TIMED AUTOMATA

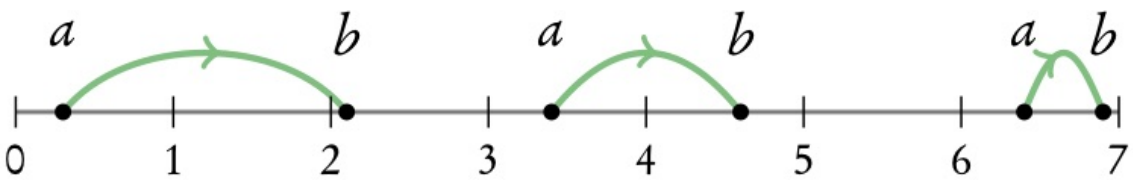
LECTURE 2

GOALS OF TODAY'S LECTURE

- 1. Languages not accepted by Timed Automata
- 2. Timed regular languages
- 3. Closure properties
- 4. Course plan

$$L_7 = \{ \, (\, (ab)^k, \tau \,) \mid \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} \text{ for each } i \geq 1 \}$$

Converging *ab* distances



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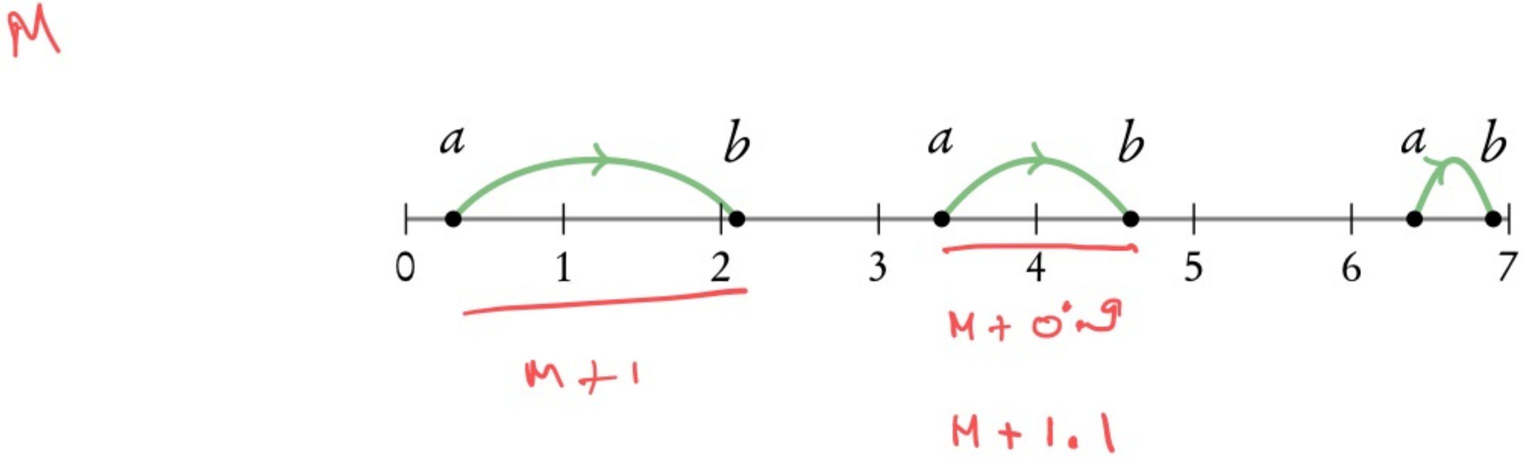
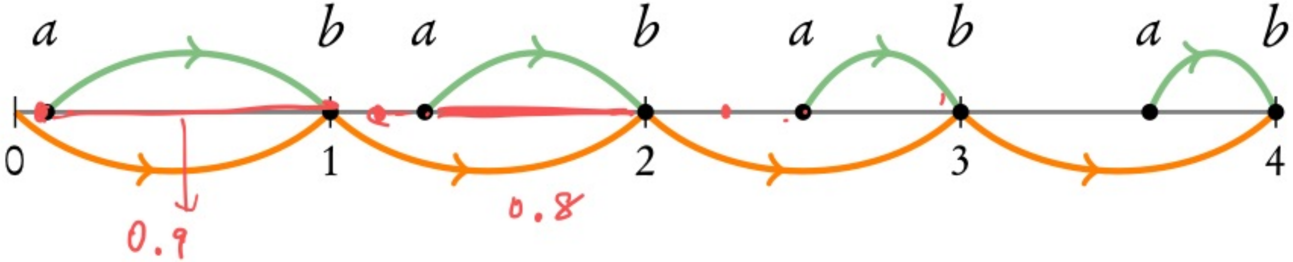


Exercise: Prove that **no timed automaton** can accept L_7

$$L_7 = \{\ (\ (ab)^k, \tau\ )\ |\ \tau_{2i} = i \ \text{ and } \ \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1}\}$$

Pivoted converging *ab* distances



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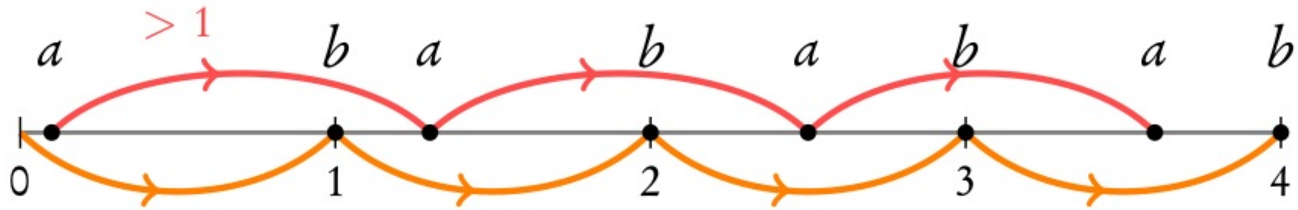

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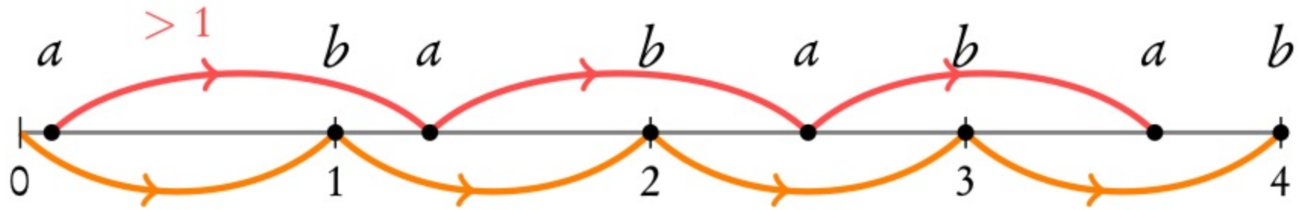

$$\tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} \;\Leftrightarrow\; \tau_{2i+2} - \tau_{2i} < \tau_{2i+1} - \tau_{2i-1}$$

$$\Leftrightarrow\; 1 < \tau_{2i+1} - \tau_{2i-1}$$

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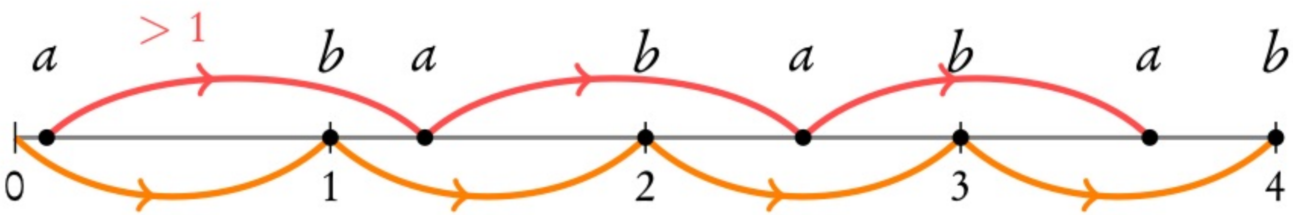

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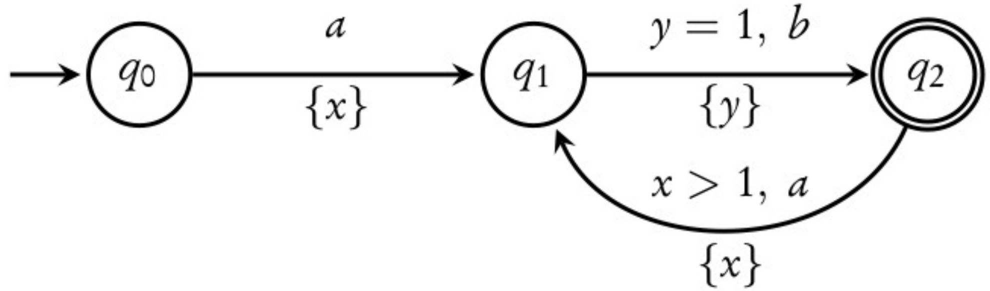

Timed automata

Runs

1 clock $<$ 2 clocks $< \ldots$

Role of max constant

Timed automata

Runs

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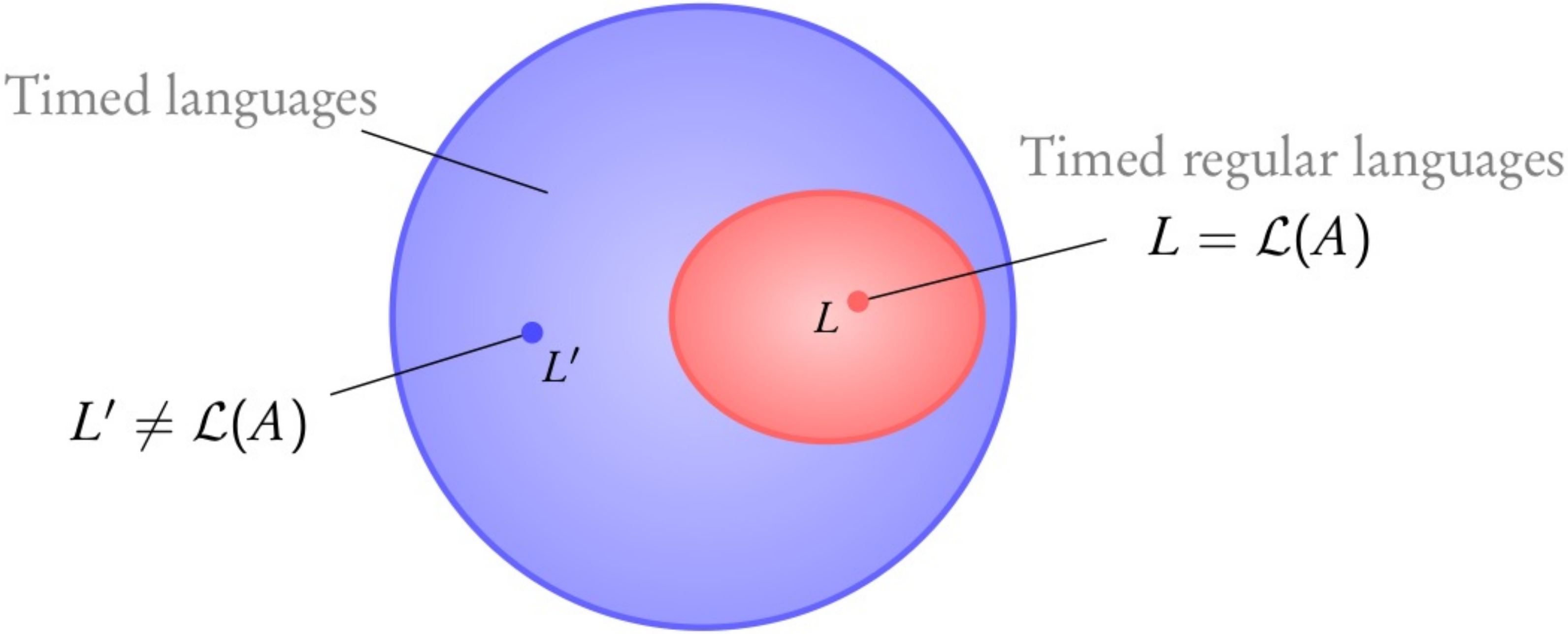
Role of max constant

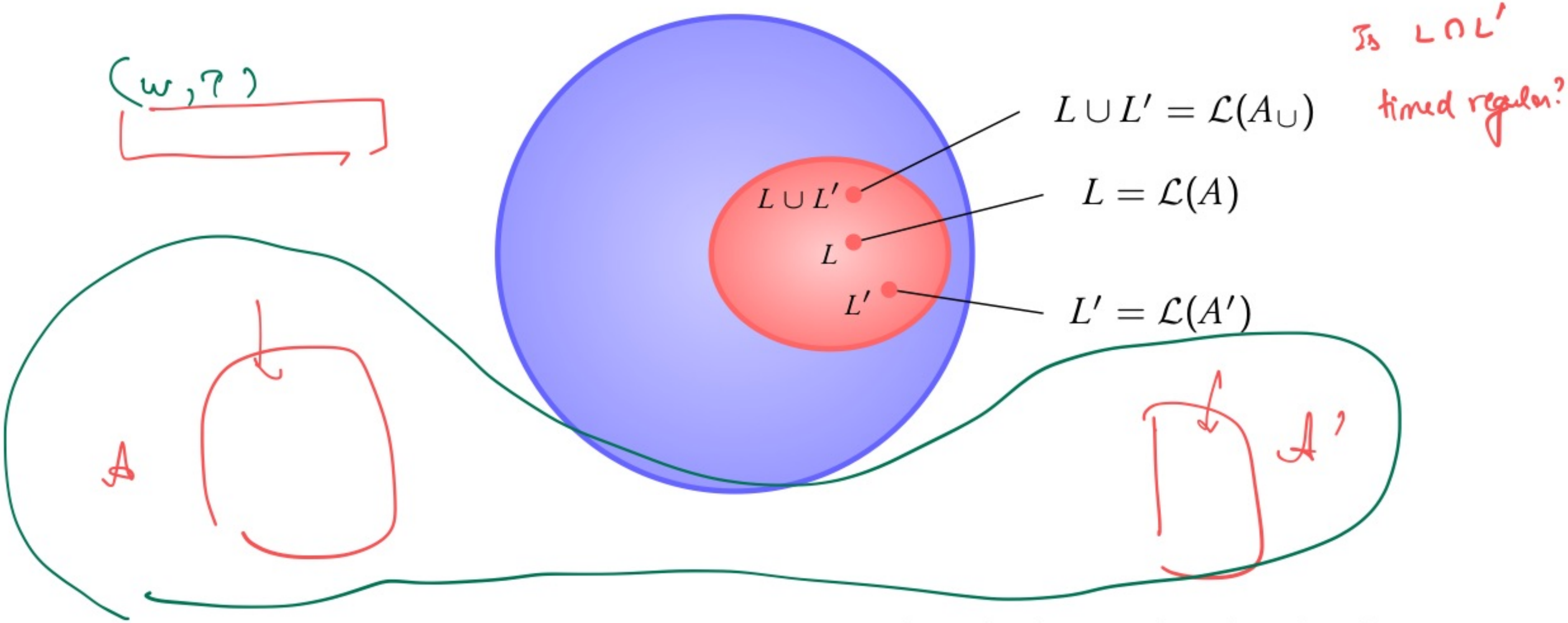
Timed regular lngs.

Timed regular languages



Definition

A timed language is called **timed regular** if it can be **accepted** by a timed automaton

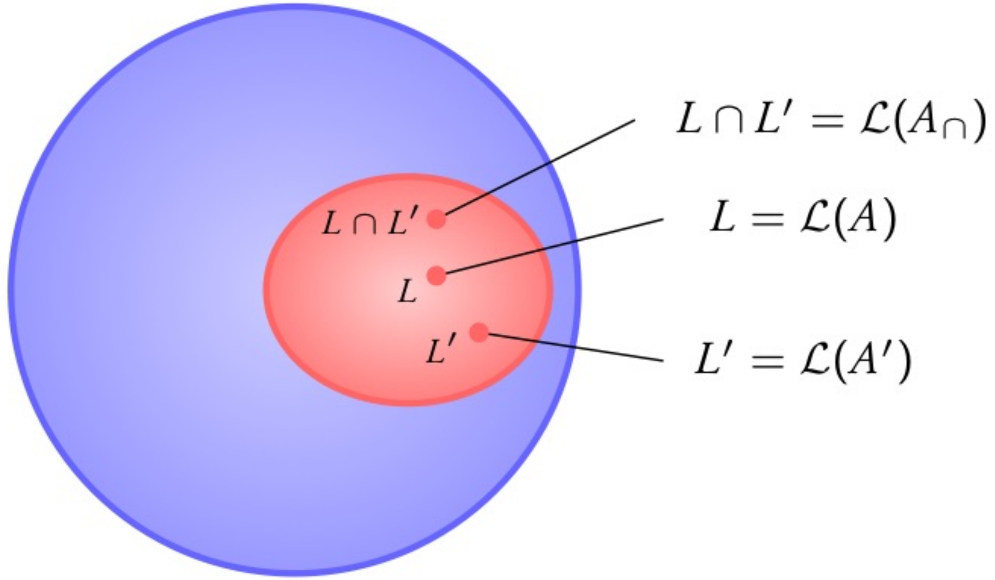


$A = (Q, \Sigma, X, T, Q_0, F)$ $\qquad$ $A' = (Q', \Sigma, X', T', Q_0', F')$

$$A_\cup = (\ Q \cup Q'\ ,\ \Sigma\ ,\ X \cup X'\ ,\ T \cup T'\ ,\ Q_0 \cup Q_0'\ ,\ F \cup F'\ )$$

$$\mathcal{L}(A) \cup \mathcal{L}(A') = \mathcal{L}(A_\cup)$$

Timed regular languages are **closed** under **union**



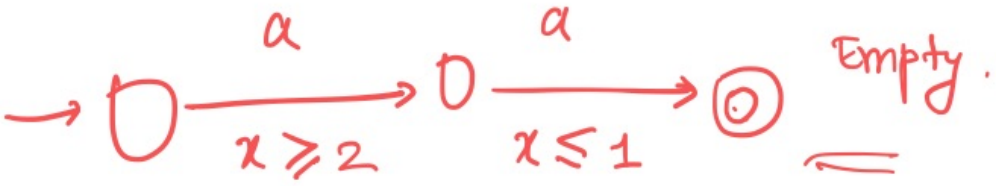
$$A = (Q, \Sigma, X, T, Q_0, F) \qquad A' = (Q', \Sigma, X', T', Q'_0, F')$$

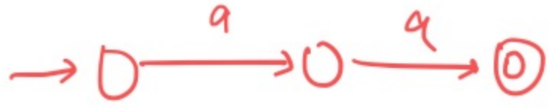
$$A_\cap = (\ Q \times Q'\ ,\ \Sigma\ ,\ X \cup X'\ ,\ T_\cap\ ,\ Q_0 \times Q'_0\ ,\ F \times F'\ )$$

$$T_\cap : \quad (q_1, q'_1) \xrightarrow[R \cup R']{a,\ g \wedge g'} (q_2, q'_2) \text{ if}$$

$$q_1 \xrightarrow[R]{a,\ g} q_2 \in T \quad \text{and} \quad q'_1 \xrightarrow[R']{a,\ g'} q'_2 \in T'$$

Timed regular languages are **closed** under **intersection**





L : a timed language over Σ

$$\text{Untime}(L) \equiv \{w \in \Sigma^* \mid \exists \tau.\ (w, \tau) \in L\}$$

Untiming construction

For **every timed** automaton A there is a **finite automaton** A_u s.t.

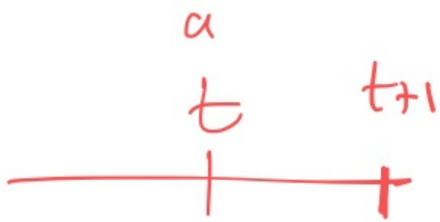
$$\text{Untime}(\ \mathcal{L}(A)\ ) = \mathcal{L}(A_u)$$

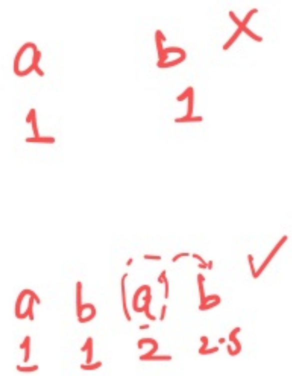
more about this later . . .

Closure under.

Complementation

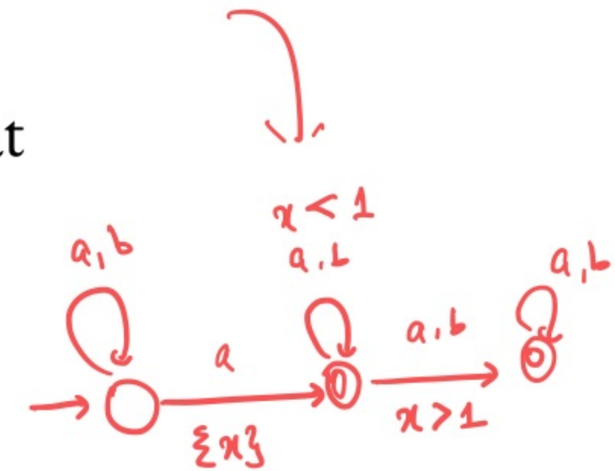
$$\Sigma : \{a, b\}$$

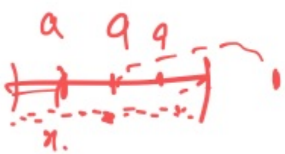




$$L = \{ (w, \tau) \mid \text{there is an } a \text{ at some time } t \text{ and no action occurs at time } t+1 \}$$

$$\overline{L} = \{ (w, \tau) \mid \text{every } a \text{ has an action at a distance 1 from it} \}$$





Complementation

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$$\overline{L} = \{\ (w, \tau) \mid \text{every } a \text{ has an action at a distance 1 from it}\ \}$$

Claim: **No timed automaton** can accept $\overline{L}$

Decision problems for timed automata: A survey

Alur, Madhusudhan. *SFM'04: RT*

Step 1: $\overline{L} = \{ (w, \tau) \mid$ every a has an action at a distance 1 from it $\}$

Suppose $\overline{L}$ is timed regular

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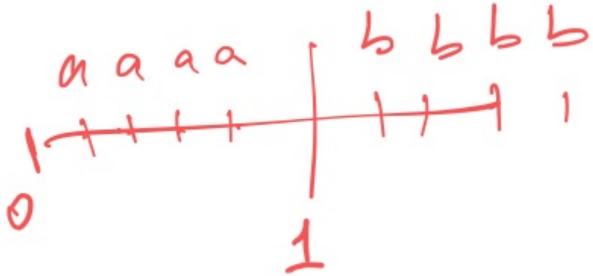
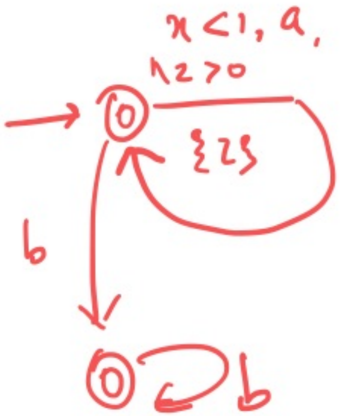
Suppose $\overline{L}$ is timed regular

Step 2: Let $L' = \{ (a^*b^*, \tau) \mid$ all a's occur before time 1 and no two a's happen at same time $\}$



Clearly L' is timed regular



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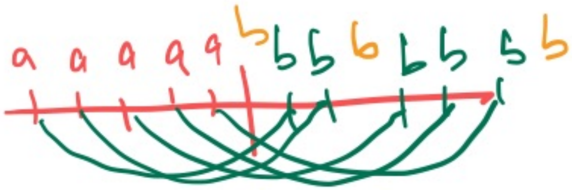

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$L' \cap \overline{L}$ should be timed regular

Step 3: Untime($\overline{L} \cap L'$) should be a regular language

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Step 4: But, Untime($\overline{L} \cap L'$) $= \{a^n b^m \mid m \geq n\}$, ***not regular!***

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Suppose $\overline{L}$ is timed regular

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Clearly L' is timed regular

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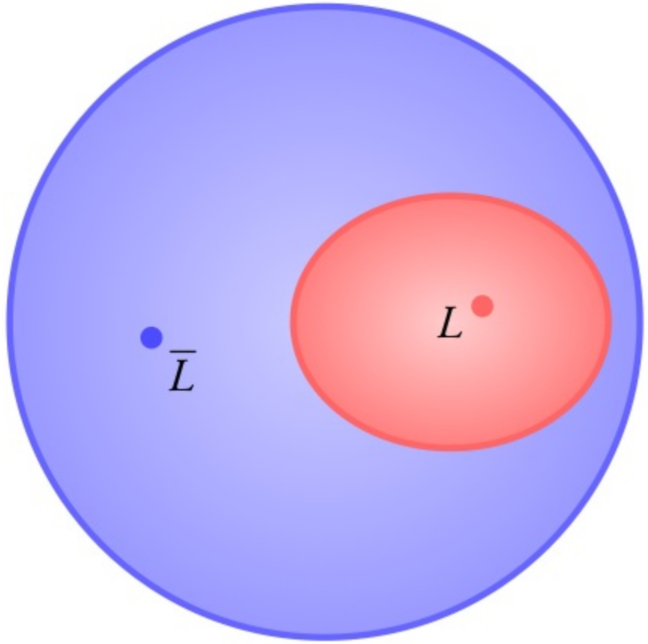
Therefore $\overline{L}$ **cannot be timed regular** $\square$



Timed regular languages are **not closed** under **complementation**

Timed automata

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1 clock $<$ 2 clocks $< \ldots$

Role of max constant

Timed regular lngs.

Closure under $\cup$, $\cap$

Non-closure under complement

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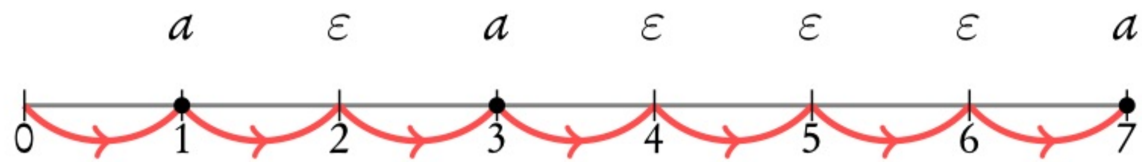
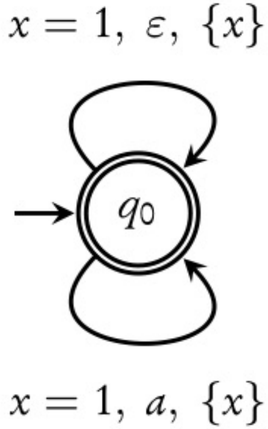
ε-transitions

$$L_6 := \{ \ (\ a^k, \tau \ ) \mid \tau_i \text{ is some integer for each } i\}$$





ε-transitions

ε-transitions **add expressive power** to timed automata.

Characterization of the expressive power of silent transitions in timed automata

Bérard, Diekert, Gastin, Petit. *Fundamenta Informaticae'98*

ε-transitions

ε-transitions **add expressive power** to timed automata. However, they add power **only** when a clock is **reset** in an ε-transition.

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ε-transitions

More expressive

$\xrightarrow{\varepsilon}$ without reset $\equiv$ TA

Recall...

Huge system → Higher-level description → (translation) → Automaton $\mathcal{A}$

Property → Higher-level description → (translation) → Automaton $\mathcal{B}$

Model-Checker

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})?$$

$$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$$

iff

$$\mathcal{L}(\mathcal{A}) \cap \overline{\mathcal{L}(\mathcal{B})} = \emptyset$$

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non-closure under complement $\Rightarrow$ the above **cannot be done** for TA!