

# Unit-9: Computation Tree Logic

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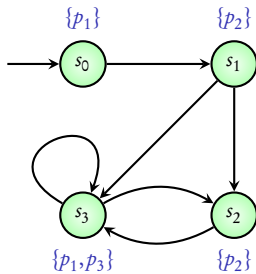
*NPTEL-course*

July - November 2015

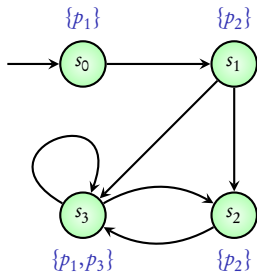
## Module 1:

# Tree behaviour of a transition system

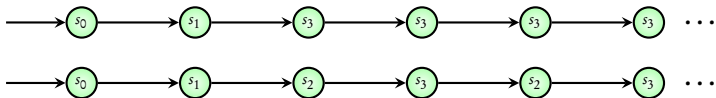
## Transition System



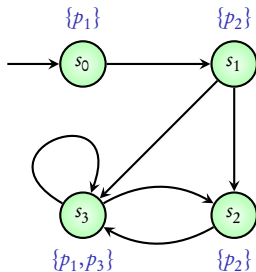
## Transition System



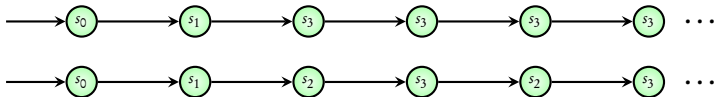
## Paths



## Transition System



## Paths



## Traces

$\{p_1\} \{p_2\} \{p_1, p_3\} \{p_1, p_3\} \{p_1, p_3\} \{p_1, p_3\} \dots$   
 $\{p_1\} \{p_2\} \{p_2\} \{p_1, p_3\} \{p_2\} \{p_1, p_3\} \{p_2\} \{p_1, p_3\} \dots$

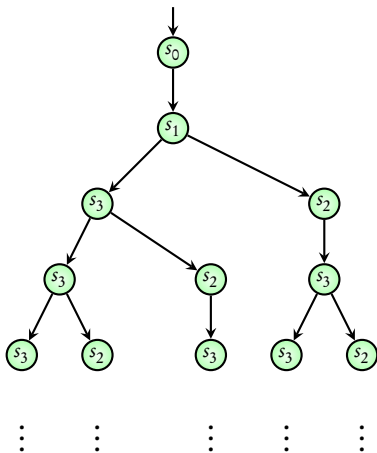
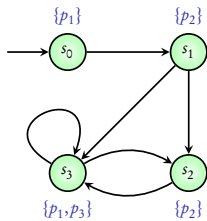
# In this unit

A **tree view** of the transition system ...

# In this unit

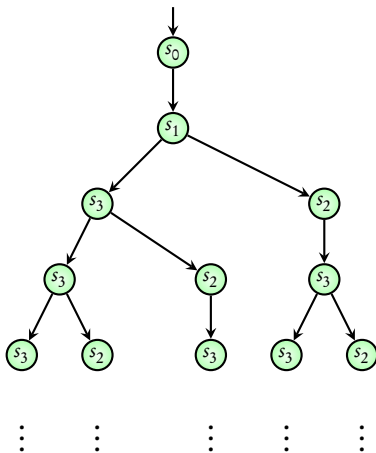
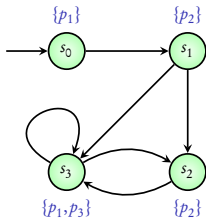
A **tree view** of the transition system ...

... obtained by repeatedly **unfolding** it





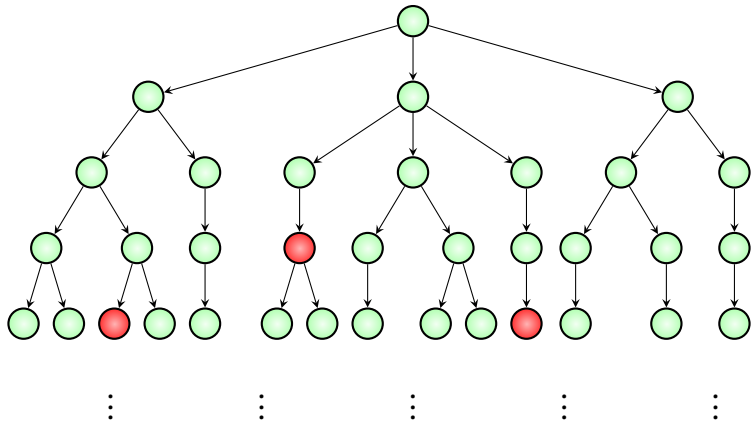
## Computation tree



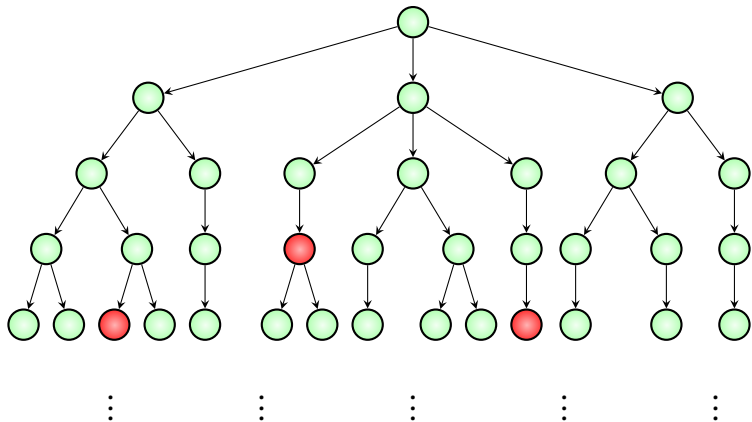
**LTL** talks about **properties of paths**

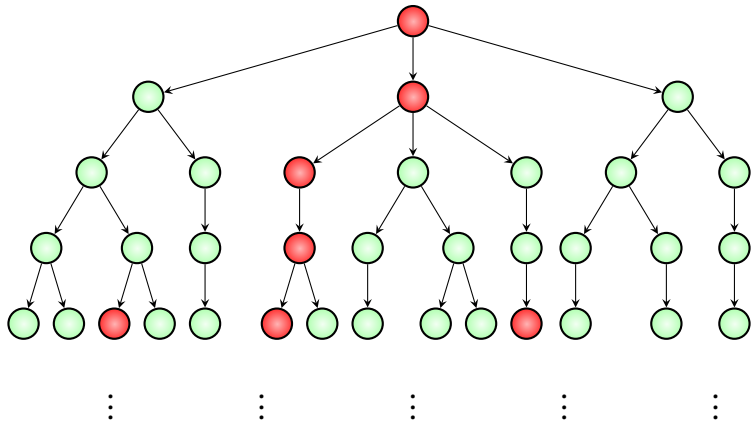
**LTL** talks about **properties of paths**

**Coming next:** Properties of **trees**

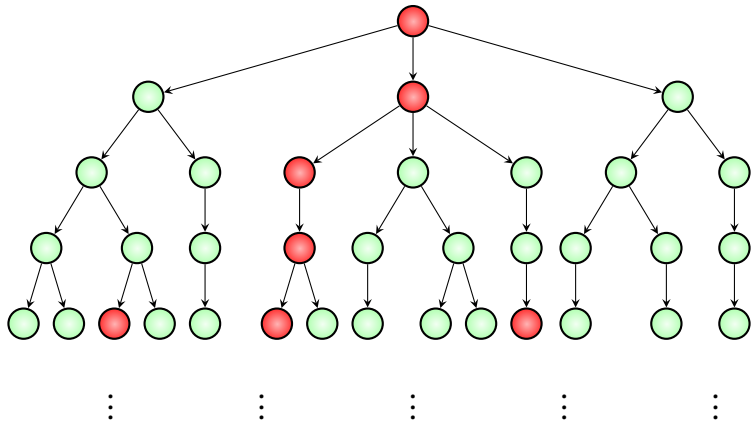


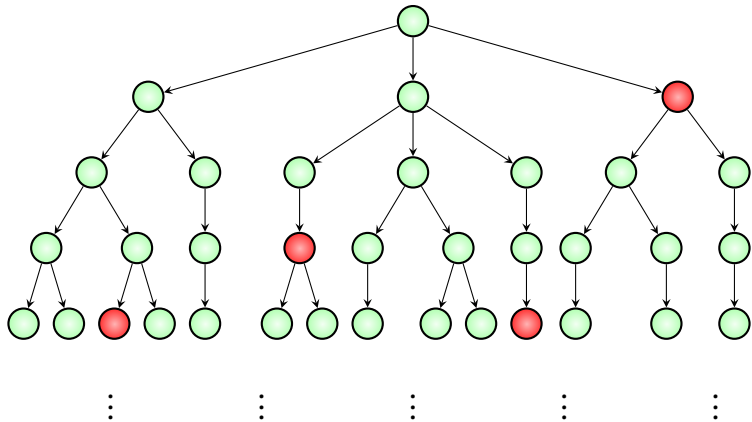
Exists a path satisfying  $F(\text{red})$





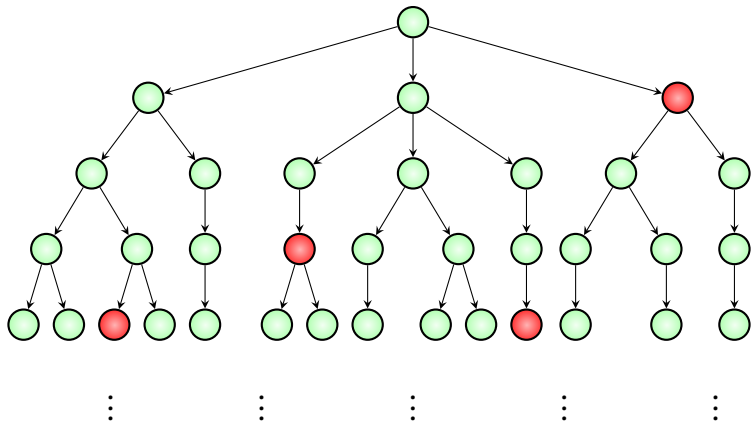
Exists a path satisfying  $G(\text{red})$

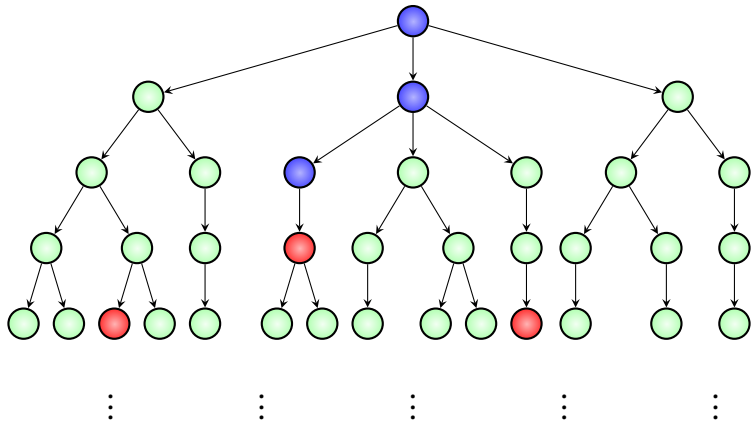




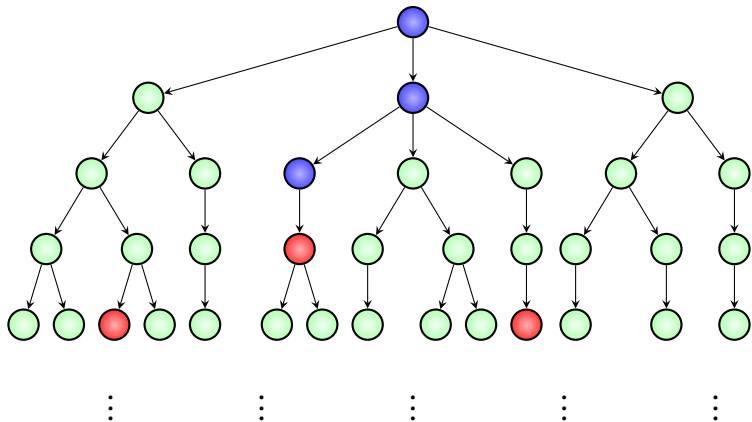


Exists a path satisfying  $X(\text{red})$





Exists a path satisfying *blue* U *red*



# Properties of trees

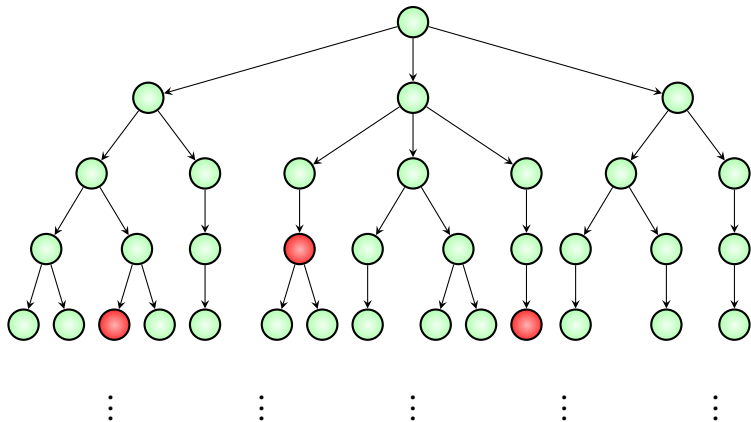
**Type 1:** Exists a path satisfying LTL formula  $\phi$

# Properties of trees

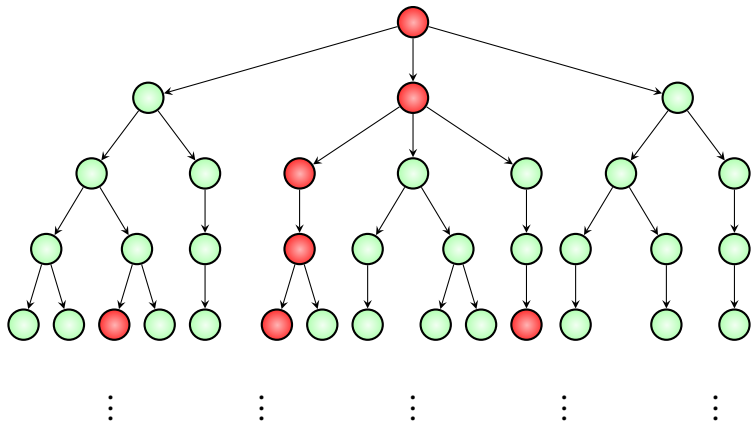
Type 1: Exists a path satisfying LTL formula  $\phi$

E operator:  $E \phi$

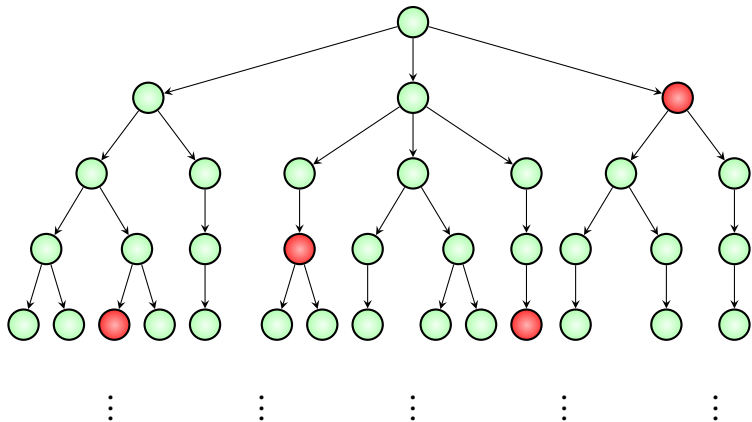
Exists a path satisfying  $F(\text{red})$ :  $E F(\text{red})$



Exists a path satisfying  $G(\text{red})$ :  $\exists G(\text{red})$

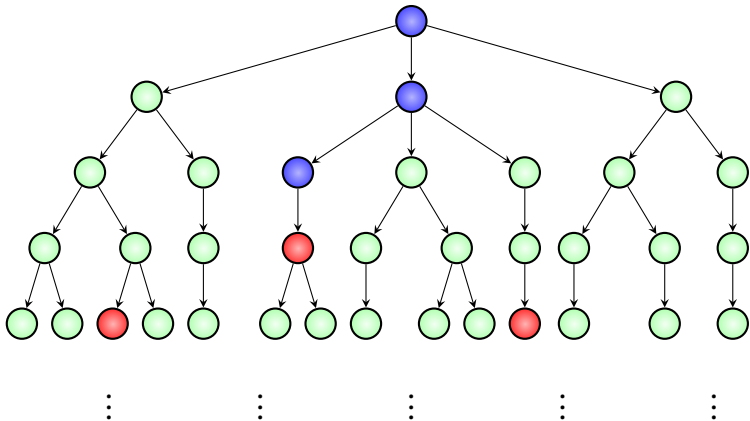


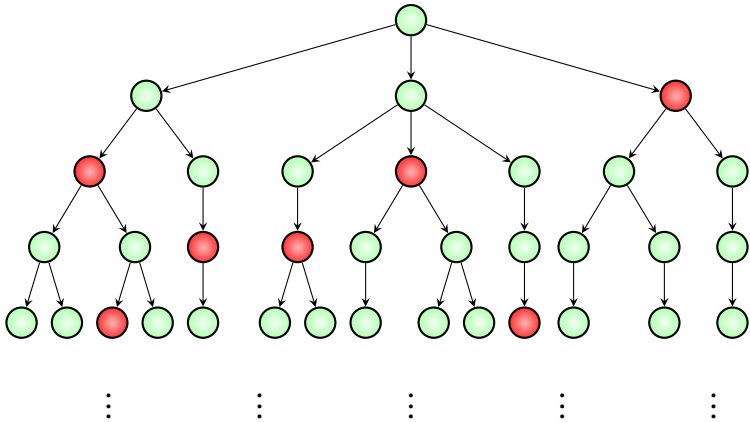
Exists a path satisfying  $X(\text{red})$ :  $E X(\text{red})$



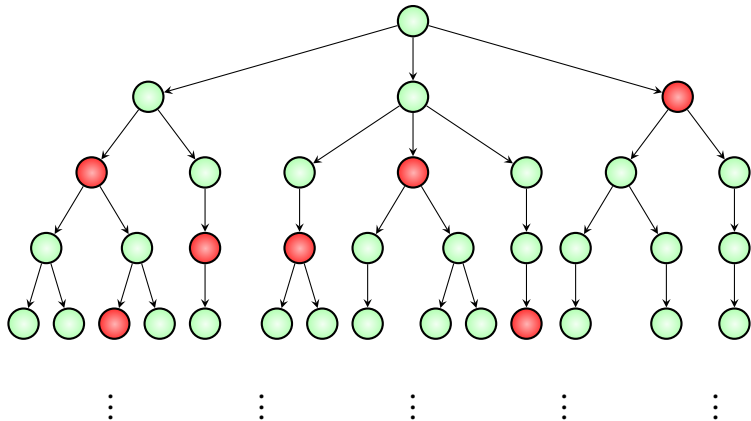


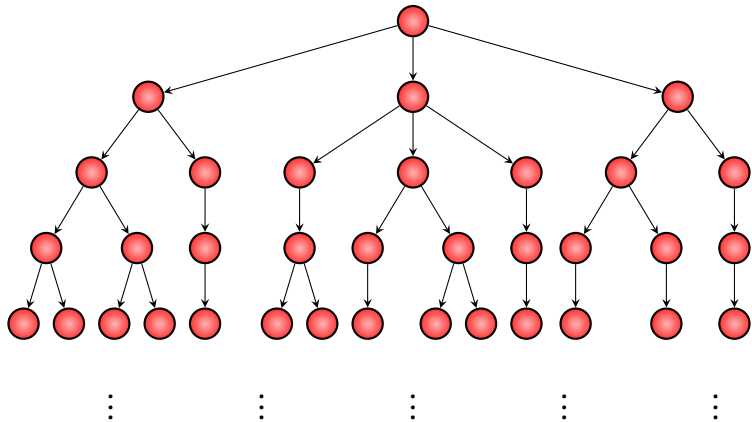
Exists a path satisfying  $blue \cup red$  :  $E ( blue \cup red )$



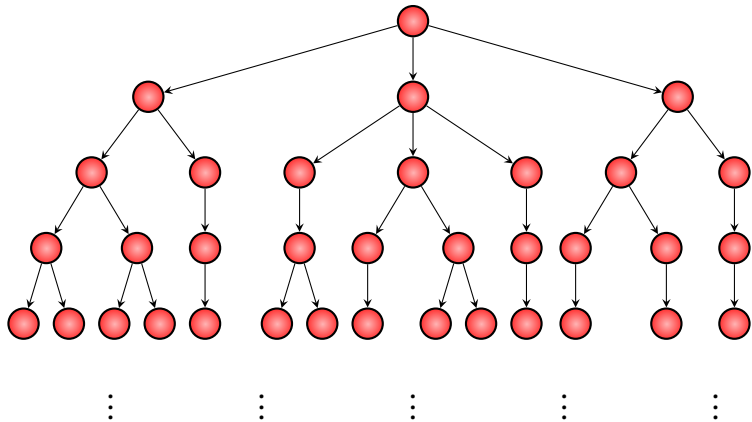


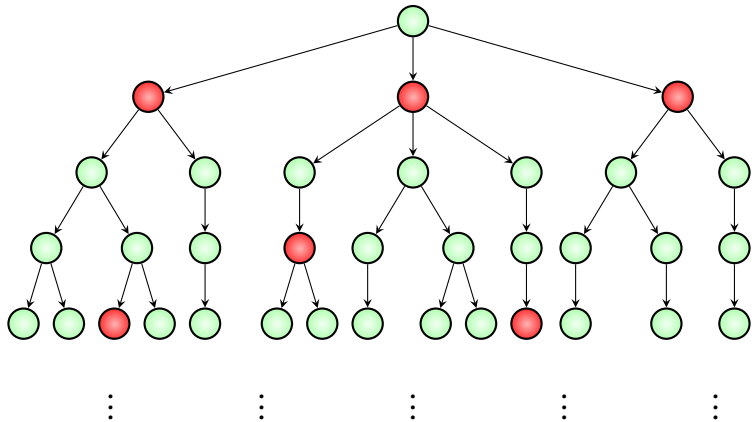
All paths satisfy  $F(\text{red})$



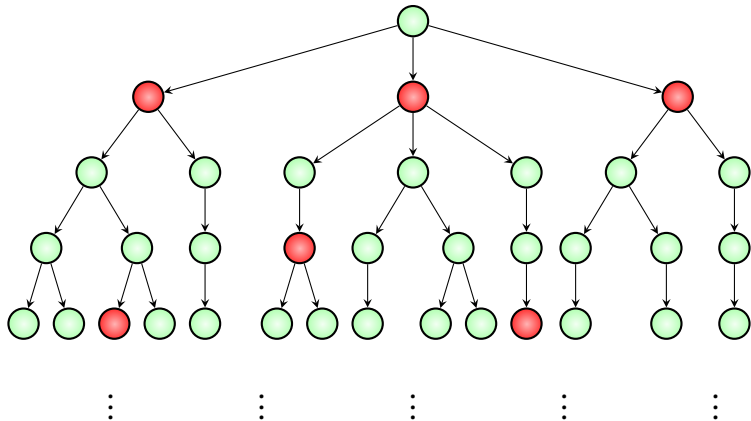


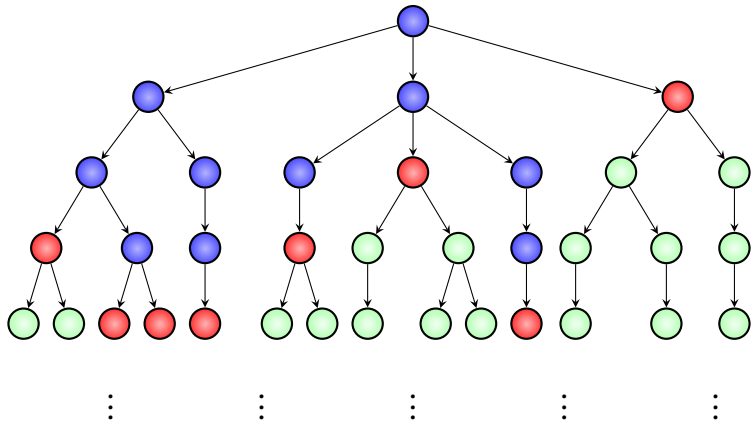
All paths satisfy  $G(\text{red})$





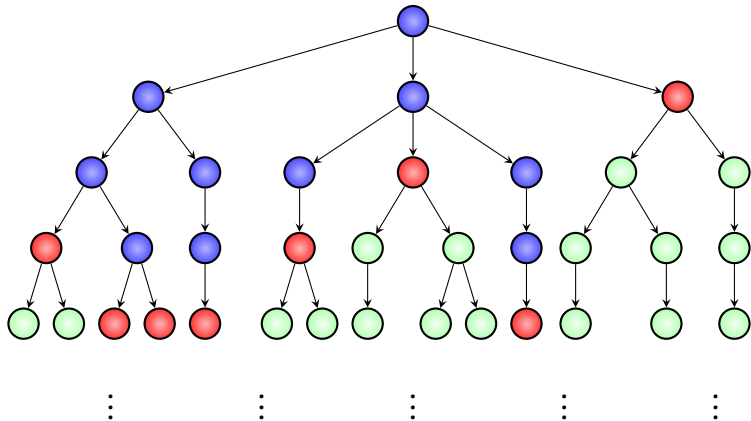
All paths satisfy  $X(\text{red})$







All paths satisfy *blue* U *red*



# Properties of trees

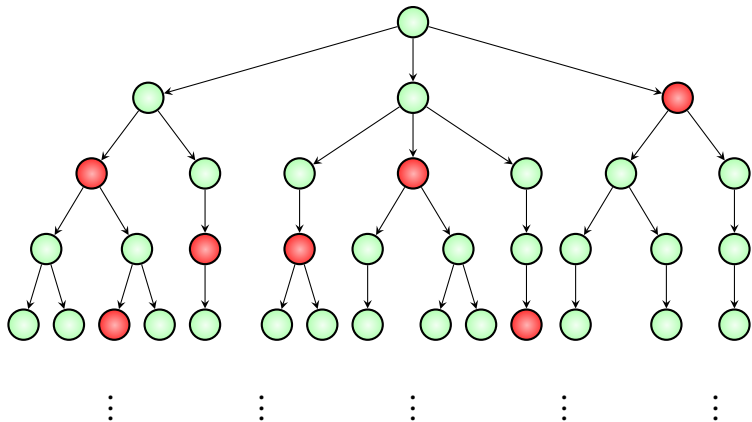
Type 2: All paths satisfy LTL formula  $\phi$

# Properties of trees

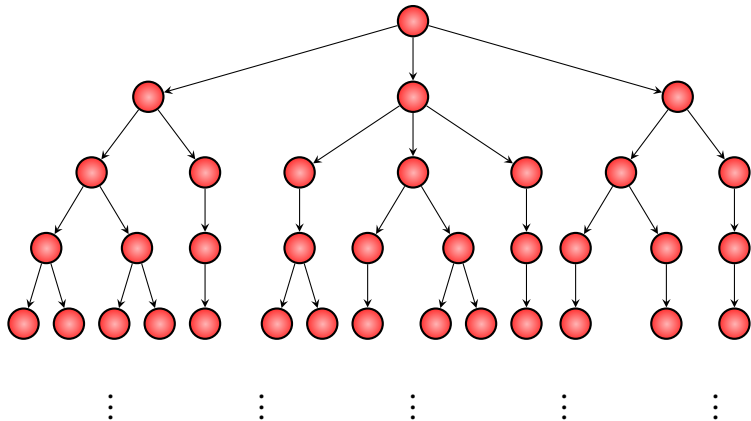
Type 2: All paths satisfy LTL formula  $\phi$

A operator:  $\mathbf{A} \phi$

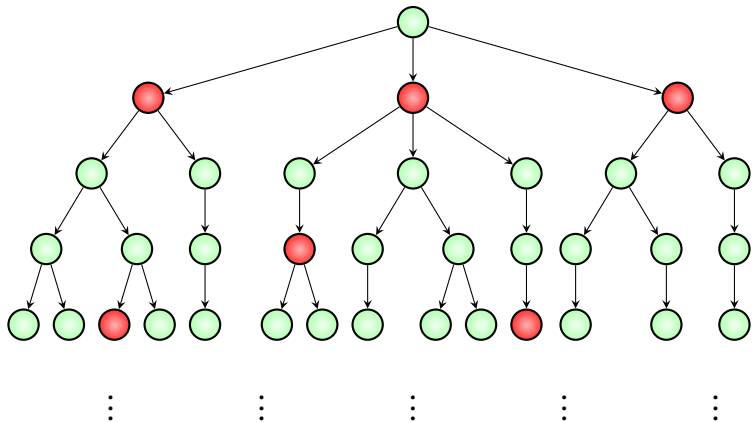
All paths satisfy  $F(\text{red})$ :  $\mathbf{A} F(\text{red})$



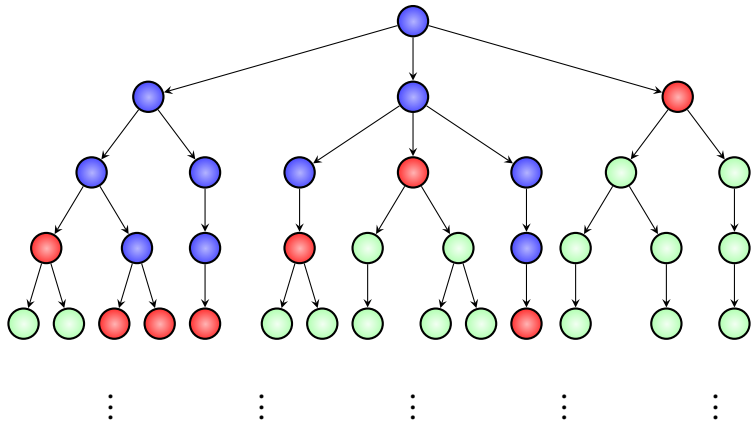
All paths satisfy  $G(\text{red})$ :  $\mathbf{A} G(\text{red})$



All paths satisfy  $X(\text{red})$ :  $\forall X(\text{red})$



All paths satisfy *blue* U *red* :  $A \text{ blue U red}$



# Properties of trees

- ▶ Exists a path satisfying **path property**  $\phi$  :  $\mathbf{E} \phi$
- ▶ All paths satisfy **path property**  $\phi$  :  $\mathbf{A} \phi$



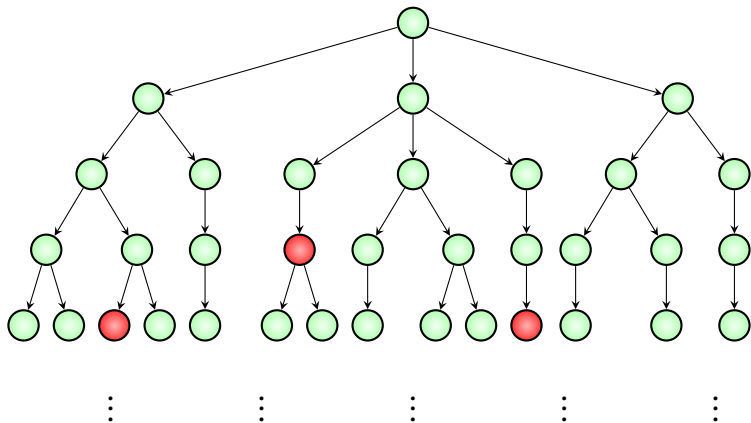
# Properties of trees

- ▶ Exists a path satisfying path property  $\phi$  :  $\exists \phi$
- ▶ All paths satisfy path property  $\phi$  :  $\forall \phi$

Coming next: Mixing  $\forall$  and  $\exists$

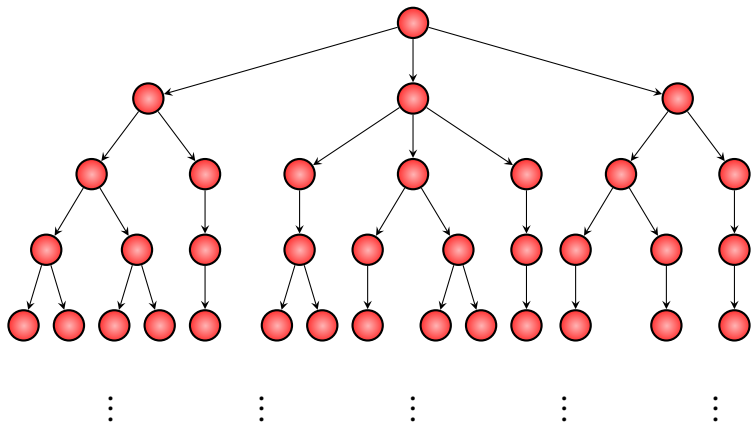
# Recall...

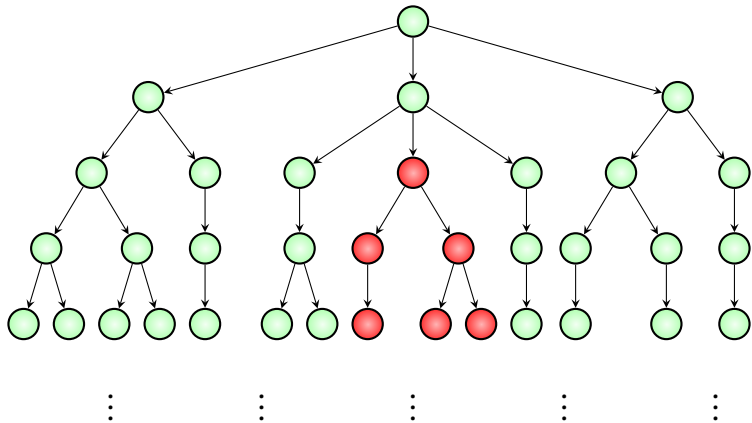
Exists a path satisfying  $F(\text{red})$ :  $E F(\text{red})$



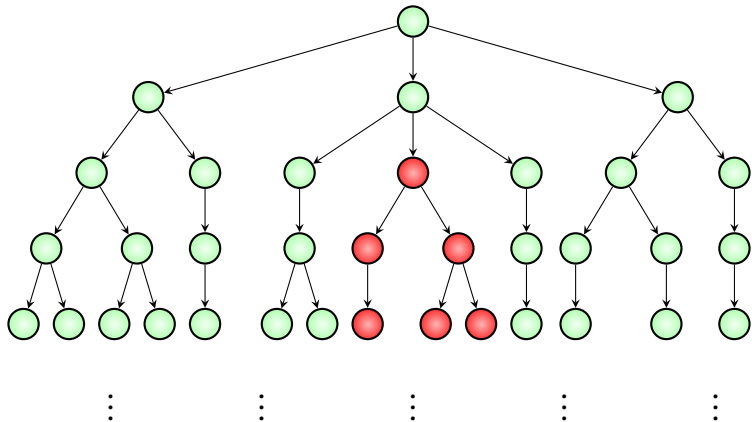
# Recall...

All paths satisfy  $G(\text{red})$ :  $\mathbf{A} G(\text{red})$

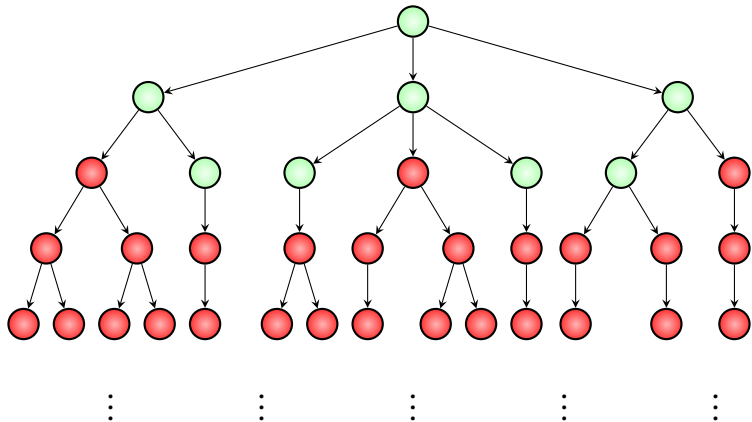




# E F A G (*red*)

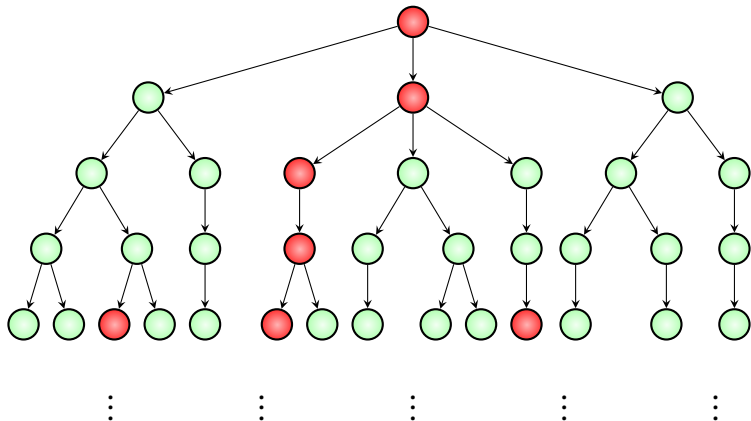


# A F A G (*red*)



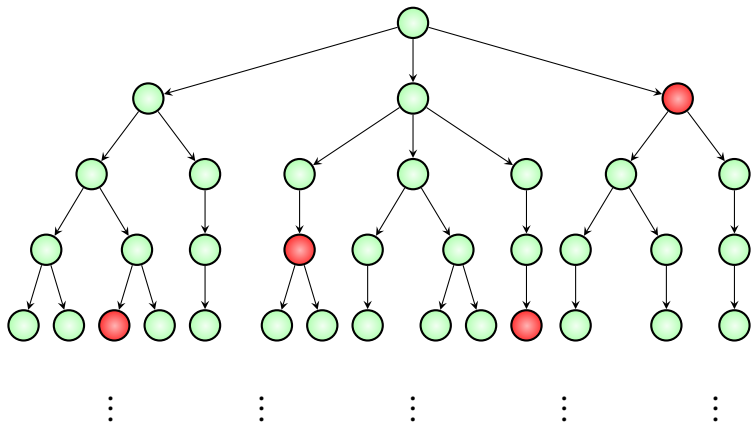
# Recall...

Exists a path satisfying  $G(\text{red})$ :  $\exists G(\text{red})$



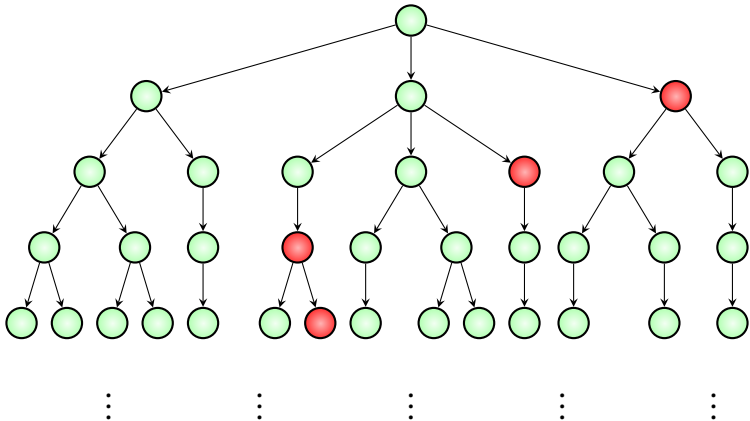
# Recall...

Exists a path satisfying  $X(\text{red})$ :  $\exists X(\text{red})$



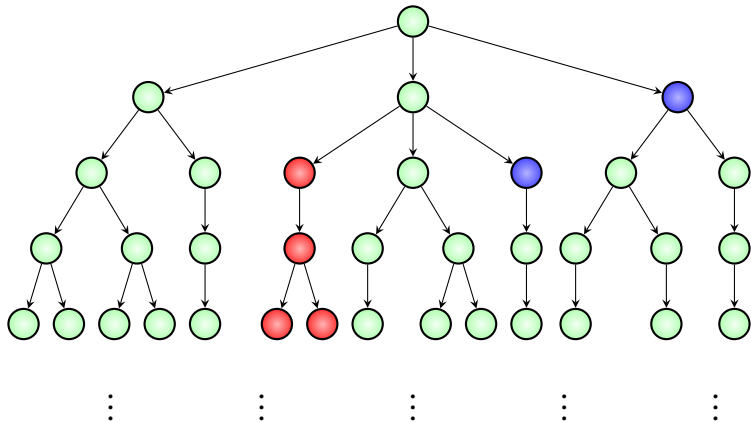


# EGEX (*red*)





E (E X *blue*) U (A G *red*)



# Summary

## Transition system as a tree

Computation tree

E and A operators