Remarks:

- In the distance graphs illustrated below, if there is no edge $x \to y$, then it denotes $y - x < \infty$.
- Normally edges in a distance graph are of the form $(\leq, c)$ or $(<, c)$. For convenience, in this tutorial we only write constants c on edges and assume that it denotes $(\leq, c)$.

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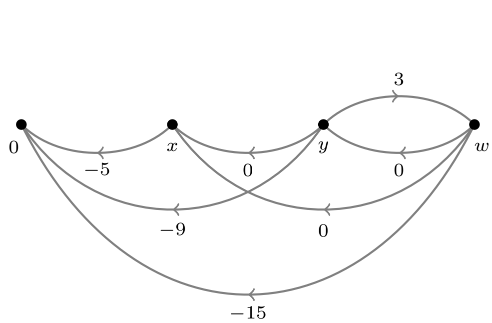
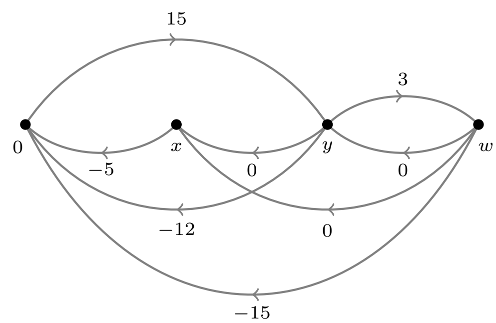
1. Which of the following distance graphs are in canonical form? If not, canonicalize them.

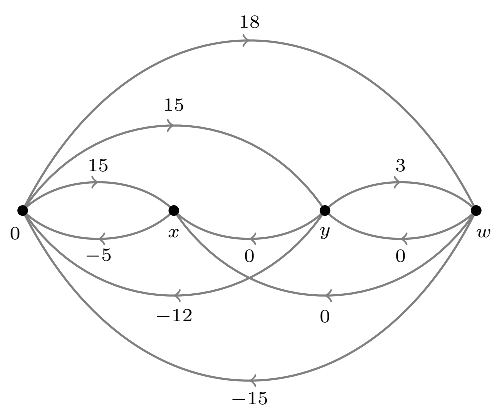




2. Is the set of solutions represented by the above graphs non-empty?

3. Construct a distance graph in canonical form with 3 clocks, in which at most 2 edges have weight 0.

4. Consider the distance graph G_Z below that represents some zone Z. Find the distance graph of $\overrightarrow{Z}$. Recall that $\overrightarrow{Z}$ denotes the zone obtained by elapsing time from Z, i.e., $\overrightarrow{Z} = \{v + \delta \mid v \in Z \text{ and } \delta \geq 0\}$



G_Z

5. Let G be a distance graph in canonical form that has no negative cycles. Suppose the edge $0 \to x$ in G is reduced to a new value so that adding this new value does not create negative cycles. Let G' be this new graph. Is G' necessarily canonical? If not, characterize the set of edges that need to be changed (reduced) in G'.

6. Same question as above, but now instead of $0 \to x$, the edge $x \to 0$ is reduced.

7. Consider a distance graph. Suppose some edges of the form $0 \to x$ and some of the form $y \to 0$ are reduced. Provide a quadratic algorithm to canonicalize this graph.

8. Consider the zone Z represented by the distance graph G_Z in the Question 4. Let Z' be the zone obtained by resetting x in all valuations of Z. Give the distance graph of Z'.

9. Extend the previous question to give an algorithm that takes Z and a set of clocks R as input, and outputs the zone $[R]Z$ which is obtained by resetting clocks in R from each valuation of Z.

10. Suppose $x \leq y \leq w$ in a zone Z. What can you say about the weight in the diagonal edges in the canonical distance graph representing Z?