A Survey of Classical, Real-Time, and Time-Bounded Verification

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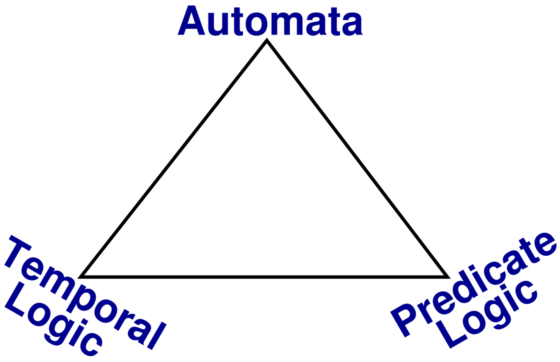
Oxford University

Quantitative Model Checking Winter School, February 2012

The Classical Theory of Verification



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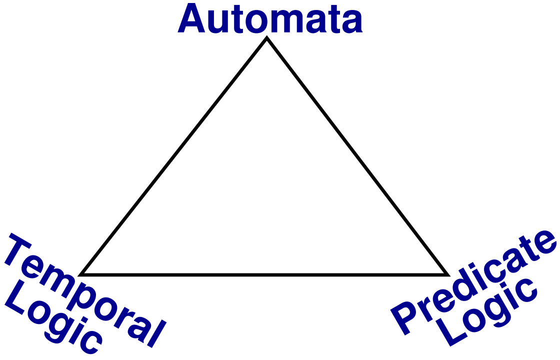

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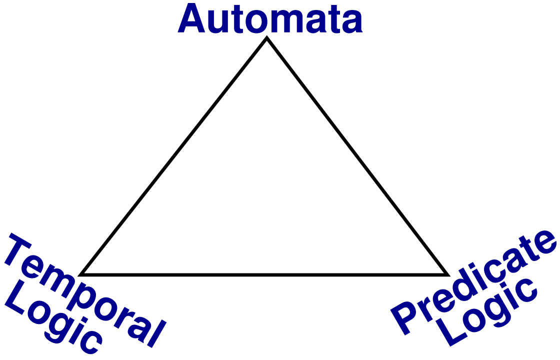

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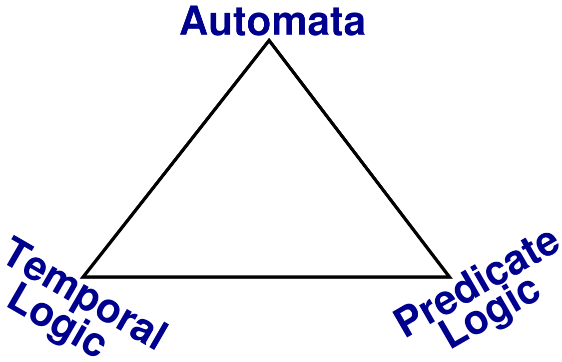

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- Note: focus on linear time (as opposed to branching time).

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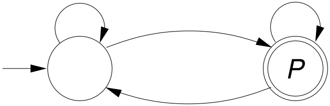

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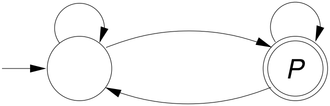

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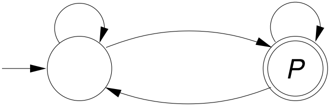

$\Box\Diamond P$ $\qquad\qquad$ $\forall x\, \exists y\, (x < y \wedge P(y))$

Specification and Verification

- Linear Temporal Logic (LTL)

$$\theta ::= P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta \mid \bigcirc\theta \mid \Diamond\theta \mid \Box\theta \mid \theta_1 \,\mathcal{U}\, \theta_2$$

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Verification is model checking: IMP $\models$ SPEC ?

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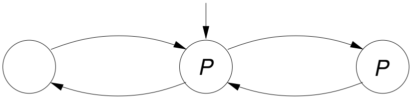

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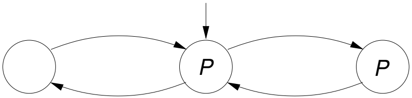

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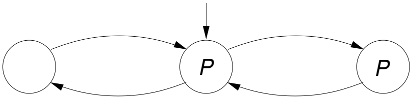

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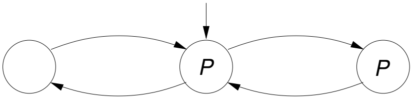

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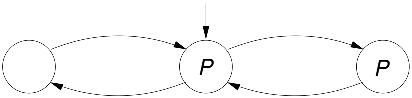

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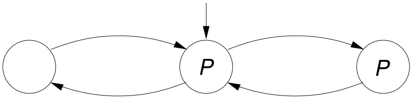

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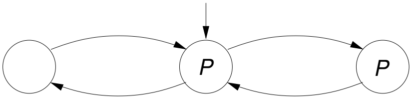

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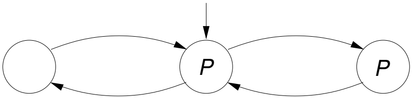

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- So one way to capture the original specification would be to write: *'Q holds precisely at even positions* **and** $\Box(Q \rightarrow P)$'.
- Finally, need to existentially quantify Q out:

$$\exists Q\ (Q \textit{ holds precisely at even positions } \textbf{and } \Box\,(Q \rightarrow P))$$

Monadic Second-Order Logic

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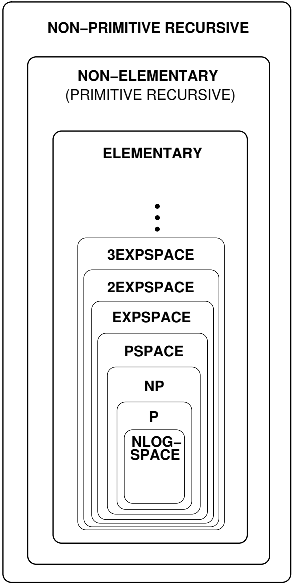
Corollary (Church 1960)

The model-checking problem for automata against MSO($<$) specifications is decidable:

$$M \models \varphi \quad \textit{iff} \quad L(M) \cap L(A_{\neg\varphi}) = \emptyset$$

Complexity

UNDECIDABLE



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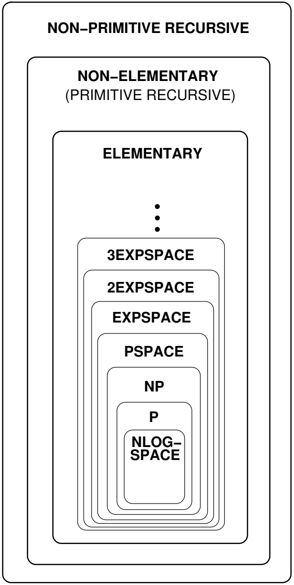

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- NON-PRIMITIVE RECURSIVE:

  Ackerman: $3, 4, 8, 2048, \underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{2048}, \ldots$

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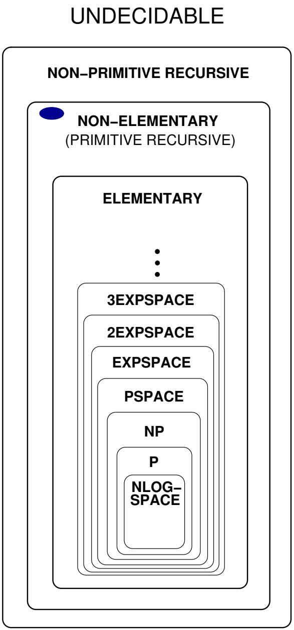

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**Theorem (Kamp 1968;
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LTL and FO($<$) have precisely the same expressive power.

But amazingly:

Theorem (Sistla & Clarke 1982)

LTL satisfiability and model checking are PSPACE-complete.

Logics and Automata

"The paradigmatic idea of the automata-theoretic approach to verification is that we can compile high-level logical specifications into an equivalent low-level finite-state formalism."

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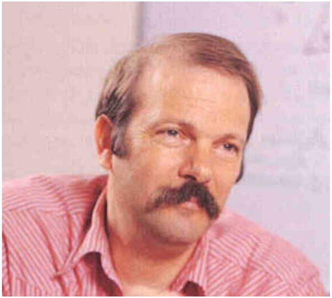

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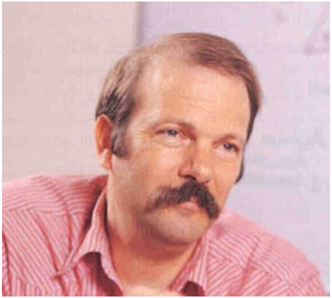

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Theorem

Automata are closed under all Boolean operations. Moreover, the language inclusion problem ($L(A) \subseteq L(B)$?) is PSPACE-complete.

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$\boxed{\mathrm{FO}(<)}$ - - - - - - - - - - - - - - - - - - $\boxed{\mathrm{LTL}}$

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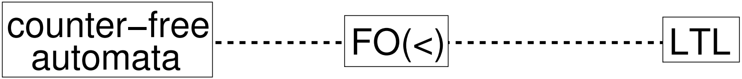

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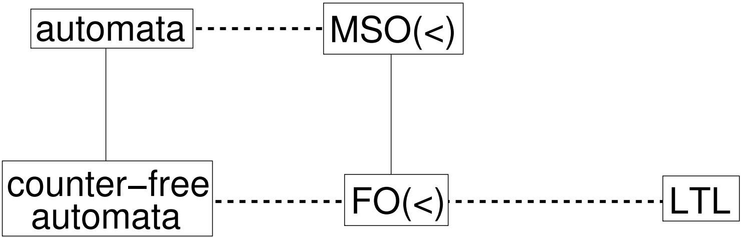

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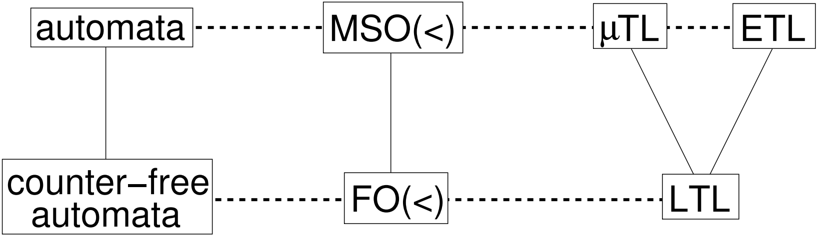

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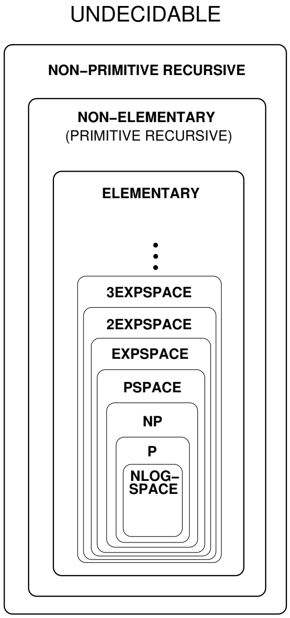

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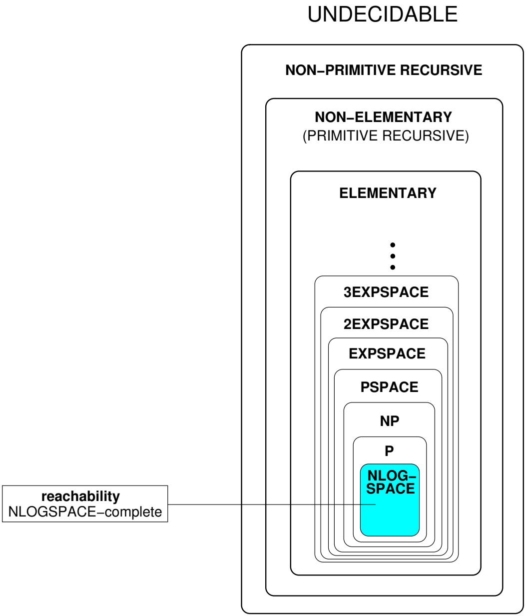

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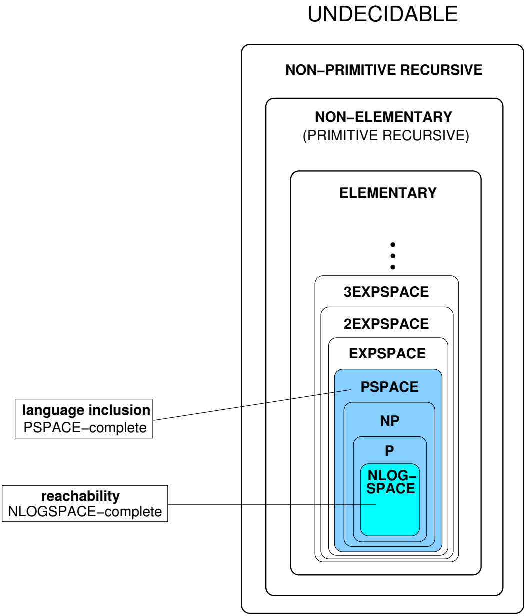

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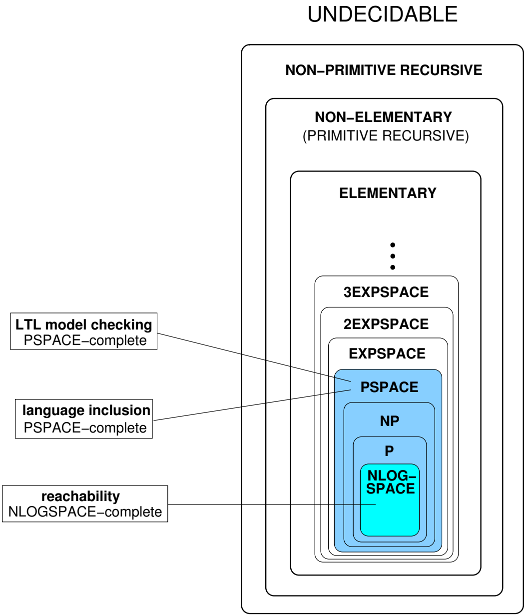

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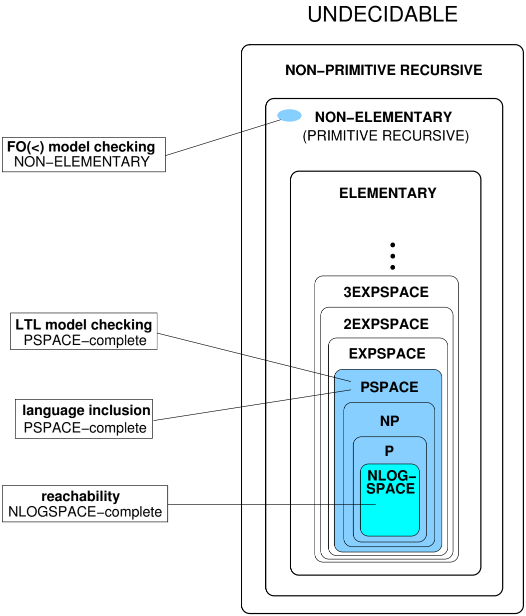

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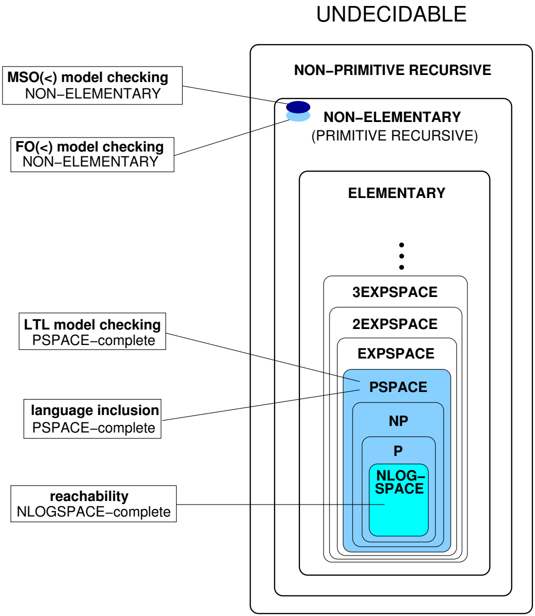


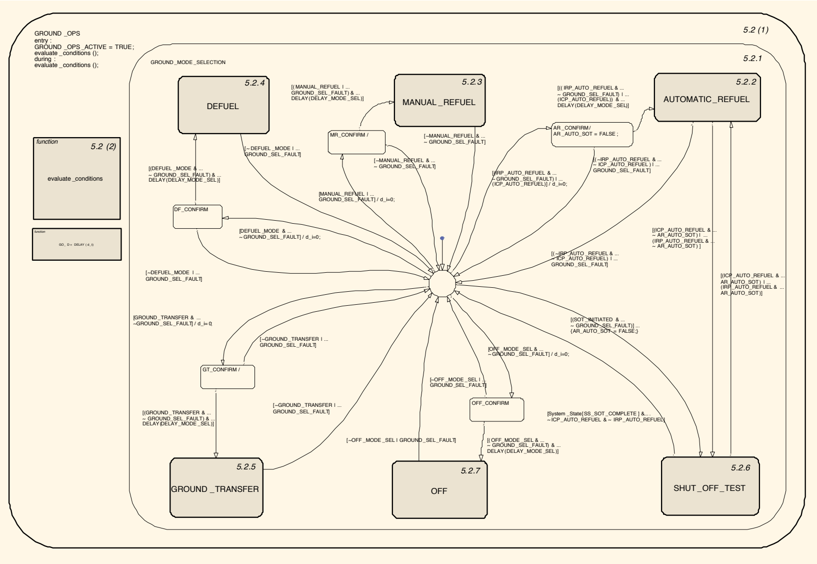
From Qualitative to Quantitative

"Lift the classical theory to the real-time world."

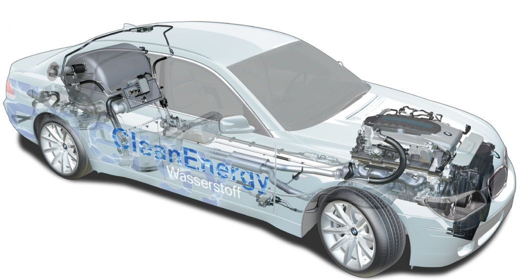
Boris Trakhtenbrot, LICS 1995

Airbus A350 XWB

A350 XWB Fuel Management Sub-System



BMW Hydrogen 7



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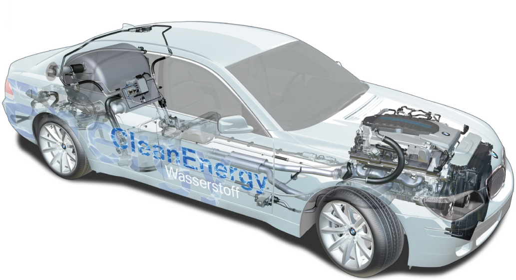

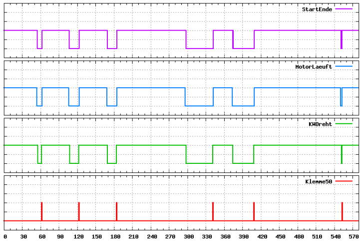

Timed Systems

Timed systems are everywhere…

- Hardware circuits
- Communication protocols
- Cell phones
- Plant controllers
- Aircraft navigation systems
- Sensor networks
- …

Timed Automata

Timed automata were introduced by Rajeev Alur at Stanford during his PhD thesis under David Dill:

- Rajeev Alur, David L. Dill: *Automata For Modeling Real-Time Systems*. ICALP 1990: 322-335
- Rajeev Alur, David L. Dill: *A Theory of Timed Automata*. TCS 126(2): 183-235, 1994

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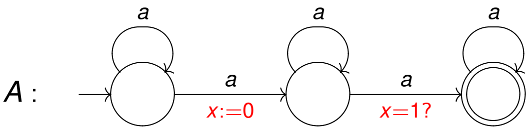
Unfortunately:

Theorem (Alur & Dill 1990)

Language inclusion is undecidable for timed automata.

An Uncomplementable Timed Automaton



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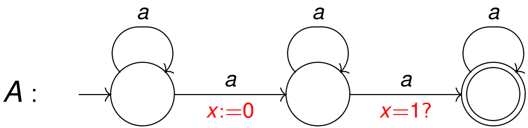

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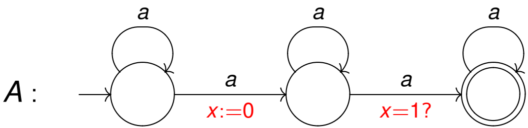

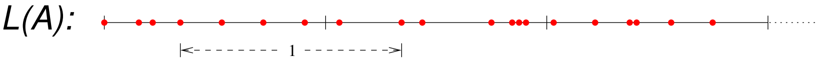

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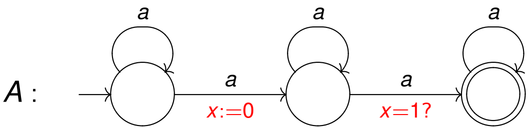

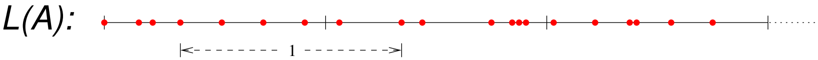

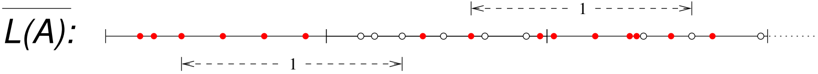

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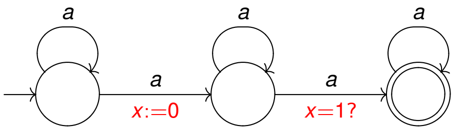
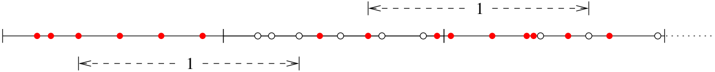


$L(A)$:

$\overline{L(A)}$:



A cannot be complemented:

There is no timed automaton B with $L(B) = \overline{L(A)}$.

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Theorem (Alur & Henzinger 1992)

MTL satisfiability and model checking are undecidable over $\mathbb{R}_{\geq 0}$.

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Metric Temporal Logic (MTL) [Koymans; de Roever; Pnueli $\sim$1990] is a central quantitative specification formalism for timed systems.

- MTL = LTL + timing constraints on operators:

$$\Box(\mathit{PEDAL} \to \Diamond_{[5,10]}\, \mathit{BRAKE})$$

- Widely cited and used (over nine hundred papers according to `scholar.google.com`!).

Unfortunately:

Theorem (Alur & Henzinger 1992)

MTL satisfiability and model checking are undecidable over $\mathbb{R}_{\geq 0}$. (Decidable but non-primitive recursive under certain semantic restrictions [Ouaknine & Worrell 2005].)

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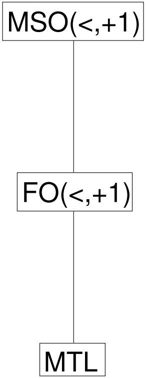
Corollary: FO($<$,$+1$) and MSO($<$,$+1$) satisfiability and model checking are undecidable over $\mathbb{R}_{\geq 0}$.

The Real-Time Theory: Expressiveness



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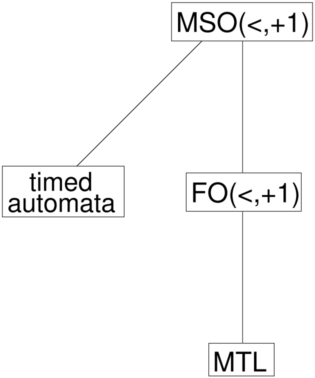
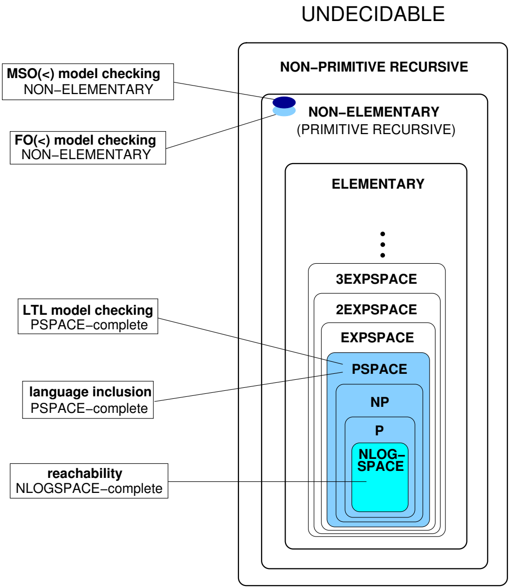


The Real-Time Theory: Complexity

Classical Theory

Real–Time Theory



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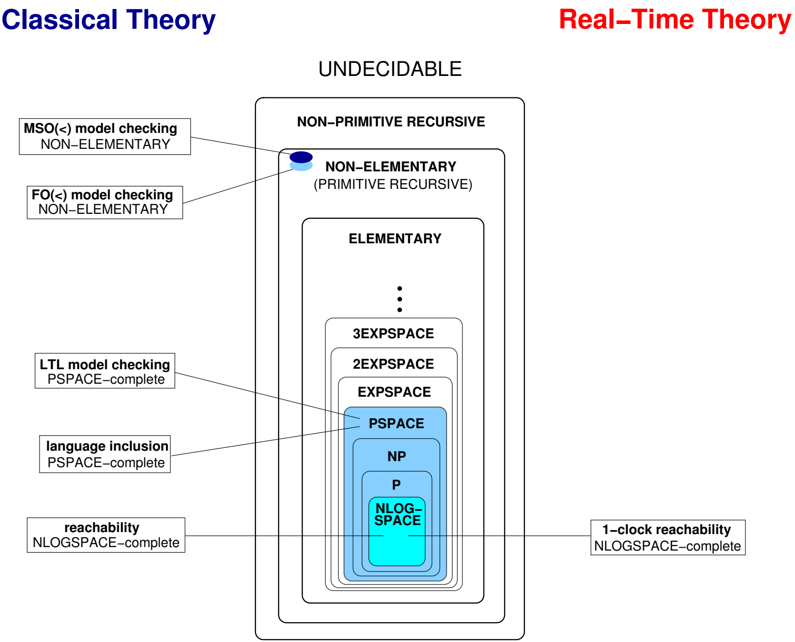

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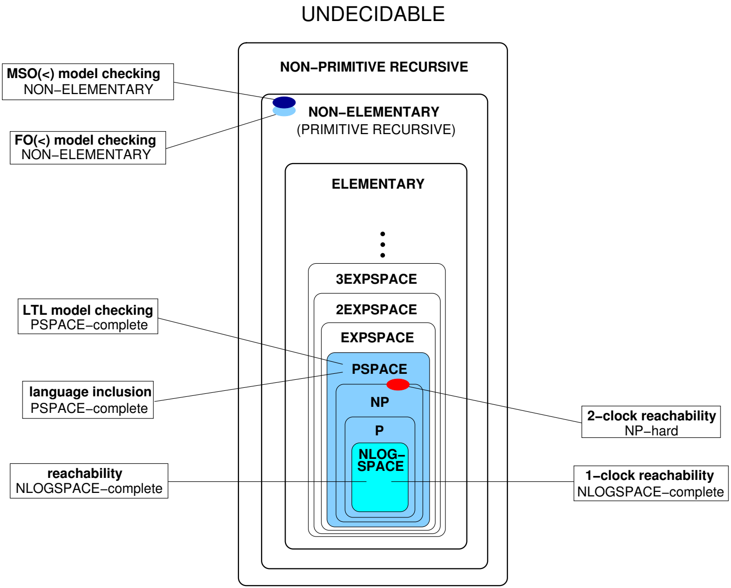

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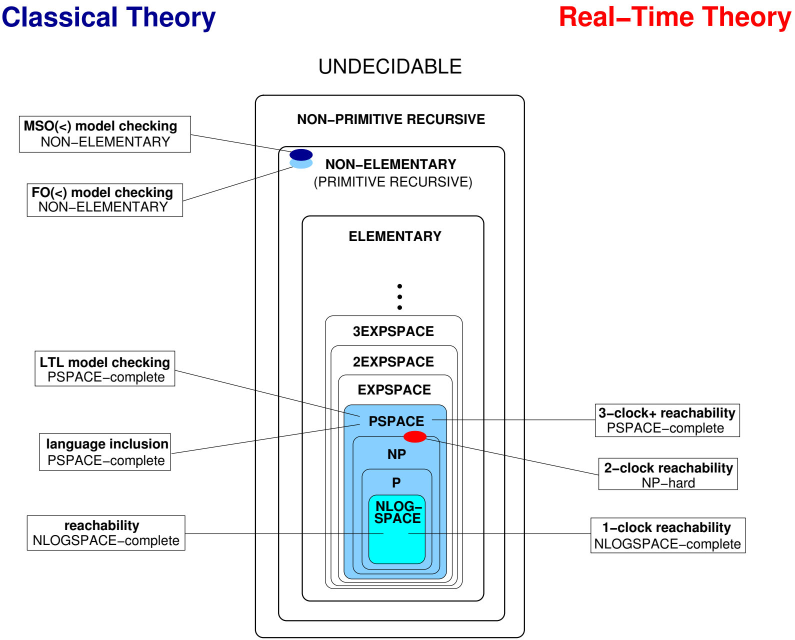

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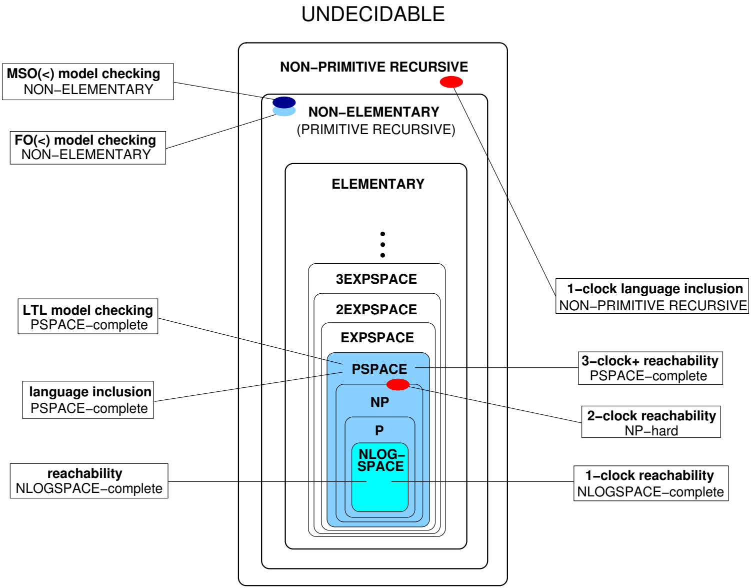

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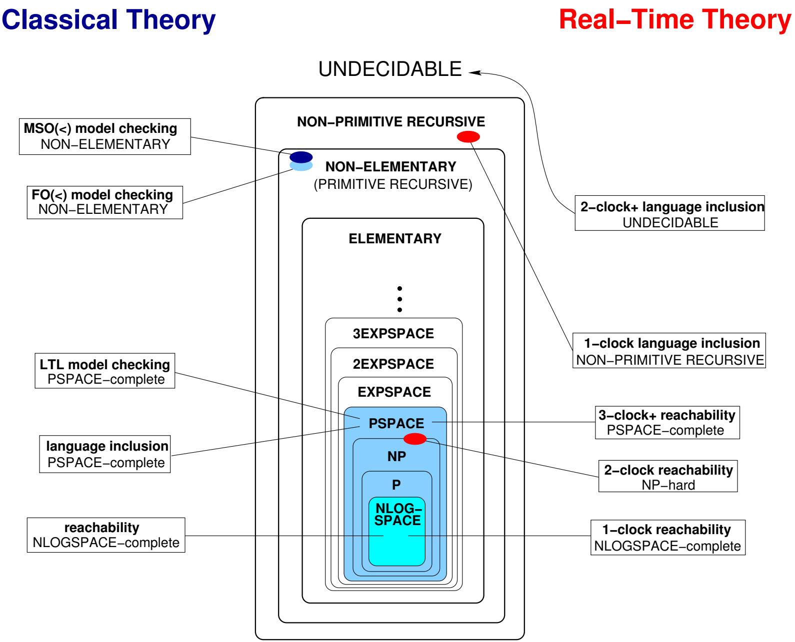

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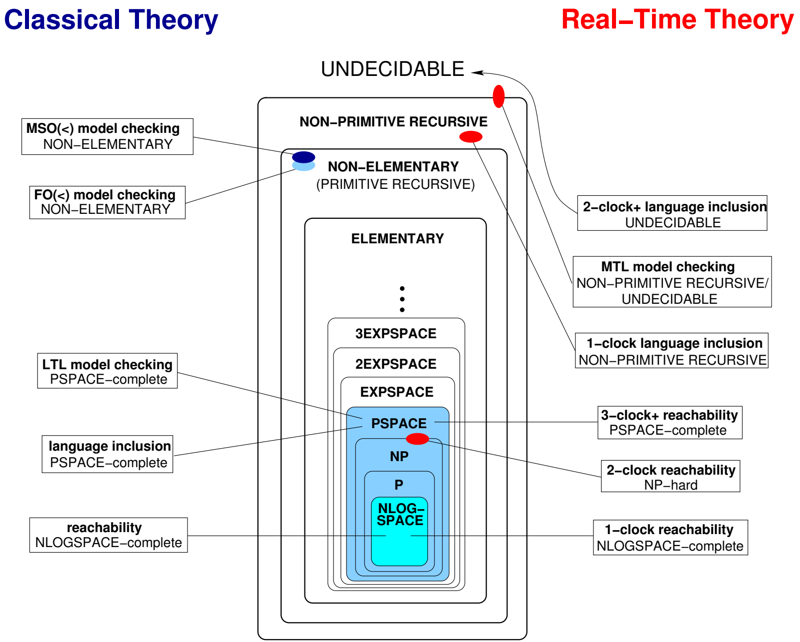

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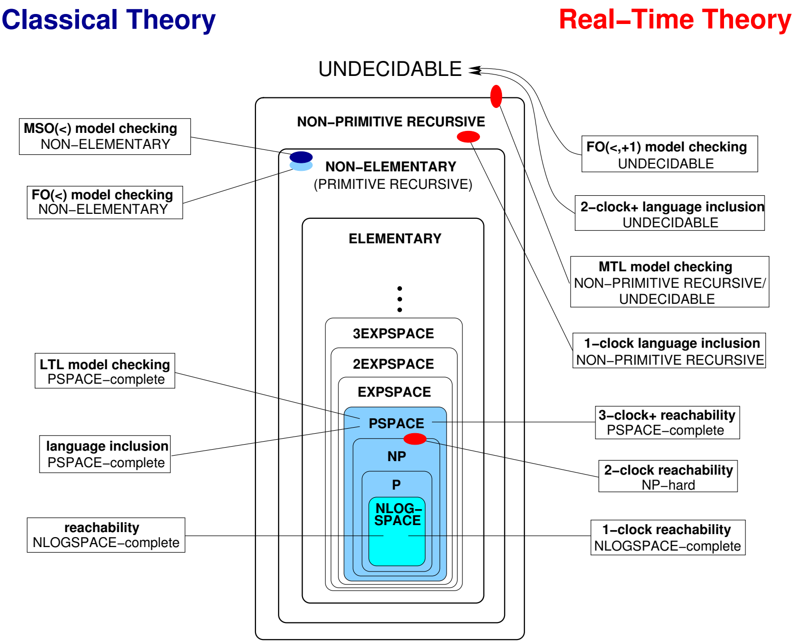

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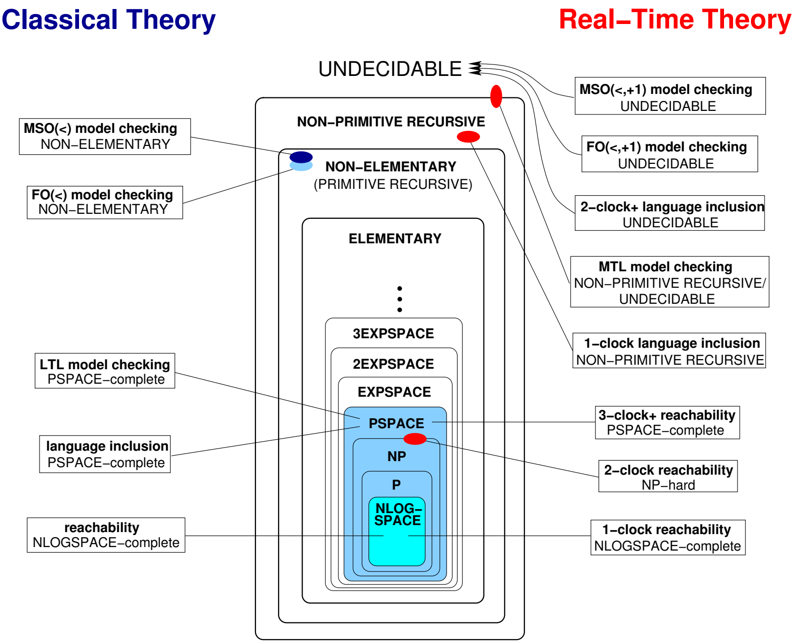

Key Stumbling Block

Theorem (Alur & Dill 1990)

Language inclusion is undecidable for timed automata.

Timed Language Inclusion: Some Related Work

- Topological restrictions and digitization techniques: [Henzinger, Manna, Pnueli 1992], [Bošnački 1999], [Ouaknine & Worrell 2003]
- Fuzzy semantics / noise-based techniques: [Maass & Orponen 1996], [Gupta, Henzinger, Jagadeesan 1997], [Fränzle 1999], [Henzinger & Raskin 2000], [Puri 2000], [Asarin & Bouajjani 2001], [Ouaknine & Worrell 2003], [Alur, La Torre, Madhusudan 2005]
- Determinisable subclasses of timed automata: [Alur & Henzinger 1992], [Alur, Fix, Henzinger 1994], [Wilke 1996], [Raskin 1999]
- Timed simulation relations and homomorphisms: [Lynch *et al.* 1992], [Taşiran *et al.* 1996], [Kaynar, Lynch, Segala, Vaandrager 2003]
- Restrictions on the number of clocks: [Ouaknine & Worrell 2004], [Emmi & Majumdar 2006]

Time-Bounded Language Inclusion

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Instance: Timed automata A, B, and time bound $T \in \mathbb{N}$

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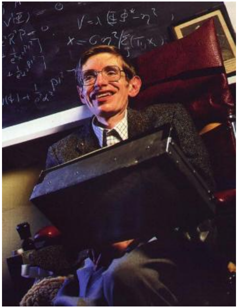
- Universe's lifetime is believed to be bounded anyway…



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- Unfortunately, timed automata cannot be complemented even over bounded time…
- Key to solution is to translate problem into logic: Behaviours of timed automata can be captured in MSO($<$,$+1$)
- This reverses Vardi's 'automata-theoretic approach to verification' paradigm!

Monadic Second-Order Logic

Theorem (Shelah 1975)

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By contrast,

Theorem

- *MSO($<$) is decidable over* $\mathbb{N}$ *[Büchi 1960]*
- *MSO($<$) is decidable over* $\mathbb{Q}$*, via [Rabin 1969]*

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Timed behaviours are modelled as flows (or signals):

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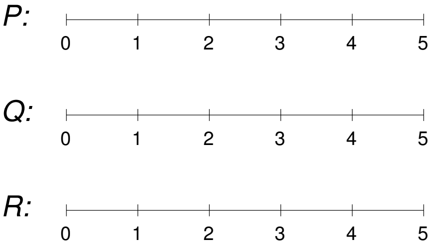

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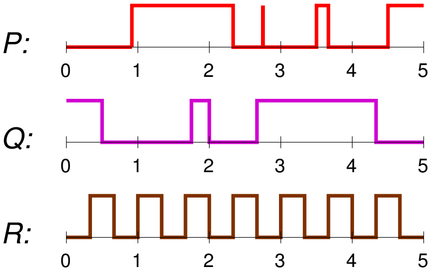

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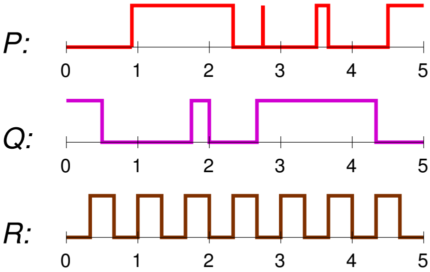

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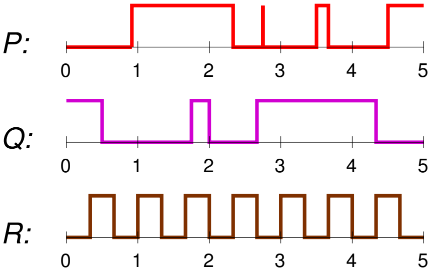

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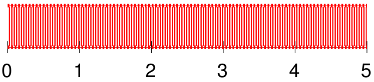

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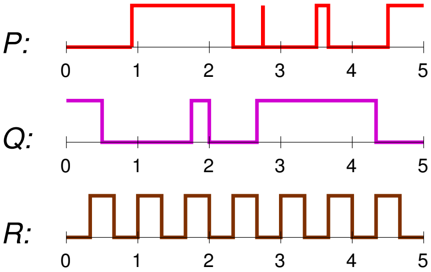

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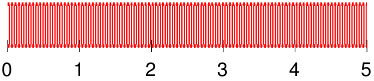

Then:

Theorem (Rabinovich 2002)

MSO(<) satisfiability over finitely-variable flows is decidable.

The Time-Bounded Theory of Verification

Theorem

For any bounded time domain $[0, T)$, satisfiability and model checking are decidable as follows:

MSO($<$,$+1$)	*NON-ELEMENTARY*
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Theorem

Given timed automata A, B, and time bound $T \in \mathbb{N}$, the time-bounded language inclusion problem $L_T(A) \subseteq L_T(B)$ is decidable and 2EXPSPACE-complete.

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- Conclude by invoking decidability of MSO($<$).

From MSO($<$,$+1$) to MSO($<$)

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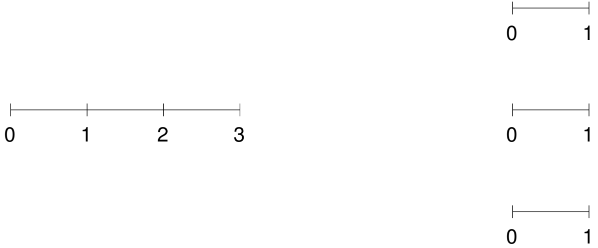

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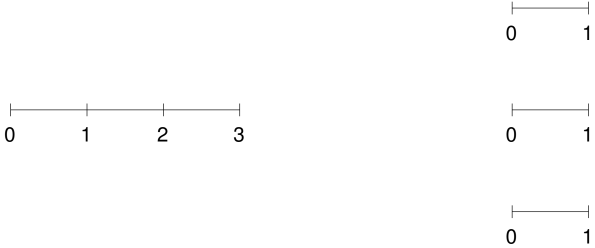

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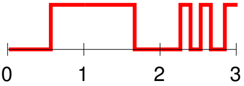

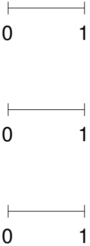

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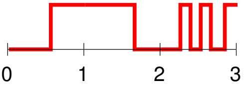

P_0:

P_1:

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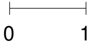

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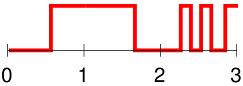

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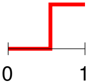

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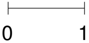

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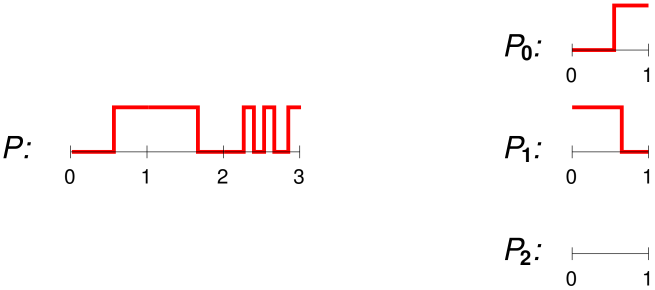

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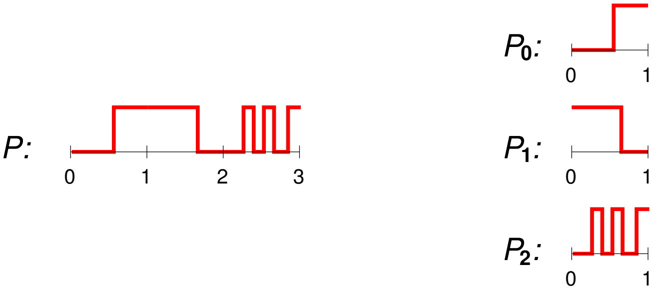

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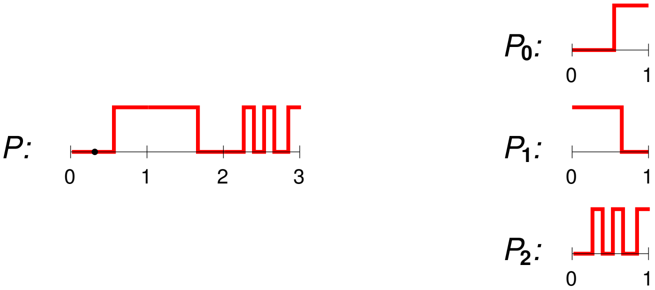

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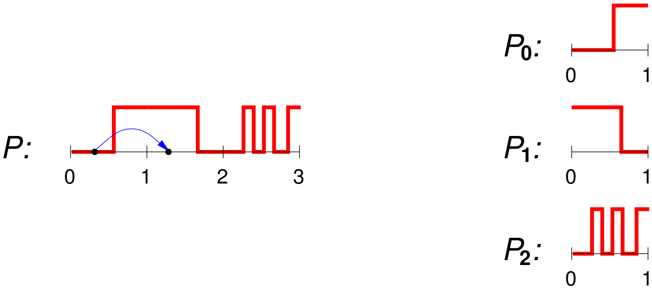

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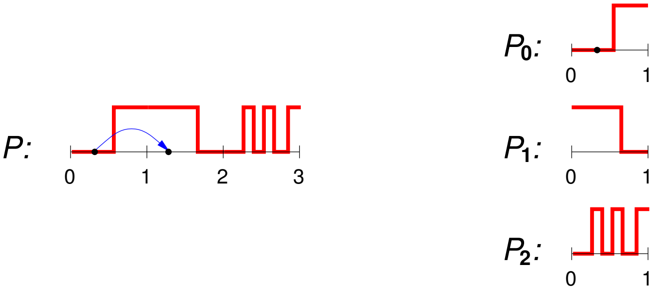

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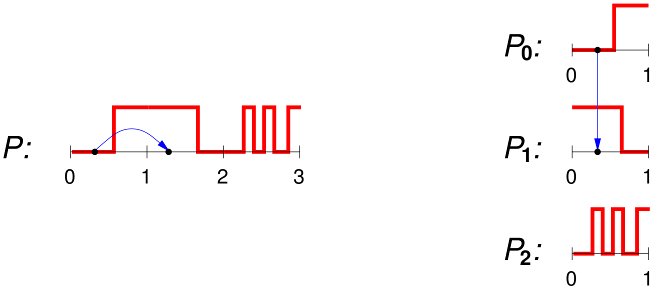

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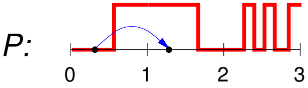

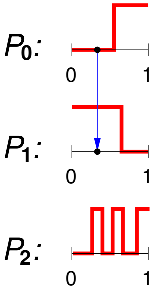

Replace every:

- $\forall x\, \psi(x)$

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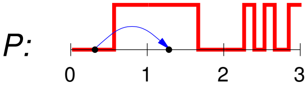

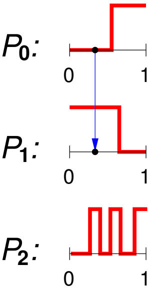

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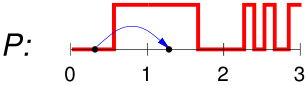

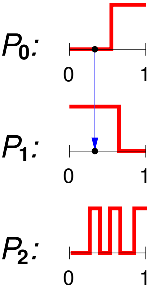

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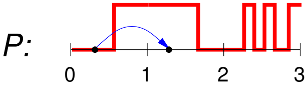

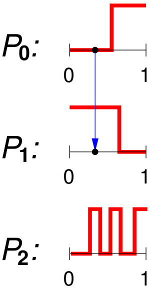

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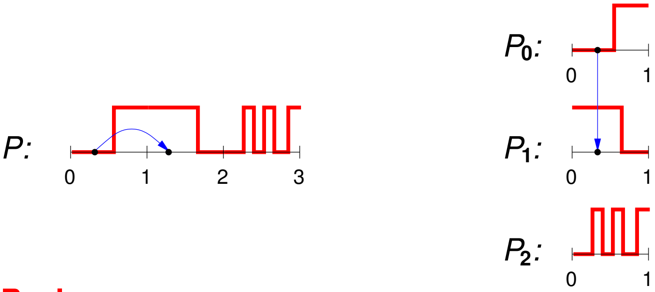

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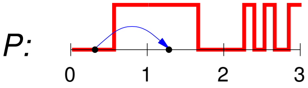
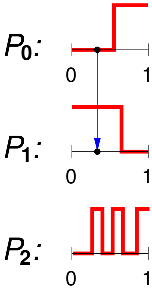
- $P(x+k)$

From MSO($<$,$+1$) to MSO($<$)

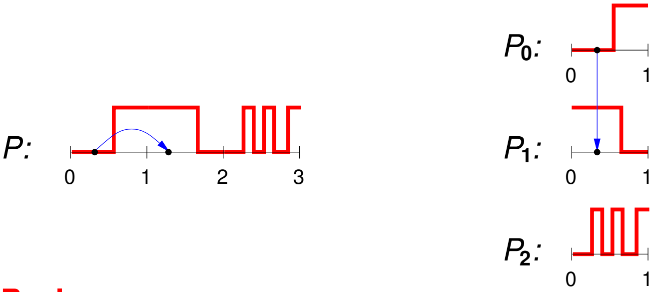




Replace every:

- $\forall x\, \psi(x)$ **by** $\forall x\ (\psi(x) \wedge \psi(x+1) \wedge \psi(x+2))$
- $x + k_1 < y + k_2$ **by** $\begin{cases} x < y & \text{if } k_1 = k_2 \\ \textbf{true} & \text{if } k_1 < k_2 \\ \textbf{false} & \text{if } k_1 > k_2 \end{cases}$
- $P(x+k)$ **by** $P_k(x)$

From MSO($<$,$+1$) to MSO($<$)



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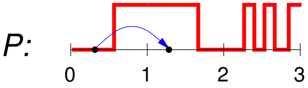
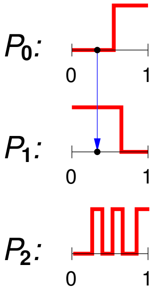
- $P(x+k)$ **by** $P_k(x)$
- $\forall P\, \psi$

From MSO(<,+1) to MSO(<)





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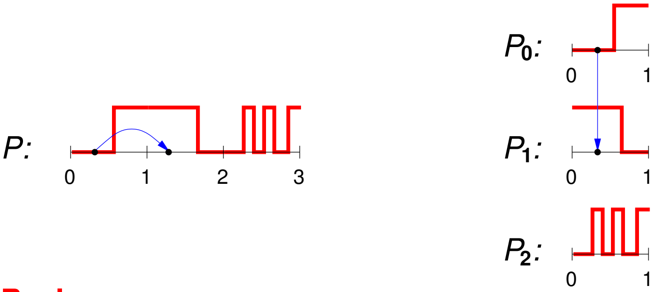
- $P(x+k)$ **by** $P_k(x)$
- $\forall P\, \psi$ **by** $\forall P_0\, \forall P_1\, \forall P_2\, \psi$

From MSO(<,+1) to MSO(<)



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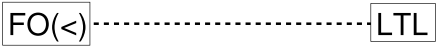
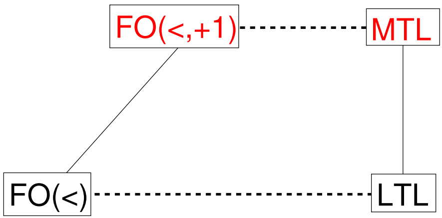
Then φ is satisfiable over $[0, T)$ $\iff$ $\overline{\varphi}$ is satisfiable over $[0, 1)$.

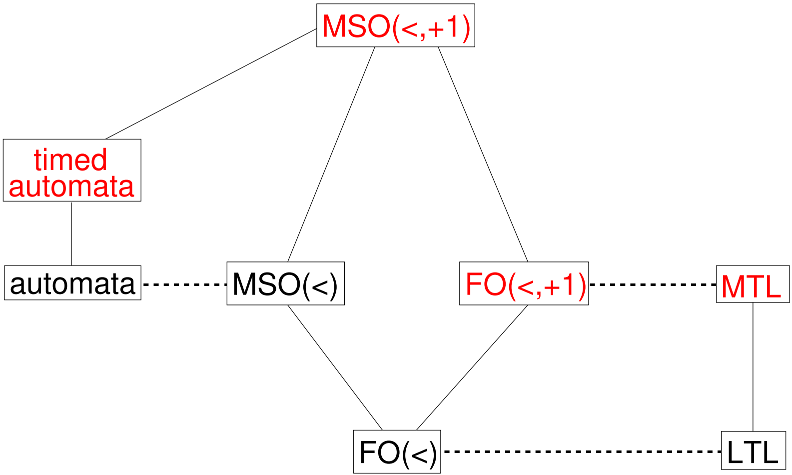
The Time-Bounded Theory: Expressiveness



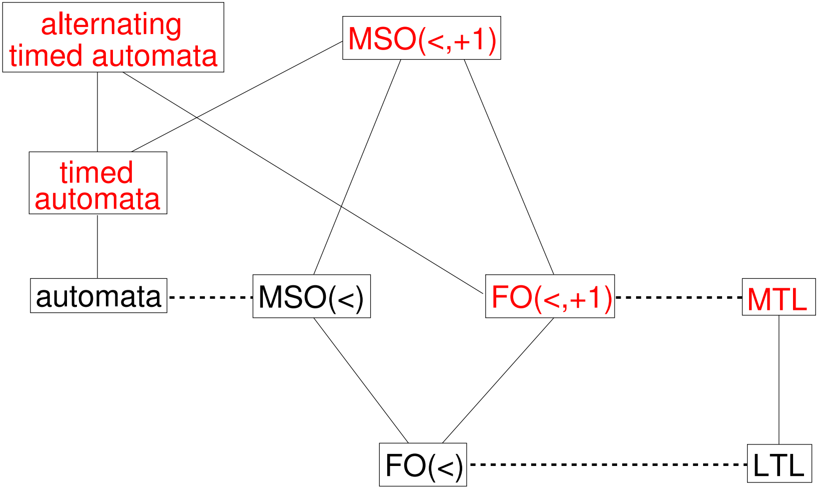
The Time-Bounded Theory: Expressiveness



The Time-Bounded Theory: Expressiveness



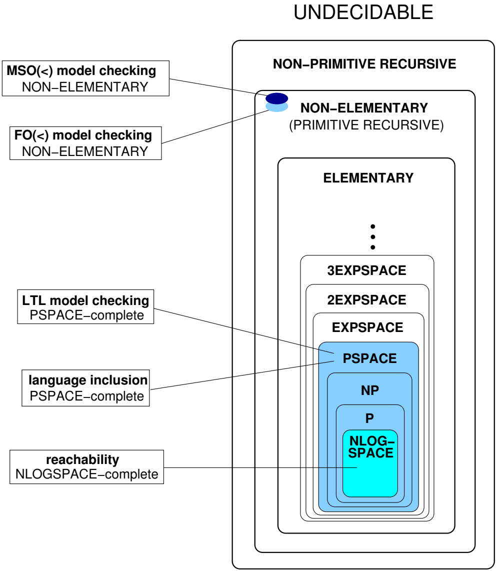
The Time-Bounded Theory: Expressiveness



The Time-Bounded Theory: Complexity

Classical Theory

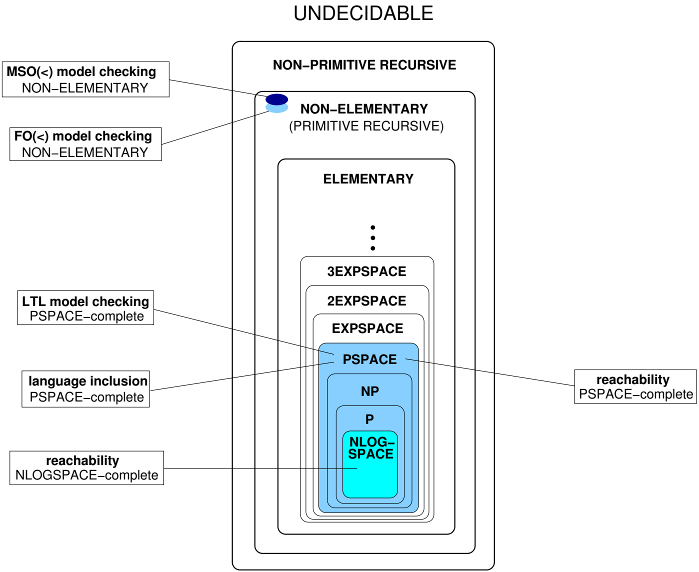
Time–Bounded Theory



The Time-Bounded Theory: Complexity

Classical Theory

Time–Bounded Theory



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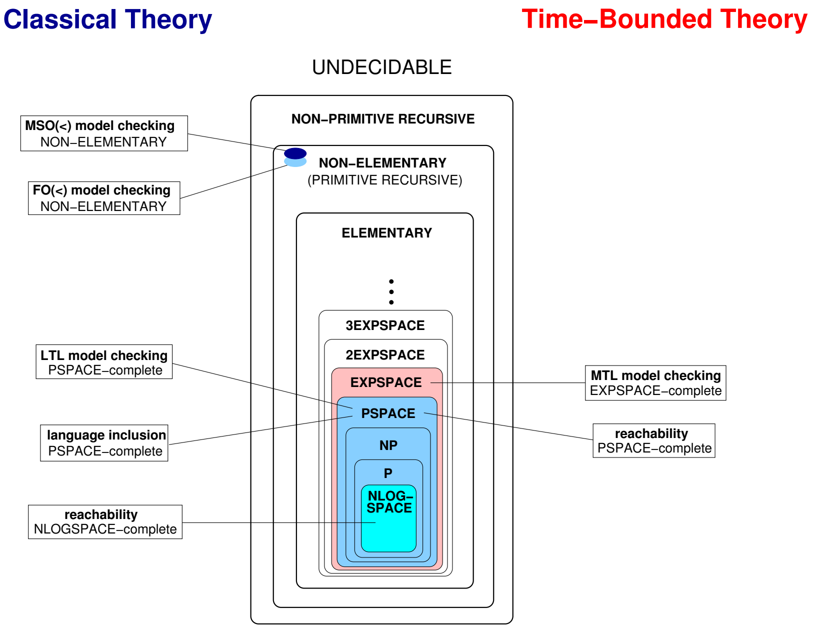
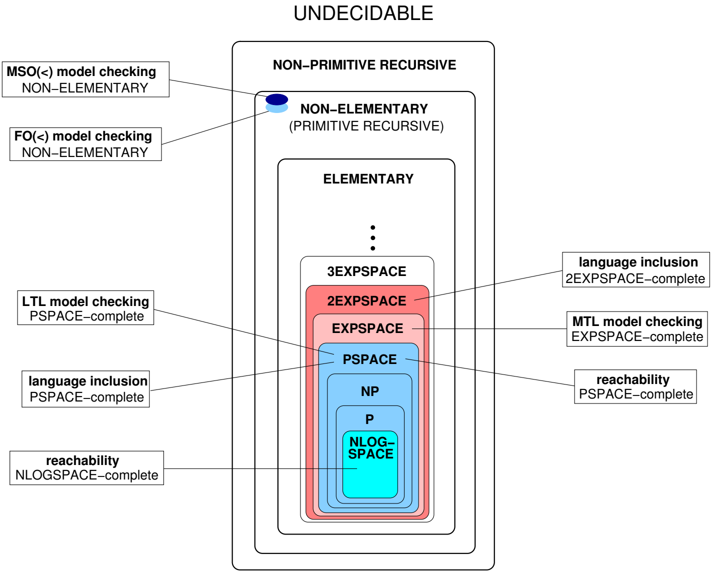
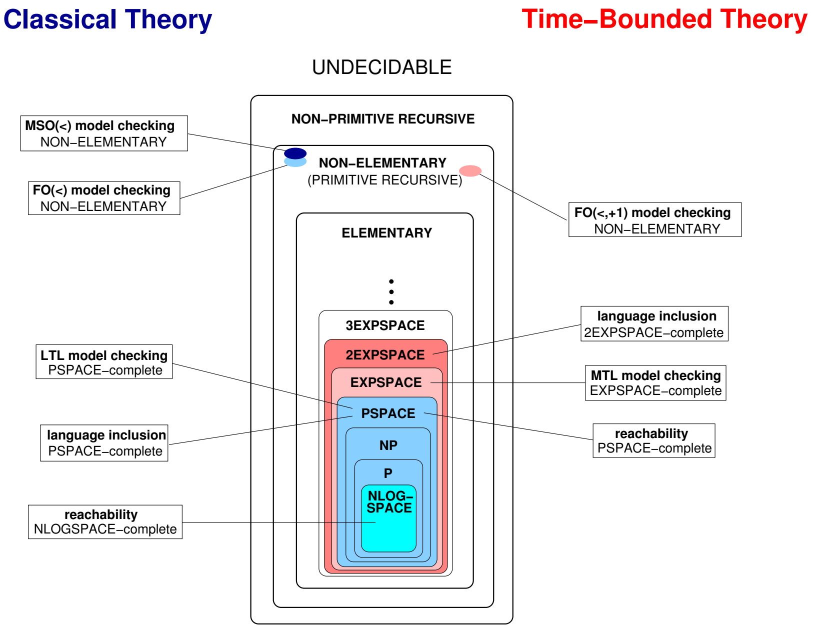


The Time-Bounded Theory: Complexity

Classical Theory **Time–Bounded Theory**



The Time-Bounded Theory: Complexity



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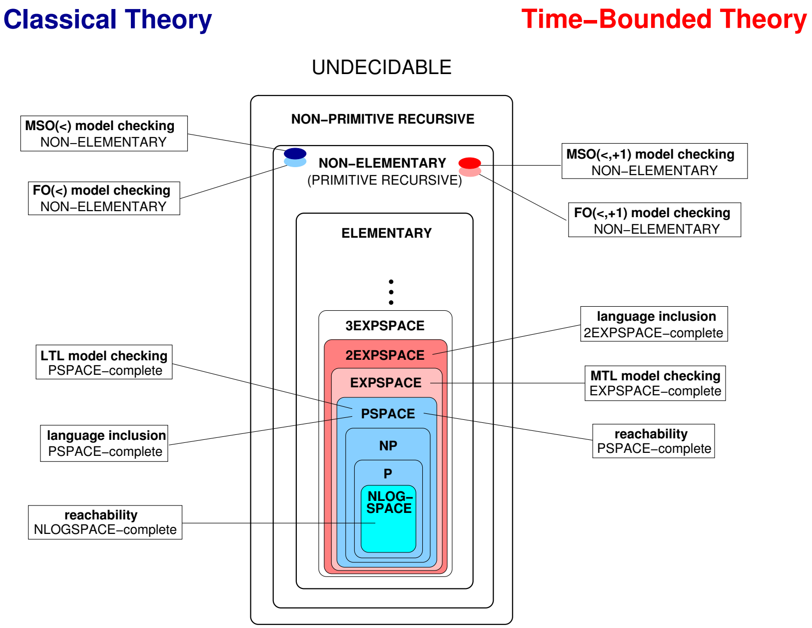
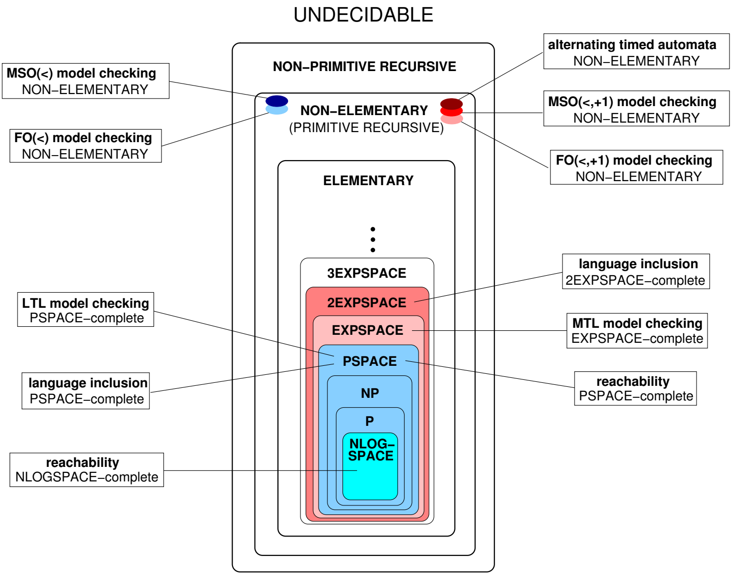


The Time-Bounded Theory: Complexity

Classical Theory **Time–Bounded Theory**



Conclusion and Future Work

- For real-time systems, the time-bounded theory is much better behaved than the real-time theory.

Conclusion and Future Work

- For real-time systems, the time-bounded theory is much better behaved than the real-time theory.

Future work:

- Extend the theory further!
  - Branching-time
  - Timed games and synthesis
  - Weighted and hybrid automata
  - …
- Algorithmic and complexity issues
- Expressiveness issues
- Implementation and case studies