PART A:

The correct options are:

$Q1 < A, B, C >$
$Q2 < B >$
$Q3 < B >$
$Q4 < A >$
$Q5 < B >$
$Q6 < C >$
$Q7 < A >$
$Q8 < A >$
$Q9 < A >$
$Q10 < D >$

PART B:

Q1: FALSE.
Example: The function $+1$ at irrationals and -1 at rational numbers.

Q2: TRUE.
Since A is of rank one, let range A be spanned by vector v. Let $Av = av$ where the number a could be zero. Let now x be any vector and say $Ax = cv$.

$$A^2x = A(Ax) = A(cv) = cAv = cav = acv = aAx.$$

Q3: TRUE.
If derivative of f is bounded by M in modulus, then mean value theorem says that

$$|f(x) - f(y)| = |f'(\theta)(x - y)| \leq M|x - y|.$$

So given $\epsilon > 0$ choose $\delta = \epsilon/(1 + M)$ to verify uniform continuity.

Q4: FALSE.
The determinant is a quadratic in λ and hence zero for at most (in this case, exactly) two values of λ. For all other values determinant is non-zero and hence invertible.

PART C:

Q1: For any polynomial P, $\int P(x)f(x)dx = 0$. Given $\epsilon > 0$, there is a polynomial P such that $|P(x) - f(x)| < \epsilon$ for all $x \in [0, 1]$ (by Weierstras). so

$$\int f^2(x)dx = \int f(x)P(x)dx + \int f(x)[f(x) - P(x)dx$$

The first integral on right is zero and the second is smaller than bound of f times ϵ. This being true for every $\epsilon > 0$, we conclude $\int f^2(x)dx = 0$. If $f^2(x_0) = a > 0$ for some x_0, then then in some interval around zero, say, $(x_0 - \delta, x_0 + \delta)$ we have $f^2(x_0).a/2$. For any partition of norm less than $\delta/2$, at least one partition interval is fully contained in the interval $(x_0 - \delta, x_0 + \delta)$ and thus Riemann sum is at least $a\delta/4$. so $\int f^2(x)dx \neq 0$.

Q2: If $h_n \neq 0$, then $[f(1 + h_n) - f(1)]/h_n$ equals $f'(\theta_n)$ for some point θ_n between $1 + h_n$ and 1. Thus if $h_n \to 0$ then, $\theta_n \to 1$ and hence by hypothesis $f'(\theta_n) \to 1$. This being true for any sequence h_n converging to zero, we conclude that the derivative exists and in fact, equals 5.

Q3: Recall that a positive definite matrix is symmetric by definition. Thus $x_0^tAy = y^tAx_0 = y^tb$ and also $y^tAx_0 = y^tb$. Using these and the hypothesis $Ax_0 = b$,

$$P(y) - P(x_0) = \frac{1}{2}y^tAy - y^tb - \frac{1}{2}x_0^tAx_0 + x_0^tb = \frac{1}{2}y^tAy - y^tb + \frac{1}{2}x_0^tAx_0.$$

$$= \frac{1}{2}(y - x_0)^tA(y - x_0) \geq 0.$$

Q4: It can be seen that in the usual method of solving linear equations (Gauss-Jordan elimination), at each step the augmented matrix has rational entries and it follows that the solution it produces has rational