path query $Q : x \xrightarrow{\downarrow r_1 . ab[r_1^{=}]} y$

$$Q(G) = \{\langle u, v \rangle\}$$

Node partitions



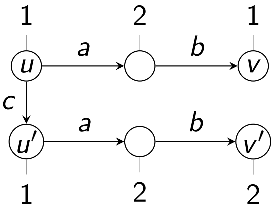
$$\text{Regular data path query } Q : x \xrightarrow{\downarrow r_1 . ab[r_1^{=}]} y$$

$$Q(G) = \{\langle u, v \rangle\}$$

$$e ::= \varepsilon \mid a \mid e + e \mid e^{+} \mid \downarrow \bar{r}.e \mid e[c]$$

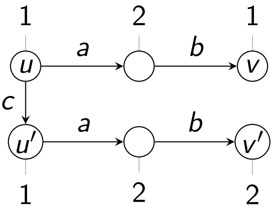
$$c ::= r^{=} \mid r^{\neq} \mid c \wedge c \mid c \vee c \mid \neg c$$

RDPQ-definability

- 

- $\{\langle u, v\rangle\}$

RDPQ-definability

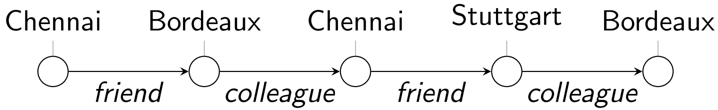
- 

- $\{\langle u, v\rangle\}$

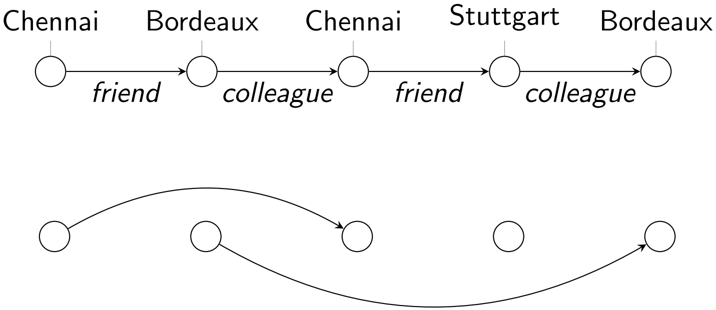
- Is there a RDPQ Q such that $Q(G) = \{\langle u, v\rangle\}$?

- We study the complexity of RDPQ-definability.

Motivation - schema mappings



Motivation - schema mappings



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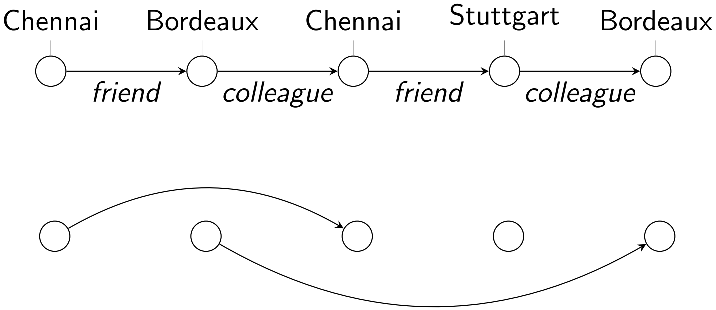


$$\downarrow r_1.(\mathit{friend} + \mathit{collegue})^*[r_1^=]$$

Related work

G.H.L. Fletcher, M. Gyssens, J. Paredaens, and D. V. Gucht.
On the expressive power of the relational algebra on finite sets of relation pairs.
IEEE Trans. Knowledge and Data Engg., 21(6):939–942, 2009.

G. Gottlob and P. Senellart.
Schema mapping discovery from data instances.
J. ACM, 57(2):6:1–6:37, 2010.

A. Das Sarma, A. Parameswaran, H. Garcia-Molina, and J. Widom.
Synthesizing view definitions from data.
In *Proceedings*, ICDT, pages 89–103, 2010.

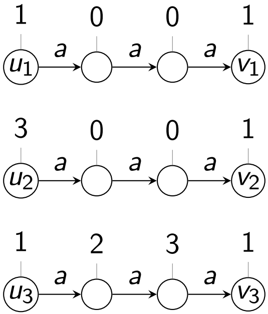
B. Alexe, B. T. Cate, P. G. Kolaitis, and W. Tan.
Designing and refining schema mappings via data examples.
In *SIGMOD*, pages 133–144, 2011.

Related work . . .

D. Calvanese, G. De Giacomo, M. Lenzerini, and M. Y. Vardi.
Simplifying schema mappings.
In *ICDT*, pages 114–125, 2011.

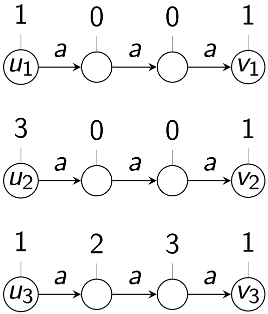
P. Barceló, J. Pérez, and J. Reutter.
Schema mappings and data exchange for graph databases.
In *ICDT*, pages 189–200, 2013.

Regular Expressions with Equality



$$Q = (a \cdot (a)_= \cdot a)_=$$

Regular Expressions with Equality



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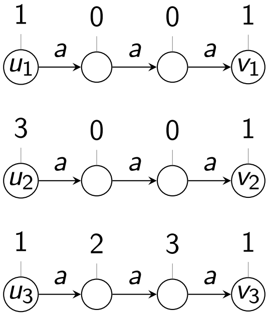

$$Q = (a \cdot (a)_= \cdot a)_=$$

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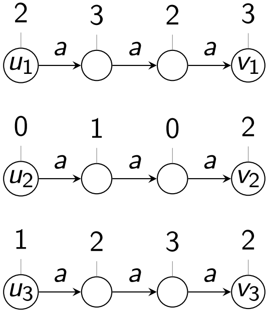
$$e ::= \varepsilon \mid a \mid e + e \mid e^+ \mid e_= \mid e_{\neq}$$

Number of registers



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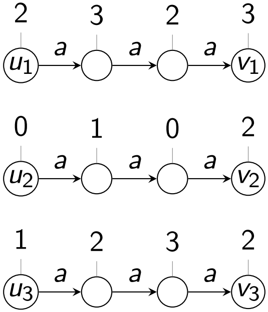
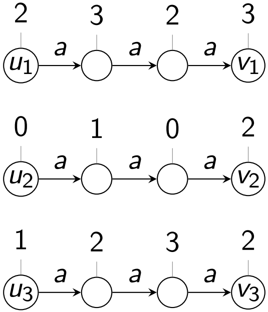


$$Q = \downarrow r_1 \cdot a \cdot \downarrow r_2 \cdot a[r_1^=] \cdot a[r_2^=]$$

Number of registers



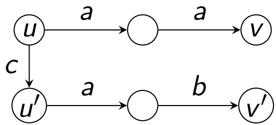
$$Q = \downarrow r_1 \cdot a \cdot \downarrow r_2 \cdot a[r_1^=] \cdot a[r_2^=]$$

$$Q(G) = \{\langle u_1, v_1 \rangle\}$$

Results

- $\mathrm{RDPQ}_{\mathrm{mem}}$-definability is $\textsc{Expspace}$-complete.
- $k - \mathrm{RDPQ}_{\mathrm{mem}}$-definability is in $\mathrm{N}\textsc{Space}(\mathcal{O}(n^2\delta^k))$.
- $\mathrm{RDPQ}_{=}$-definability is $\textsc{Pspace}$-complete.
- UCRDPQ-definability is $\textsc{co}\mathrm{NP}$-complete.

Witnesses for RPQ-definability



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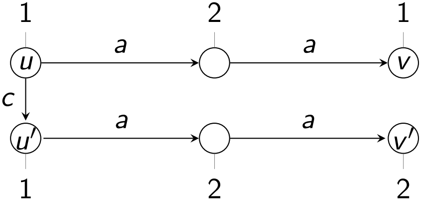

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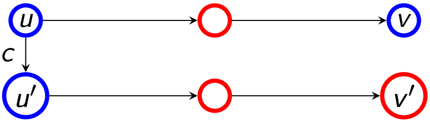

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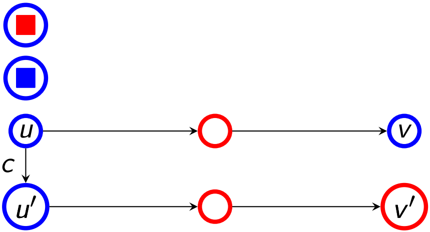

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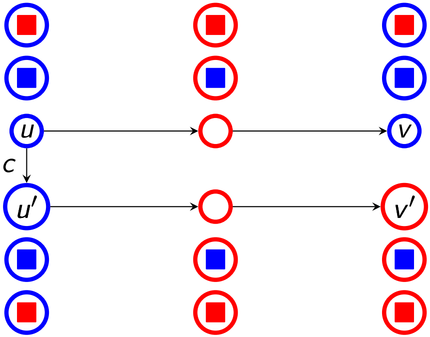

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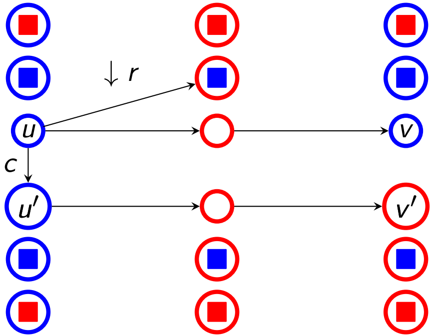

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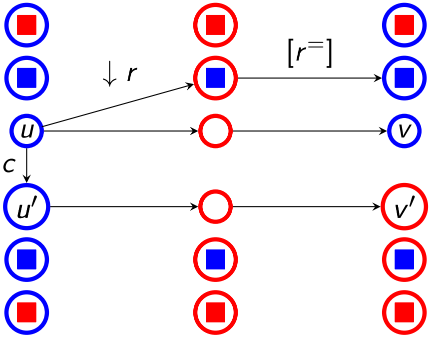

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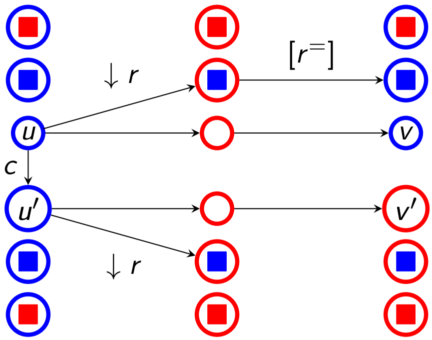

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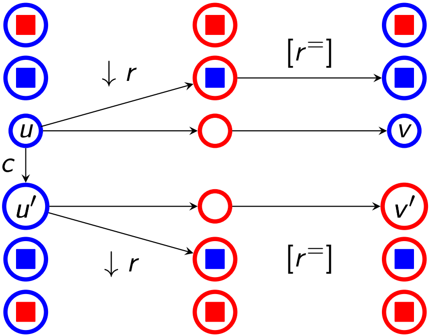
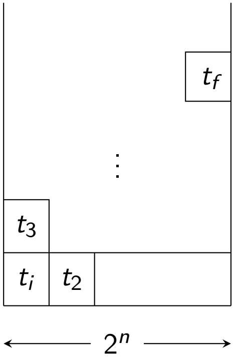


EXPSPACE Lower Bound



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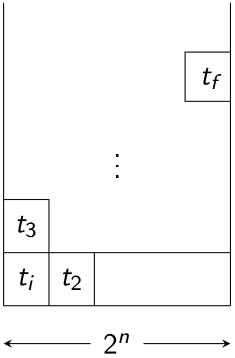

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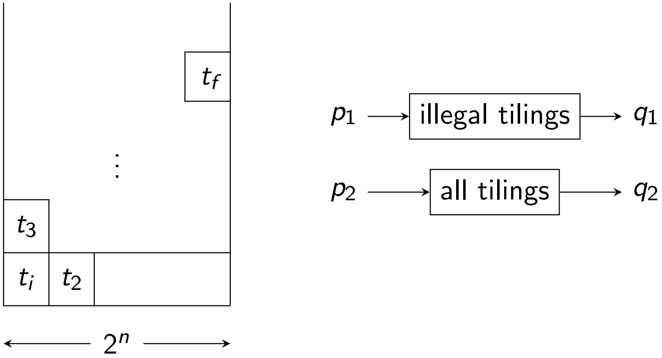
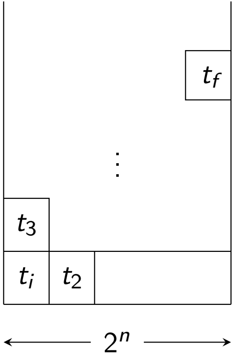


There exists a legal tiling iff $\{\langle p_2, q_2 \rangle\}$ is definable.

Expspace Lower Bound



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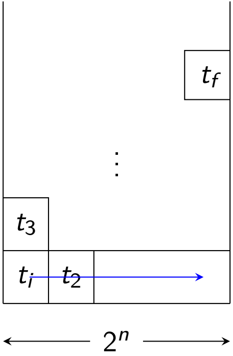
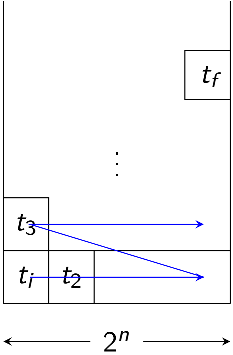


t_i

t_2

$\vdots$

EXPSPACE Lower Bound



t_i

t_2

$\vdots$

t_3

$\vdots$

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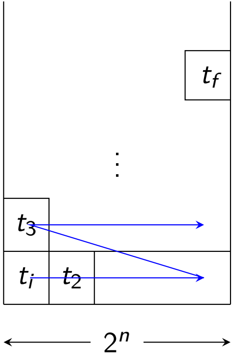


$\downarrow r_n \quad \cdots \quad \downarrow r_2 \quad \downarrow r_2 \quad t_i$

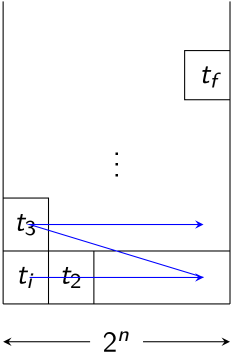
t_2

$\vdots$

t_3

$\vdots$

Expspace Lower Bound



$\downarrow r_n$	$\cdots$	$\downarrow r_2$	$\downarrow r_2$	t_i
$r_n^=$	$\cdots$	$r_2^=$	$r_1^{\neq}$	t_2
				$\vdots$
$r_n^=$	$\cdots$	$r_2^=$	$r_1^=$	t_3
				$\vdots$

Conclusion

- $\mathrm{RDPQ}_{\mathrm{mem}}$-definability is $\textsc{Expspace}$-complete.
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Thank you. Questions?