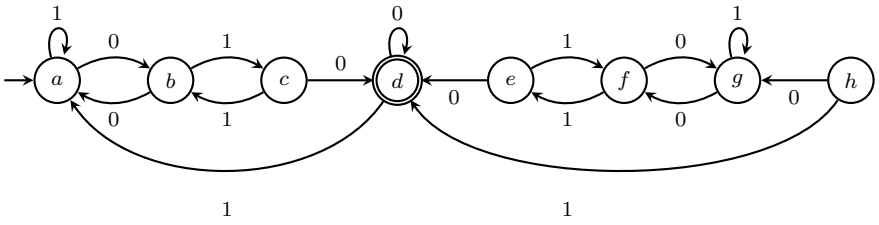
1. Find the minimum state DFA equivalent to the following DFA:



2. Use Myhill Nerode theorem to prove non-regularity of the following languages
   (a) $\{\ ww \mid w \in (0+1)^*\}$
   (b) $\{\ a^i b^j c^k \mid i,j,k \geq 0$ and if $i = 1$ then $j = k\}$

3. One of the following subsets of $\{a, b, \$\}$ is regular and the other is not. Which is which? Give proofs.

$$\{xy \mid x, y \in \{a,b\}^*, \#a(x) = \#b(y)\}$$
$$\{x\$y \mid x, y \in \{a,b\}^*, \#a(x) = \#b(y)\}$$

4. Prove that NFAs are exponentially more succinct than DFAs: for any m, there exists an NFA with m states such that any equivalent DFA has at least 2^{m-1} states.

5. For a language $L \subseteq \Sigma^*$ define $\text{LOG}(L) = \{x \in \Sigma^* \mid \exists y \text{ s.t. } |y| = 2^{|x|} \text{ and } xy \in L\}$. Show that if L is regular, so is $\text{LOG}(L)$.