1. Find grammars for $\Sigma = \{a, b\}$ that generate the sets of

   (a) all strings with exactly one a,
   (b) all strings with at least one a,
   (c) all strings with no more than 2 a's,
   (d) all strings with at least 2 a's,
   (e) the empty set.

   Are your grammars right-linear? If not, give right-linear grammars.

2. Find grammars for the following languages on $\Sigma = \{a\}$.

   (a) $\{w \mid |w| \mod 3 = 0\}$
   (b) $\{w \mid |w| \mod 3 > 0\}$

3. Find grammars for languages below, with $\Sigma = \{a, b\}$ (we write $n_a(w)$ to denote the number of a's in w).

   (a) $\{w \mid n_a(w) = n_b(w) + 1\}$
   (b) $\{w \mid n_a(w) = 2n_b(w)\}$
   (c) $\Sigma^* - \{\text{palindromes}\}$

4. Construct a dfa that accepts the language generated by the grammar

$$
\begin{aligned}
S &\rightarrow abA \\
A &\rightarrow baB \\
B &\rightarrow aA \mid bb
\end{aligned}
$$

5. What is the language generated by the following grammar?

$$
\begin{aligned}
S &\rightarrow aSa \mid aBa \\
B &\rightarrow bB \mid b
\end{aligned}
$$

6. Explain why the grammar below is ambiguous.

$$
\begin{aligned}
S &\rightarrow 0A \mid 1B \\
A &\rightarrow 0AA \mid 1S \mid 1 \\
B &\rightarrow 1BB \mid 0S \mid 0
\end{aligned}
$$

7. Given the following ambiguous CFG

$$
\begin{aligned}
S &\rightarrow Ab \mid aaB \\
A &\rightarrow a \mid Aa \\
B &\rightarrow b
\end{aligned}
$$

   (a) Find a string s generated by the grammar that has two leftmost derivations. Show the derivations.
   (b) Show the two derivation trees for the string s.
   (c) Find an equivalent unambiguous CFG.
   (d) Give the unique leftmost derivation and derivation tree for the string s above using this unambiguous CFG.

8. Let $L = \{a^n b^n \mid n \geq 0\}$.

(a) Show that L^2 is context-free.

(b) Show that L^k is context-free for any $k \geq 1$.

(c) Show that $\overline{L}$ (the complement of L) and L^* are context-free.

9. Show that a regular language cannot be inherently ambiguous.

10. Give a context-free grammar that generates the language

$$A \;=\; \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$$

Is your grammar ambiguous? Why or why not?