1. Show that r.e. languages are closed under union and intersection. Are they closed under complementation?

2. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over alphabet Σ such that:

   1. For all $i \neq j$, $L_i \cap L_j = \emptyset$, no string is in two of the languages.
   2. $L_1 \cup L_2 \cup \cdots \cup L_k = \Sigma^*$, i.e., every string is in one of the languages.
   3. Each of the languages L_i, for $i = 1, 2, \ldots, k$ is recursively enumerable.

   Prove that each of languages is therefore recursive.

3. Classify the following languages to be either recursive, r.e. but not recursive, or non r.e.

   (a) $\{ M \mid L(M)$ contains at least two strings $\}$

   (b) $\{ M \mid$ M halts on all inputs$\}$

   (c) $\{ M \mid$ M fails to halt on at least one input$\}$

   (d) The set of Turing Machine codes that accept all inputs that are palindromes (possibly along with some other inputs)

   (e) The set of TM codes that when started with a blank tape would eventually write some nonblank symbol on its tape

   (f) The set of TM codes that when started with a blank tape eventually write a 1 somewhere on the tape

   (g) The set of TM codes that never make a move left

   (h) $\{ M\#w \mid M,$ started with input w never scans any tape cell more than once $\}$

   (i) $\{ M \mid L(M) = \textbf{rev}\ L(M)\}$

4. Let C be a language. Prove that C is r.e. iff there exists a decidable language D such that

$$C = \{ x \mid \exists y \text{ s. t. } \langle x, y\rangle \in D\}$$

5. For $A, B \subseteq \Sigma^*$, define

$$A/B := \{ x \mid \exists y \in B\ xy \in A \}$$
$$A \leftarrow B := \{ x \mid \forall y \in B\ xy \in A \}$$

   (a) Show that if A is regular and B is any set whatsoever, then A/B and $A \leftarrow B$ are regular.

   (b) Show that even if we are given a finite automaton for A and a Turing machine for B, we cannot necessarily construct an automaton for A/B or $A \leftarrow B$ effectively (in other words, there is no algorithm that can construct these automata).