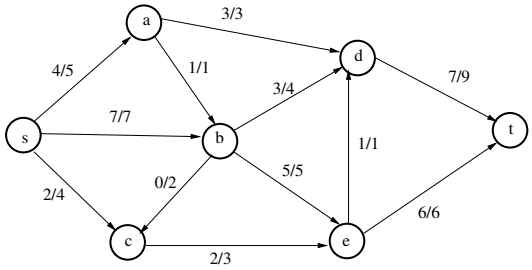
NCM IST, Mathematics for Computer Science
Problems on network flows

20 June, 2018

1. Figure below shows a flow network, the flow values, and capacities specified on each edge. Construct the residual network w.r.t. the given flow. Using the ford fulkerson method, find the max-flow from s to t in the network. Show a cut C which gives the certificate of optimality of your max-flow.



2. Consider the statement : If G is a flow network on which (A, B) is a minimum s-t cut. If we add a capacity of 1 to each edge, (A, B) must necessarily remain a minimum cut. Is it true? If yes, prove it, if not give a counter example.

3. Suppose we have a flow network on which we want to compute the maximum flow. The node s has an edge (u, s) going into it from a node u. Argue that there is a max flow which does not use this edge. Is it necessary that all max flows should not be using this edge?

4. There are k non-moving cellphones in an area, with some coordinates. There are b base stations, also with some coordinates given to you. Each phone must connect directly (without going through other phones or base stations) to a base station which is no more than d distance away. Each station can handle at most p phones. Give and analyze an efficient algorithm to decide whether it is possible to assign cell phones to base stations. You can assume that your algorithm is given all cell phones and base stations as well as their coordinates.

5. We are given a bipartite graph $G = (A \cup B, E)$. Associated with a vertex v is a non-negative value $b(v)$. The goal is to compute a maximum cardinality subset of edges $M \subseteq E$ such that a vertex v has at most $b(v)$ edges incident on it in M. Design an efficient algorithm for the problem. What is the running time of your algorithm?

6. Given positive integers $r_1, \ldots, r_n$ and $c_1, \ldots, c_n$, design an algorithm to construct a graph that has the following property:

   For all i, the in-degree of ith vertex is r_i, and out-degree of ith vertex is c_i. If such a graph does not exist, your algorithm must recognize it.