

# NCM IST, Mathematics for Computer Science

## Systems of linear equations

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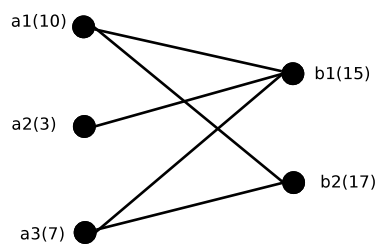
1. Let  $x_1, x_2, \dots, x_n$  be  $n$  distinct variables. Consider the  $n \times n$  matrix whose  $(i, j)$  term is  $x_i^{j-1}$ . Compute the determinant of this matrix.
2. We are given two  $n \times n$  matrices  $A$  and  $B$  with rational entries. We wish to determine if there is a matrix  $X$  such that  $AX=XB$ . Give an algorithm for this problem. Show that the set of all such  $X$  forms a vector space. How will you determine the dimension of the space of solutions?
3. Let  $A$  be a matrix on which we perform Gaussain elimination. Let  $a_{ij}^k$  denote the entries of the resulting matrix at the  $k$ -th iteration. Give a recurrence equation for  $a_{ij}^k$  in terms of determinants and sub-determinants of the original matrix.
4. Formulate the max flow problem as a Linear program.
5. Consider the following LP. Construct a dual for this:

$$\begin{array}{rcll}
 \min & 10y_1 & + & 6y_2 \\
 \text{Subject to:} & y_1 & + & 5y_2 \leq 7 \\
 & -y_1 & + & 2y_2 = 1 \\
 & 3y_1 & - & y_2 \leq 5 \\
 & & & y_1, y_2 \geq 0
 \end{array}$$

What can you say about the dual variable corresponding to the equality constraint?

6. **Weighted vertex cover problem on a bipartite graph:** Given a graph  $G(V = A \cup B, E)$ , and a weight function  $W : V \rightarrow \mathbb{R}$ . Find a vertex cover  $X \subseteq V$ , such that  $\sum_{v \in X} W(v)$  is minimum.

Consider this example: The vertices are  $a_1, a_2, a_3, b_1$  and  $b_2$ . The weights are given in the bracket.



Construct a LP for the above mentioned problem. Construct the dual. How can you interpret the dual?

Write a LP for a general bipartite graph.