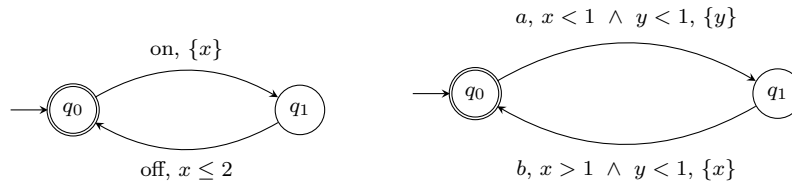


1. Consider an automaton with 2 clocks $\{x, y\}$. Let the maximum bounds function M for the automaton be given by: $M(x) = 3, M(y) = 4$. Draw the division of the xy -plane into regions.
2. Given 3 clocks $\{x, y, z\}$ and $M(x) = 3, M(y) = 4, M(z) = 2$, enumerate the set of regions.
3. Let R be a region over clock set X and bound function M . Give an algorithm to compute the time-successors of a region R .
4. Draw the region graph for the following automata:



5. Suppose R is a region over clock set X and bound function M . Let x, y be two arbitrary clocks in X . Is the projection of R on to the xy -plane a region over $\{x, y\}$ with the bounds function M restricted to x and y ?
6. A *deterministic 2-stack machine* has a finite control, 2 stacks and a read-only two way bounded tape. On a transition, the tape head reads a letter on the input tape and the topmost letters on the 2 stacks, pops both the topmost letters and pushes a chosen string on each of them. It can then either move left, right or *stay put* in the same cell. Show that a deterministic Turing machine can be simulated by a deterministic 2-stack machine.
7. Show that a deterministic 2-stack machine can be simulated by a deterministic 2-stack machine whose stack alphabet has only 2 symbols.
8. A *deterministic counter machine* has a finite control, a read-only two way bounded tape and a finite set of counters. On a transition, the machine can check if one or more of the counters are zero, and can increment or decrement some of them. After the transition, the tape head can move left, right or *stay-put*. Show that deterministic 2-stack machines can be simulated by deterministic 4 counter machines.
9. Show that deterministic 4 counter machines can be simulated by deterministic 2 counter machines. Deduce that the membership problem for deterministic 2 counter machines is undecidable.
10. We discussed in Lecture 4 that the undecidability for language inclusion in timed automata proceeds by reducing the membership problem for deterministic 2-counter machines to language inclusion. Complete the proof and note where we actually use the fact that the 2-counter machine is *deterministic*.