Instructions

- Enter your *Registration Number* here: **CMIPG ID** | | | | | | |

  Enter your *Examination Centre* here: | | | | | | | | | | | | |

  - The time allowed is 3 hours.
  - Total Marks: 100
  - Each question carries 20 marks.

- Answer all questions.
- *Rough Work:* The coloured blank pages are to be used for rough work only.

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1	
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Total	

CMI Ph. D. Physics Entrance Exam 2014

(1) Consider a non-relativistic particle of mass m moving along the x-axis subject to the potential $V(x) = g(x^2 - a^2)^2$ with $g, a > 0$.
(a) Find the physical dimensions of a and of the coupling constant g. [2 mks]
(b) Plot the potential and mark the value of potential energy at $x = 0$ and $x = \pm a$. [3 mks]
(c) Write the Lagrangian, obtain the equation of motion. Identify the independent and dependent variables and state whether it is a linear or non-linear equation. [5 mks]
(d) Find the time-independent trajectories (static solutions) and classify them according to their stability with respect to small perturbations. [5 mks]
(e) Find the time period T of small oscillations about any stable static solution. [5 mks]

(2) (a) Consider an infinite slab of material occupying the region $z \leq 0$ with permittivity ϵ and permeability μ. The region $z > 0$ is vacuum. If there is a magnetic field $\vec{B}(x, y, z, t)$ in the region $z > 0$ what can be said about the field just inside the slab ($z \approx -\epsilon < 0$), using appropriate boundary conditions at $z = 0$? (ϵ is a small distance compared to other physical dimensions) [5 mks]
(b) If we replace the slab by a superconductor (which has the property that the magnetic field is zero inside the superconductor, which in our case is the region $z < 0$) , what can be said about the magnetic field on the surface of the superconductor $z = 0$? [5 mks]
(c) Now consider a small magnet of magnetic moment $m\,\hat{k}$ kept at the point $(0\,,\,0,\,h > 0)$. In order to satisfy the boundary condition found in (b), assume there is another magnet kept at the mirror image $(\,0,\,0,\,-h\,)$ of magnitude m. What should be its direction for getting the appropriate boundary condition? Give reasons. Draw an appropriate diagram. [5 mks]
(d) Does this lead to a surface current on the surface of the superconductor ($z = 0$)? If so, why and what is its direction at $(x > 0, 0, 0)$. If there are no surface currents, why? [5 mks]

(3) Consider two spin-$\frac{1}{2}$ degrees of freedom interacting through the Hamiltonian

$$H = \mu(\sigma_1^z + \sigma_2^z)B - J\sigma_1^x\sigma_2^x.$$

The first term represents the interaction of the spins with an external magnetic field $B\hat{k}$. The second term is the spin-spin interaction between the two spins (assume $J \geq 0$ and $\mu \geq 0$).

(a) If the two spins are non-interacting, i.e. $J = 0$ (but B is nonzero), find the ground state of this system and its energy. [5mks]
(b) Now imagine that the magnetic field is turned off, i.e. $B = 0$, but the spins interact with nonzero J. Find the ground state of the system and its energy. [10mks]
(c) Now starting with non-interacting spins as in (a), imagine turning on a small interaction, i.e. small J. Find the correction to the ground state energy in first order perturbation theory. [5mks]

(4) N free nonrelativistic identical particles of spin 1/2 and mass m are in a two-dimensional domain of area A.
(a) Consider the system to be at temperature $T = 0\ K$. Define the Fermi energy ϵ_F. Write down N as an appropriate integral over the phase space of the system and hence find an expression for ϵ_F in terms of the number density $n = N/A$, m and $\hbar$. [7 mks]
(b) Consider the system at a finite temperature $T \neq 0$. Take the energy of the particles to be given by $p^2/2m$. Write down N as an appropriate integral over the phase space of the system using the Fermi distribution. Evaluate the resulting integral exactly. This will lead to a relation between ϵ_F, T, and the chemical potential μ. Solve this relation for μ in terms of ϵ_F and T. [10 mks]
(c) Show that in the limit $T = 0$, $\mu = \epsilon_F$. [3 mks]

(5) (a) Using Power Series method (or otherwise) solve the differential equation

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y = 0.$$

to obtain $y(x)$. (Take $y(0) = 1$ and $y'(0) = 0$.) [10 mks]
(b) The equation of a surface is

$$x^2 + 2xy + 2y^2 + 2yz + z^2 = 1.$$

Determine the type of surface this represents, orientation of its principal axes, and relevant lengths in the directions of these axes. (Hint: Write down the equation of the surface in matrix form. The eigenvalues and eigenvectors of the matrix will help in addressing the questions.) [10 mks]