NCM IST, Mathematics for Computer Science
Problems on bipartite graphs, flows, NP-completeness

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1. Prove that a graph is bipartite if and only if it contains no cycles of odd length.

2. We proved the max-flow min-cut theorem in the lecture. Recall the way we constructed the set (S, T) for exhibiting a cut such that the value of the flow f (obtained at the termination of Ford Fulkerson method) was equal to the capacity of the (S, T) cut. Consider the following alternate method of constructing a cut (S', T'). Let f be the flow obtained when the Ford Fulkerson method terminates. Let G_f be the corresponding residual network. Let T be the set of vertices such that $u \in T$ iff there exists a path from u to t in G_f. The set $S' = V \setminus T'$. Prove that (S', T') is a min-cut in G. That is, show that the capacity of (S', T') is equal to the value of the flow f.

3. The edge connectivity of an undirected graph $G = (V, E)$ (here $|V| = n$ and $|E| = m$) is the minimum number k of edges that must be removed to disconnect the graph. For instance the edge connectivity of a tree is 1; the edge connectivity of a cycle is 2. Show how to compute the edge connectivity of an undirected graph by at most n flow computations each having $O(n)$ vertices and $O(m)$ edges.

4. We are given a flow network G with integer capacities. We wish to find amongst all minimum cuts in G, the one that contains the smallest number of edges. Show how to modify the capacities of G to create a new flow network G' in which any minimum cut in G' is a minimum cut in G with the smallest number of edges.

5. The input to the **subgraph-isomorphism** problem consists of two graphs G_1 and G_2, and the goal is to determine whether G_1 has a subgraph isomorphic to G_2. Show that this problem is NP-complete.

6. Show that a polynomial-time algorithm for the longest path problem also implies a polynomial-time algorithm for the Hamiltonian cycle problem. Hence conclude that the longest path problem is NP-complete.