1. Give a grammar with no ϵ or unit productions generating the set $L(G) - \{\epsilon\}$ where G is the grammar:

$$S \to aSbb \mid T$$
$$T \to bTaa \mid S \mid \epsilon$$

2. Give grammars in Chomsky Normal form for the following context-free languages:
   (a) $\{a^n b^{2n} c^k \mid k, n \geq 1\}$
   (b) $\{a, b\}^*-$ palindromes

3. Use the pumping lemma to show that the following languages are not context-free:
   (a) $\{w\#x \mid w$ is a substring of x, where $w, x \in \{a, b\}^* \}$
   (b) $\{a^p \mid p$ is prime$\}$

4. Construct NPDA for the set $\{a, b\}^*-$ palindromes. Specify clearly if your NPDA accepts by empty stack or by final state.

5. Let $b(n)$ denote the binary representation of $n \geq 1$, leading zeros omitted. For example, $b(5) = 101$ and $b(12) = 1100$. Let \$ be another symbol not in $\{0, 1\}$.
   (a) Show that the set
   $$\{b(n)\$b(n+1) \mid n \geq 1\}$$
   is not a CFL.
   (b) Suppose we reverse the first numeral; that is, consider the set
   $$\{\textbf{rev } b(n)\$b(n+1) \mid n \geq 1\}.$$
   Show that this set is a CFL.

6. **Closure properties of Context-Free Languages:**

   A CFL is a language that can be generated by a context-free grammar. Equivalently, a CFL is a language that can be recognized by an NPDA.
   (a) Show that CFLs are closed under union, concatenation and Kleene star.
   (b) Show that CFLs are not closed under intersection and complementation.
   (c) If L is a CFL and R is a regular language, show that $L \cap R$ is a CFL.
   (d) Show that CFLs are closed under homomorphisms and inverse homomorphisms.

7. Recall the *shuffle* operator from Tutorial 2. Show that CFLs are not closed under shuffle.

8. Given a CFG G, provide an algorithm to check if $L(G)$ is empty.