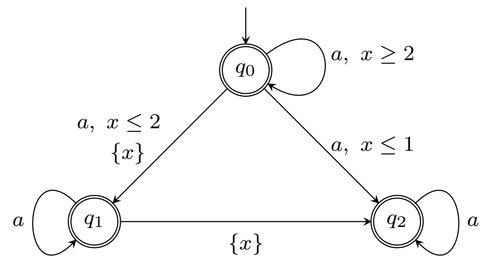
1. The purpose of this question is to understand the algorithm by Ouaknine and Worrell for checking the universality of one-clock timed automata. Let us call it the OW-algorithm. The definitions and notations used in this question are identical to those in Lecture 5.

   Consider the following automaton over the singleton alphabet $\{a\}$:



   The set of regions $REG = \{r_0, r_{01}, r_1, r_{12}, r_2, r_{2\infty}\}$

   a) What is the set Λ? Recall that nodes in the graph built by the OW-algorithm would be labeled by words in Λ^*.

   b) Consider the word $W = \{(q_1, r_0)\}\{(q_2, r_{01})\}$. List two different configurations C_1 and C_2 of the automaton that map to this word by the encoding H defined in Lecture 5, i.e., $H(C_1) = H(C_2) = W$.

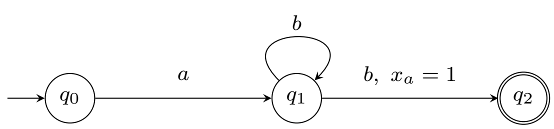
   c) For the C_1 and C_2 that you have chosen, prove that for $\delta_1 = 1.3$, there exists a $\delta_2 \in \mathbb{R}_{\geq 0}$ such that

$$\text{if } C_1 \xrightarrow{\delta_1, a} C_1' \qquad \text{then } C_2 \xrightarrow{\delta_2, a} C_2'$$

   such that $H(C_1') = H(C_2')$. In the above, the value 1.3 did not have a special role. Prove that for every value of δ_1 there exists a δ_2 such that the above is true.

   d) Run the OW-algorithm on the above automaton and check if it accepts all timed words.

2. Determinize the following event recording automaton over the alphabet $\{a, b\}$:



3. Determinize the following Integer Reset timed automaton: