GOALS OF TODAY'S LECTURE

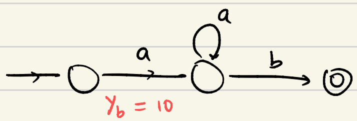
- 1. Event - Predicting Automata
- 2. Event - Clock Automata
- 3. Expressive power

Event-Predicting clocks and Event-Predicting Automata (EPA)

Σ : alphabet

$Y_\Sigma = \{ y_a \mid a \in \Sigma \}$

y_a gives the time to the "next" a.

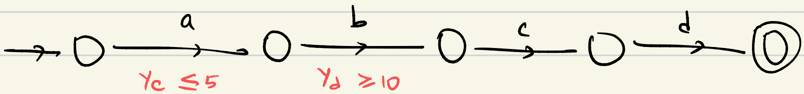
- 

$\{ aa^*b \mid$ time between first 'a' and the 'b' is 10 t.u. $\}$

↳ cannot give an ERA

- $\{ (abcd, \tau_1\tau_2\tau_3\tau_4) \mid \tau_3 - \tau_1 \leq 5, \ \tau_4 - \tau_2 \geq 10 \}$

→ can also give an ERA for this language.



Semantics of predicting clocks:

Given a timed word, what is the value of y_a at each step.

	a	a	b	a	b	b	a
	0.5	1.7	3.2	4.5	6.7	8.0	10.0
y_a	1.2	2.8	1.3	3.5	3.3	2.0	$\perp$
y_b	2.7	1.5	3.5	2.2	1.3	$\perp$	$\perp$

Semantics on a time word is given by functions γ_i :

$w : (a_1 a_2 \dots a_k , \tau_1 \tau_2 , \dots \tau_k)$

$$\gamma_i(y_a) = \begin{cases} t_j - t_i & \text{if } \exists j > i.\ a_j = a \\ & \text{and } \forall m.\ i < m < j,\ a_m \neq a \\ \perp & \text{otherwise} \end{cases}$$

Guards: $y_a \sim c \mid \phi \wedge \phi \mid \phi \vee \phi$

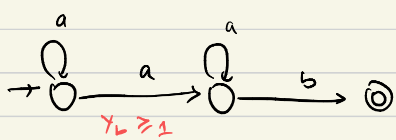
$c \in \mathbb{N} \cup \{\perp\}$

Event-Predicting Automata (EPA)

→ Analogous to defn. of ERA, with use of y-clocks instead of x-clocks.

Example: $\{ a^k b \mid \exists a$ s.t. time between b and this 'a' is at least $1 \}$

$$\text{a a a } \underbrace{\text{a a} \dots \text{a b}}_{\geq 1}$$



Determinization of EPA

→ same subset construction as done for ERAs.

Closure properties:

- same as ERA
- closed under union, intersection and complementation.

Event - clock Automata:

Example: $\{\underbrace{a\ a\ b}_{=1}\} \cup \{\underbrace{a\ b\ b}_{=1}\}$

Can we write an ERA? We can give an ERA for $\{\underbrace{a\ b\ b}_{=1}\}$

But we cannot give an ERA for $\{\underbrace{a\ a\ b}_{=1}\}$

Because an ERA cannot constrain the difference between the first 'a' and 'b' in aab.

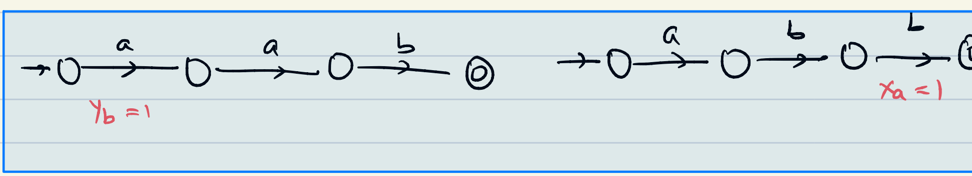
Similarly, we cannot give an EPA.

Because EPA cannot constrain the difference between a and last 'b' in $\{\underbrace{a\ b\ b}_{=1}\}$

Now we need use of both x-clocks and y-clocks. Such automata using both x and y-clocks will be called Event-Clock-Automata (ECA)

$\{\underbrace{a\ a\ b}_{=1}\} \cup \{\underbrace{a\ b\ b}_{=1}\}$

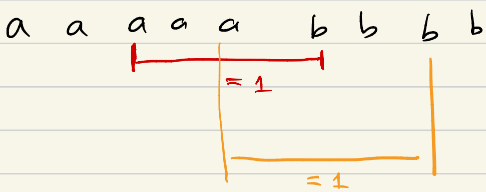
disjoint union

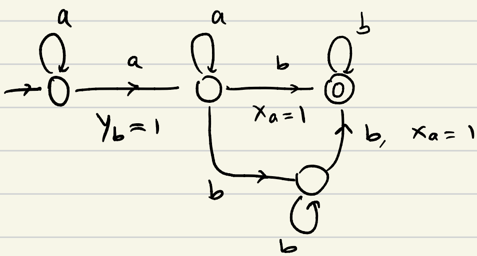


Examples:

$$k, m \geq 1$$

$\{ a^k b^m \mid \exists a$ which is at distance 1 from the first 'b'
and
$\exists b$ which is at distance 1 from the last 'a' $\}$



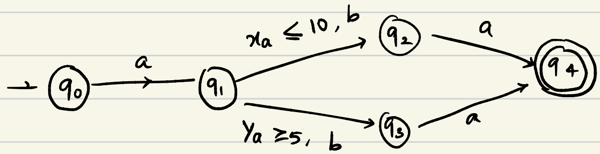


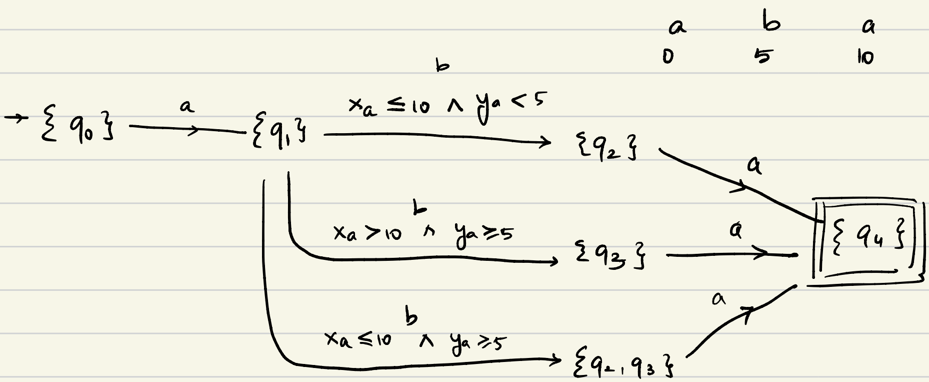
Determinizing ECA: Subset construction

Closure properties: Closed under union, intersection, complement.

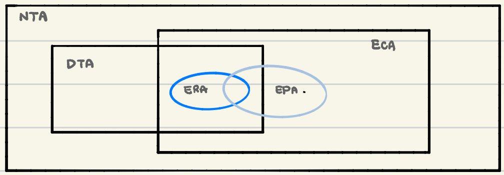
Examples:

- Determinize following ECA:





Expressive Power of different classes :



1) ERA $\subseteq$ DTA : conversion preserves determinism.

2) ERA $\not\subseteq$ EPA

3) EPA $\not\subseteq$ ERA

$\underbrace{a\ a\ b}_{=1}$ → EPA, but not ERA

$\underbrace{a\ b\ b}_{=1}$ → ERA, but not EPA

4) DTA $\not\subseteq$ EPA ($\exists$ L in DTA which is not EPA recog.)

$\underbrace{abb}_{=1}$

5) EPA $\not\subseteq$ DTA

$\exists$ a L which EPA but not by DTA

$\{ a^k b \mid k \geq 1,$ $\exists$ some 'a' which is at distance 1 from 'b' $\}$

(see next page)

5) DTA $\not\subseteq$ ECA ($\exists$ L in DTA which is not ECA recog.)

$\overbrace{a\ a\ a}^{=1}$

6) ECA $\not\subseteq$ DTA

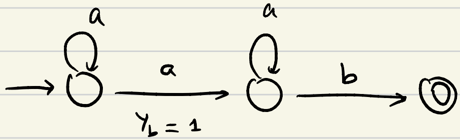
$\exists$ L in ECA which is not DTA recog.

7) ECA $\not\subseteq$ EPA $\cup$ ERA : $\{ a\underbrace{ab}_{\geq 1} \} \cup \{ \underbrace{ab}_{\geq 1}b \}$

8) ECA $\subseteq$ NTA : conversion algorithms as seen before

EPA $\not\subseteq$ DTA

$\{ a^k b \mid k \geq 1,$ there exists an 'a' which is at distance 1 from 'b' $\}$



There is no DTA for this language. Intuitively, we cannot guess the 'a' deterministically for which the b is at distance 1.

Exercise: Prove this formally.