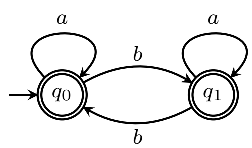
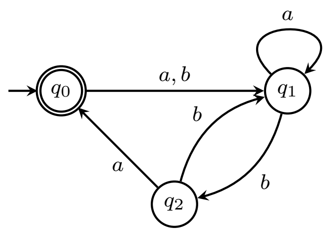
1. Give regular expressions for the following languages over $\Sigma = \{0, 1\}$:
   (a) $\{\ w \mid w$ begins with a 0 and ends with a $1\}$
   (b) $\{\ w \mid w$ contains at least three 1s$\}$
   (c) $\{\ w \mid w$ has length at most $5\}$

2. Convert the following automata to regular expressions:





3. Using pumping lemma, show that the following languages are not regular:
   (a) $\{\ 0^{2^n} \mid n \geq 1\}$
   (b) $\{\ ww \mid w \in \{0, 1\}^*\}$
   (c) $\{\ 0^{n!} \mid n \geq 1\}$

4. Let $h : \Sigma^* \mapsto \Gamma^*$ be a homomorphism. Let $A \subseteq \Sigma^*$ be a language. We know that if A is regular, then $h(A)$ is regular. Construct an example of a language $A' \subseteq \Sigma^*$ such that $h(A')$ is regular, but A' is not.

5. Using closure properties of regular languages, show that the following languages are non-regular.
   (a) $\{0^n 1^m \mid m \geq n \geq 0\}$
   (b) $\{0^i 1^j \mid i \neq j\}$
   (c) $\{0^n 1^n 2^n \mid n \geq 1\}$

6. Consider the language $\{\ a^i b^j c^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k\}$. Does it satisfy the three conditions of the pumping lemma? Is this language regular?

7. The *Hamming distance* $H(x, y)$ between two bit strings x and y is the number of places at which they differ, for example $H(110, 011) = 2$. If $|x| \neq |y|$, then their Hamming distance is infinite. If x is a string and A is a language, the Hamming distance between x and A is the distance from x to the closest string in A:

   $$H(x, A) := \min_{y \in A} H(x, y)$$

   For a language $A \subseteq \{0, 1\}^*$ and a $k \geq 0$, define the set of strings of Hamming distance at most k from A:

   $$N_k(A) := \{x \mid H(x, A) \leq A\}$$

   Prove that if A is regular, so is $N_k(A)$.