

Unit-9: Computation Tree Logic

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NPTEL-course

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Module 3:

CTL

In this module...

Restrict to a **subset** of CTL* which has **efficient model-checking algorithms**

CTL*

State formulae

$$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid E\alpha \mid A\alpha$$

$p_i \in AP$ ϕ_1, ϕ_2 : State formulae α : Path formula

Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\alpha_1 \mid \alpha_1 U \alpha_2 \mid F\alpha_1 \mid G\alpha_1$$

ϕ : State formula α_1, α_2 : Path formulae

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CTL

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Legal CTL formulae

Illegal CTL formulae

State formulae

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Legal CTL formulae

$E F p_1$

Illegal CTL formulae

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Legal CTL formulae

$EF p_1$

$EFAG p_1$

Illegal CTL formulae

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Legal CTL formulae

$EF p_1$

$EFAG p_1$

$AX p_2$

Illegal CTL formulae

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$EF p_1$

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$AX p_2$

$AF p_1 \wedge AG p_2$

Illegal CTL formulae

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Illegal CTL formulae

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Ap_1

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Illegal CTL formulae

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$A p_1$

$EGF p_1$

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Legal CTL formulae

$EF p_1$

$EFA G p_1$

$AX p_2$

$AF p_1 \wedge AG p_2$

Illegal CTL formulae

$AFG p_1$

$A p_1$

$EGF p_1$

$A(F p_1 \wedge G p_2)$

State formulae

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Path formulae

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Legal CTL formulae

$$EF p_1$$
$$EFA G p_1$$
$$AX p_2$$
$$AF p_1 \wedge AG p_2$$
$$A(p_1 U (EG p_2))$$

Illegal CTL formulae

$$AFG p_1$$
$$A p_1$$
$$EGF p_1$$
$$A(F p_1 \wedge G p_2)$$

State formulae

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$$EFA G p_1$$
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Illegal CTL formulae

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$$A(p_1 U (G p_2))$$

Every temporal operator X, U, F, G has a corresponding A or E

CTL

Syntax: Restricted form of CTL*

Semantics: Same as seen in CTL*

Example

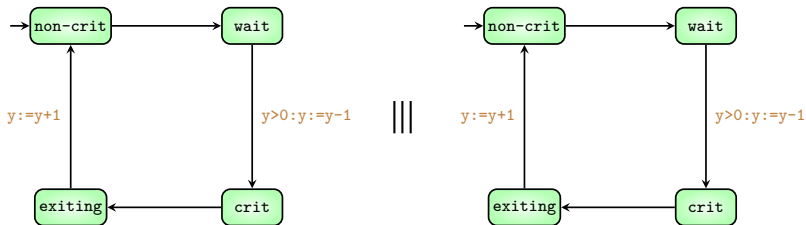
Atomic propositions $AP = \{p_1, p_2, p_3, p_4\}$

p_1 : pr1.location=crit

p_2 : pr1.location=wait

p_3 : pr2.location=crit

p_4 : pr2.location=wait



Mutual exclusion: $\mathbf{A G} \neg (p_1 \wedge p_3)$

Can LTL properties be written using CTL?

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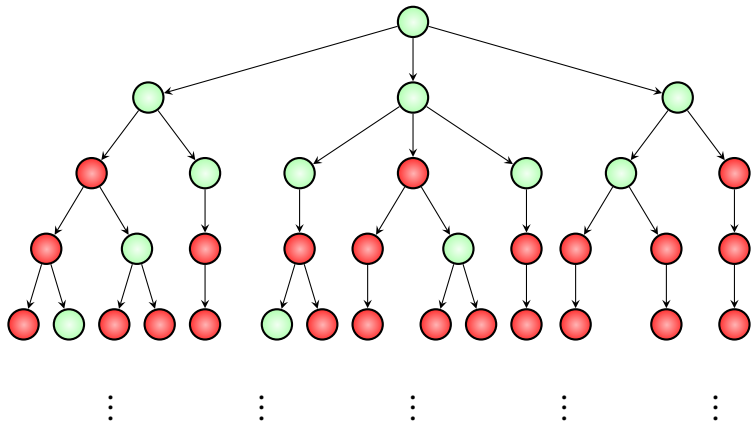
Answer: No

Can LTL properties be written using CTL?

Answer: No

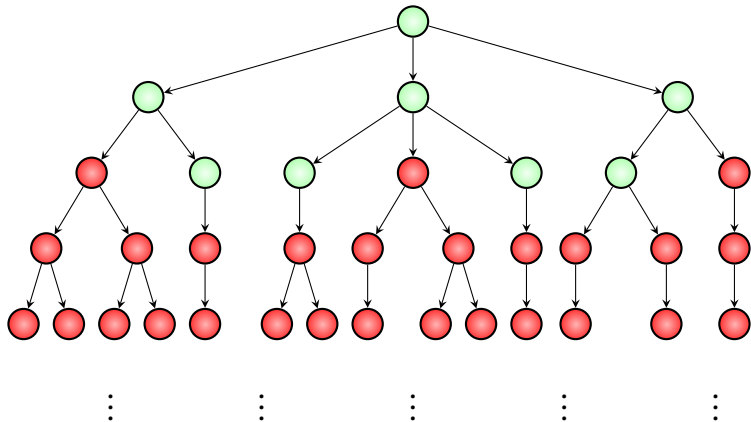
Property **A F G** p_1 cannot be expressed in CTL

A F G (*red*)

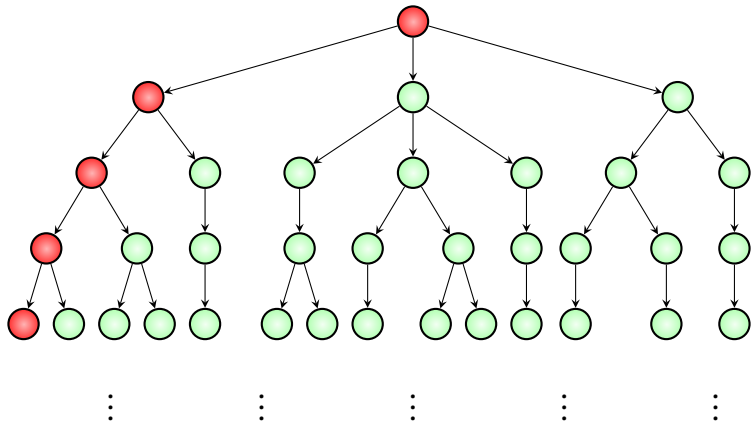


In all paths, eventually *red* is true forever

A F A G (*red*)



A F E G (*red*)



Can LTL properties be written using CTL?

Answer: No

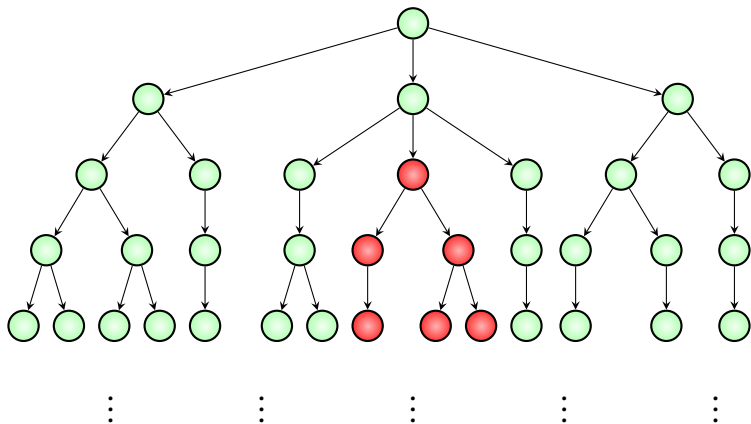
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Can CTL properties be written using LTL?

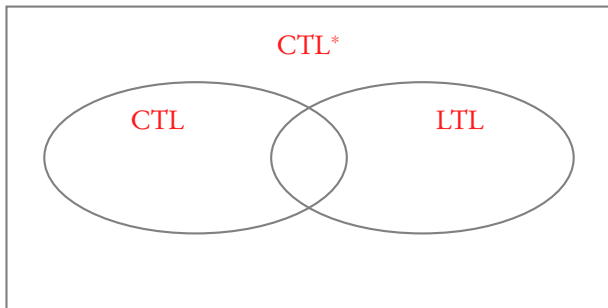
Can CTL properties be written using LTL?

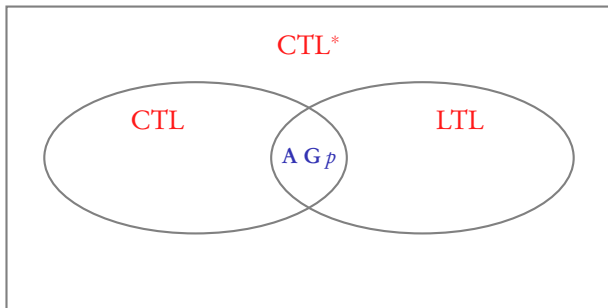
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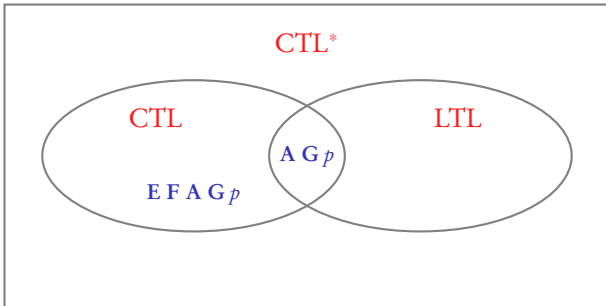
E F A G (*red*)

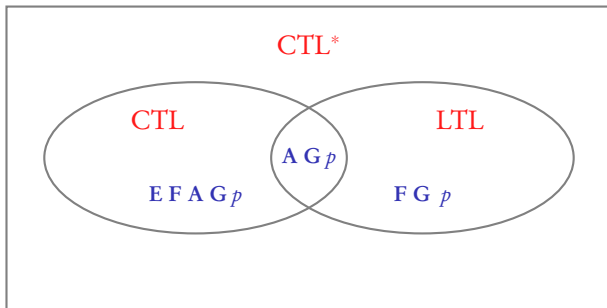


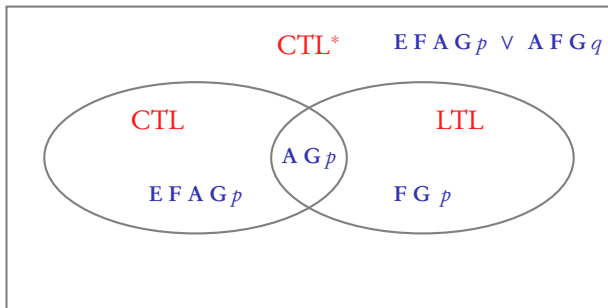
Cannot be expressed in LTL











Summary

CTL

Subset of CTL*

Paired temporal and A-E operators

Expressive powers