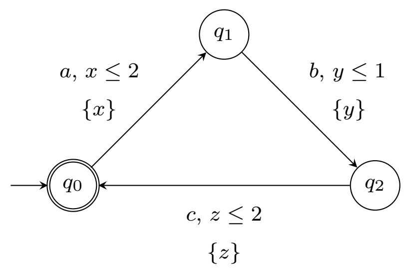
1. Give a timed automaton over $\Sigma = \{a, b\}$ that accepts all timed words.

2. Let $\mathcal{A} = (\{q\}, \{a, b\}, \{x\}, T, \{q\}, \{q\})$ be a timed automaton with a single state q and a single clock x. Note that q is also an accepting state. Let T be the set of transitions. Give an instance of T that makes $\mathcal{A}$ reject at least one timed word.

3. What is the timed word accepted by the following accepting run of some timed automaton with two clocks x and y?

$$\begin{array}{c} q_0 \\ x:0 \\ y:0 \end{array} \xrightarrow{\delta_0} \begin{array}{c} q_0 \\ x:0.6 \\ y:0.6 \end{array} \xrightarrow{a} \begin{array}{c} q_1 \\ x:0.6 \\ y:0 \end{array} \xrightarrow{\delta_1} \begin{array}{c} q_1 \\ x:1.9 \\ y:1.3 \end{array} \xrightarrow{a} \begin{array}{c} q_F \\ x:0 \\ y:1.3 \end{array}$$

4. Let $\mathcal{B}$ be the following timed automaton:



Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

a) Does $\mathcal{B}$ accept s? If so, write down the accepting run of $\mathcal{B}$ on s.

b) For a timed word (w, τ) we define the *time span* of (w, τ) to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of (w, τ) is τ_n.
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than k and whose time span is lesser than 1.

5. Consider the following algorithm that checks for the emptiness of a timed automaton:

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Input: A timed automaton A = (Q, Σ, X, T, Q_0, F)
Output: Is L(A) empty?

Visited = {}
Waiting = Q_0

while Waiting ≠ ∅
      pick q ∈ Waiting
      if q ∈ F
           print L(A) is not empty
           exit
      else
           for each (q, a, g, R, q') ∈ T
                if q' ∉ Visited
                     add q' to Waiting
           end for
           add q to Visited
end while

print L(A) is empty
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a) Give an example of a timed automaton for which the above algorithm works correctly.

b) Provide an example for which the above algorithm is wrong.

6. For the purpose of this question, we need a few definitions.

Let X be a set of clocks. A *clock valuation* is a function $v : X \mapsto \mathbb{R}_{\geq 0}$ that associates a non-negative real value to each clock.

For $\delta \in \mathbb{R}_{\geq 0}$, we write $v + \delta$ for the clock valuation that associates $v(x) + \delta$ to each clock $x \in X$. Essentially, $v + \delta$ denotes the valuation that is reached if δ time elapses from v.

For a set of clocks $S \subseteq X$, we define $[S := 0]v$:

$$[S := 0]v = \begin{cases} 0 & \text{if } x \in S \\ v(x) & \text{if } x \notin S \end{cases}$$

Here, $[S := 0]v$ denotes the valuation that is reached from v a transition is taken that resets clocks in S.

A *configuration* of a timed automaton is given by a pair (q, v) where q is a state of $\mathcal{A}$ and v is a clock valuation that gives the value of each clock. Consider the following transition θ:

$$\theta : \quad (q, v) \xrightarrow[R]{a,\ g} (q', v')$$

where g is the guard of the transition and R is the reset set.

We say that a transition θ is *enabled* from a configuration (q, v) if there exists a non-negative duration $\delta \in \mathbb{R}_{\geq 0}$ such that $v + \delta$ satisfies the guard g. The transition could then be decomposed as a time followed by an action transition:

$$(q, v) \xrightarrow{\delta} (q, v + \delta) \xrightarrow[R]{a,\ g} (q', v')$$

where $v' = [R := 0]v$.

We now come to our question. Let $X = \{x, y, z\}$. Let θ be an arbitrary transition in the above form such that the maximum constant that can be used in the guard g is 10.

a) Show that if θ is enabled from $(q, \langle 21, 15.3, 43.3 \rangle)$, then θ would be enabled from $(q, \langle 100, 40, 22.44 \rangle)$ too.

b) Given an example of δ, g and R so that θ is enabled from (q, v_1) but not from (q, v_2) when:

   i) $v_1 = \langle 5.5, 6, 7.8 \rangle$ and $v_2 = \langle 6.6, 6.7.9 \rangle$

   ii) $v_1 = \langle 5.5, 6, 7.8 \rangle$ and $v_2 = \langle 5.5, 6.2, 7.8 \rangle$

   iii) $v_1 = \langle 5.5, 6, 7.8 \rangle$ and $v_2 = \langle 5.8, 6.7.5 \rangle$