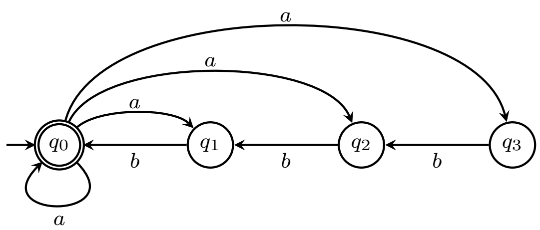
1. Consider the following NFA.



   (a) Give a string beginning with a that is not accepted.

   (b) Contruct an equivalent DFA. Omit inaccessible states.

2. Give a k-state NFA or ϵ-NFA for the set of strings w over $\Sigma = \{a_1, a_2, \ldots, a_k\}$ such that at least one letter of Σ does not occur in w.

3. Give an NFA or an ϵ-NFA for the set of strings over $\{0, 1\}$ such that at least one of the last seven positions is a 1.

4. Show that every ϵ-NFA can be converted to an ϵ-NFA with a single accepting state that accepts the same language.

5. Show that if an NFA with k states accepts any string at all, then it accepts a string of length $k - 1$ or less.

6. For a string $w = w_1 w_2 \ldots w_k$, the *reverse* of w denoted as $w^{\mathcal{R}}$ is the string $w_k w_{k-1} \ldots w_2 w_1$ obtained by writing w from the back. For a language A let $\text{rev}(A)$ be $\{w^{\mathcal{R}} \mid w \in A\}$. Show that if A is regular, then $\text{rev}(A)$ is regular.

7. For any set of strings A, define the set:

$$\text{FirstHalves}(A) = \{x \mid \exists y \ |x| = |y| \text{ and } xy \in A\}$$

   Prove that if A is regular, so is $\text{FirstHalves}(A)$.

8. For languages A and B, let the *perfect shuffle* of A and B be the language:

$$\{w \mid w = a_1 b_1 \ldots a_k b_k, \text{ where the string } a_1 \ldots a_k \in A \text{ and the string } b_1 \ldots b_k \in B \text{ and each } a_i, b_i \in \Sigma\}$$

   Show that the class of regular languages is closed under perfect shuffle.

9. For languages A and B, let the *shuffle* of A and B be the language:

$$\{w \mid w = a_1 b_1 \ldots a_k b_k, \text{ where the string } a_1 \ldots a_k \in A \text{ and the string } b_1 \ldots b_k \in B \text{ and each } a_i, b_i \in \Sigma^*\}$$

   Show that the class of regular languages is closed under shuffle.