Automata for Real-time Systems

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Theorem (Lecture 7)

Deterministic timed automata are **closed under complement**

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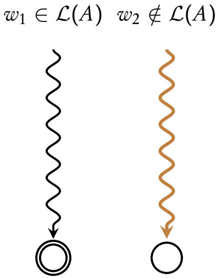
1. **Unique** run for every timed word

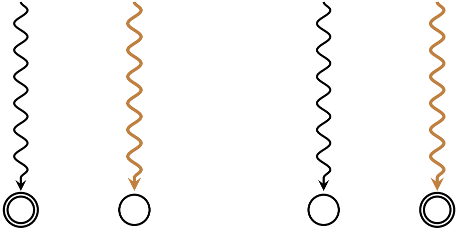


Theorem (Lecture 7)

Deterministic timed automata are **closed under complement**

1. **Unique** run for every timed word
2. **Complementation: Interchange** acc. and non-acc. states

$w_1 \in \mathcal{L}(A)$ $w_2 \notin \mathcal{L}(A)$ $w_1 \notin \overline{\mathcal{L}(A)}$ $w_2 \in \overline{\mathcal{L}(A)}$

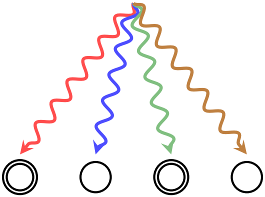


Theorem (Lecture 1)

Non-deterministic timed automata are **not closed under complement**

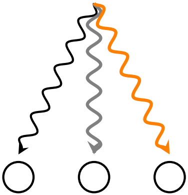
Many runs for a timed word

$w_1 \in \mathcal{L}(A)$



Exists an acc. run

$w_2 \notin \mathcal{L}(A)$



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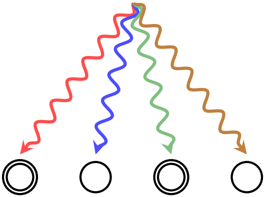

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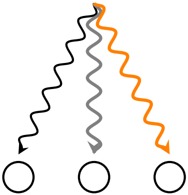

All runs non-acc.

Complementation: **interchange** acc/non-acc + ask are **all runs acc.** ?

A timed automaton model with **existential** and **universal** semantics for acceptance

Lecture 9:

Alternating timed automata

Lasota and Walukiewicz. *FoSSaCS'05*, *ACM TOCL'2008*

Section 1:

Introduction to ATA

- X : set of **clocks**
- $\Phi(X)$: set of clock constraints σ (**guards**)

$$\sigma : \; x < c \mid x \leq c \mid \sigma_1 \wedge \sigma_2 \mid \neg\sigma$$

  c is a non-negative **integer**

- Timed automaton A: $(Q, Q_0, \Sigma, X, T, F)$

$$T \; \subseteq \; Q \times \Sigma \times \Phi(X) \times Q \times \mathcal{P}(X)$$

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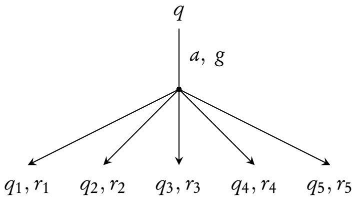
$$\downarrow$$

$$T : Q \times \Sigma \times \Phi(X) \mapsto \mathcal{P}(Q \times \mathcal{P}(X))$$



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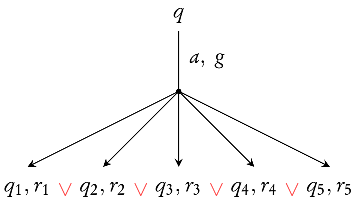

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$\downarrow$ $\mathcal{B}^+(S)$ is all $\phi ::= S \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2$

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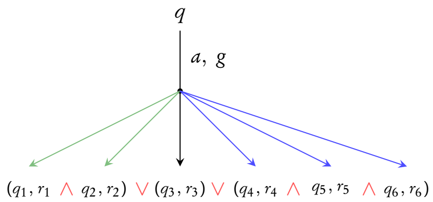

Alternating Timed Automata

An **ATA** is a tuple $A = (Q, q_0, \Sigma, X, T, F)$ where:

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is a **finite partial function.**

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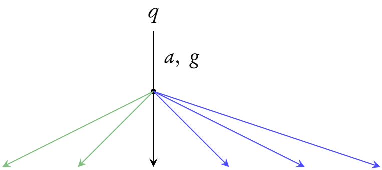
is a **finite partial function.**

Partition: For every q, a the set

$$\{\ [\sigma] \mid T(q, a, \sigma) \text{ is defined } \}$$

gives a finite partition of $\mathbb{R}^X_{\geq 0}$

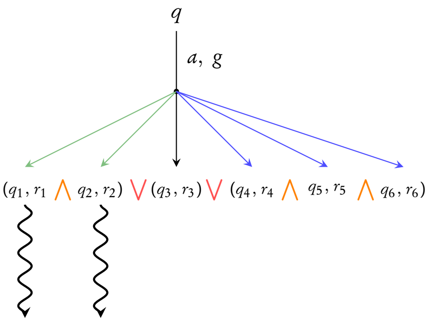
Acceptance



$(q_1, r_1 \bigwedge q_2, r_2) \bigvee (q_3, r_3) \bigvee (q_4, r_4 \bigwedge q_5, r_5 \bigwedge q_6, r_6)$

Accepting run from q iff:

Acceptance



Accepting run from q iff:

- accepting run from q_1 **and** q_2,

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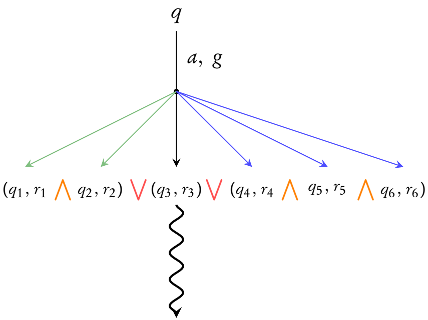

Accepting run from q iff:

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- **or** accepting run from q_3,

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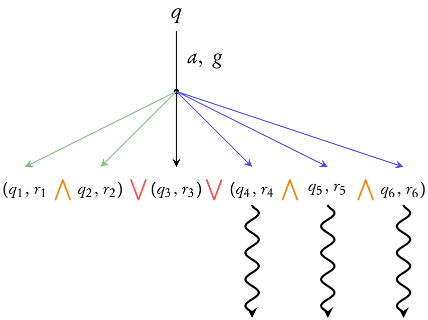

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- accepting run from q_1 **and** q_2,
- **or** accepting run from q_3,
- **or** accepting run from q_4 **and** q_5 **and** q_6

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ATA:

$$q_0, a, tt \mapsto (q_0, \emptyset) \wedge (q_1, \{x\})$$

$$q_1, a, x = 1 \mapsto (q_2, \emptyset)$$

$$q_1, a, x \neq 1 \mapsto (q_1, \emptyset)$$

$$q_2, a, tt \mapsto (q_2, \emptyset)$$

q_0, q_1 are acc., q_2 is non-acc.

Closure properties

- Union, intersection: use disjunction/conjunction
- Complementation: **interchange**
  1. acc./non-acc.
  2. conjunction/disjunction

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No change in the number of clocks!

Section 2:

The 1-clock restriction

- Emptiness: given A, is $\mathcal{L}(A)$ empty
- Universality: given A, does $\mathcal{L}(A)$ contain all timed words
- Inclusion: given A, B, is $\mathcal{L}(A) \subseteq \mathcal{L}(B)$

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Decidable for **one clock** (via Lecture 5)

Restrict to one-clock ATA

Theorem

Languages recognizable by 1-clock ATA and (many clock) TA are **incomparable**

$\rightarrow$ proof on the board

Next class

Complexity of emptiness of 1-clock ATA