Solutions:

Part A:

$$1:(AD)\quad 2:(D)\quad 3:(AC)\quad 4:(ACD)\quad 5:(BC)$$
$$6:(BCD)\quad 7:(ABD)\quad 8:(ABC)\quad 9:(BD):\quad 10(B)$$

Part B:

1. for each of the n elements of the set Γ there are m choices from the set Λ. Answer: m^n.

List the elements of Γ. For the first element you can associate any one of the m elements of Λ. Having done this we can associate with the second element of Γ any one of the remaining $m-1$ elements of Λ. etc. Answer: $m(m-1)(m-2)\cdots(m-n+1)$.

Since we associate only one element of Λ with each element of Γ and there are more elements in Λ than in Γ; there can not be any onto map.
There can not be any bijective map.

2. Let $f(x) = e^x - x - 1$ then $f(0) = 0$. Also $f'(x) = e^x - 1$. if $x \geq 0$ then $e^x \geq 1$ so that $f'(x) \geq 0$. so f is increasing. In particular, $f(x) \geq f(0) = 0$ if $x \geq 0$. In other words $f(x) \geq x + 1$.

If $x \leq 0$, then $e^x \leq 1$ so that $f'(x) \leq 0$ or f is decreasing. in particular, $f(x) \geq f(0) = 0$ if $x \leq 0$. In other words $f(x) \geq 1 + x$.

3. We know that the sequence $1/n$ converges to zero. Since $a < b$, we can get N so that $1/N < b - a$. Consider the numbers $\{k/N\}$ where k runs over all integers, positive as well as negative. k/N decreases to $-\infty$ as k decreases to $-\infty$ and k/N increases to $+\infty$ as k increases to ∞. Let K be the largest integer (positive or negative) such that $K/N < a$. Thus $(K+1)/N > a$. But then

$$\frac{K+1}{N} = \frac{K}{N} + \frac{1}{N} < a + (b-a) = b$$

This shows $a < (K+1)/N < b$ and $(K+1)/N$ is rational.

We know $\sqrt{2}$ is an irrational number. Using the above result, get a rational number between $a/\sqrt{2} < r < b/\sqrt{2}$. If $r \neq 0$, then $r\sqrt{2}$ is irrational and is between a and b. However, if $r = 0$, then we take a rational number s such that $0 < s < b/\sqrt{2}$. then $\sqrt{2}s$ will complete the proof.

4. Fix any integer $k > 1$. we prove the result by induction on r. For $r = 1$ we need to show

$$\binom{k-1}{k-1} + \binom{k}{k-1} = \binom{1+k}{k}.$$

Do it. If it is true for r.

$$\sum_{m=0}^{r+1} \binom{m+k-1}{k-1} = \sum_{m=0}^{r} \binom{m+k-1}{k-1} + \binom{r+1+k-1}{k-1}$$

$$= \binom{r+k}{k} + \binom{r+k}{k-1} = \binom{r+1+k}{k}$$

5. We show $f'(0) = 0$. let $\epsilon > 0$ be given. Take any $\delta < \epsilon \wedge 1$. Let $h \in R$ with $|h| < \delta$. If h is rational then

$$|\frac{f(h) - f(0)}{h}| = |\frac{h^2}{h}| = |h| < \epsilon.$$

If h is irrational then using $\delta < 1$ we see

$$|\frac{f(h) - f(0)}{h}| = |h^3| < |h| < \epsilon.$$

Thus for an h with $|h| < \delta$ we have $|\frac{f(h)}{h}| < \epsilon$ and hence derivative of f at zero exists and equals zero.

6. Fix $a \in [0,1]$. Let $\epsilon > 0$ be given. Since $f_n \to f$ uniformly, choose N such that $n \geq N$ implies $|f_n(y) - f(y)| < \epsilon/3$ for all $y \in [0,1]$. For this fixed integer N using continuity of f_N get $\delta > 0$ such that $|x - a| < \delta$ implies $|f_N(x) - f_N(a)| < \epsilon/3$. If now $|x - a| < \delta$ then

$$|f(x) - f(a)| \leq |f(x) - f_N(x)| + |f_N(x) - f_N(a)| + |f_N(a) - f(a)|$$

$$< \epsilon/3 + \epsilon/3 + \epsilon/3 = \epsilon$$

7. Rank of a matrix M is the dimension of range of M; that is, dimension of the vector space $\{Mx : x \in R^n\}$. Thus if $V = \{Ay : y \in R^n\}$ and $W = \{Bx : x \in R^n\}$ then rank$(A) = \dim(V)$ and rank$(B) = \dim(W)$. Similarly if $U = \{ABx : x \in R^n\}$ then rank$(AB) = \dim(U)$.

Since $W \subset R^n$ we conclude $U = \{Ay : y \in W\} \subset \{Ay : y \in R^n\} = V$ and so rank$(AB) \leq$ rank(A).
Also, since $\dim\{Ay : y \in W\} \leq \dim(W)$ we conclude that rank$(AB) \leq$ rank(B).
Thus rank(AB) is smaller than both.

8. Take any nonsingular 2×2 matrix A and $B = -A$. Let $c = d$ be the column vector $(1, 0)$. Then A and B being non-singular we can solve for $Ax = c$ and $Bx = d$. However $c + d$ is the column vector $(2, 0)$ but AB is the zero matrix.