Unit-8: Algorithms for LTL

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NPTEL-course

July - November 2015

Module 2:

LTL to NBA

Goal: Understand the **evaluation** of an LTL formula on an infinite word

$p_1 \ \mathbf{U} \ p_2$

$p_1 \; \mathbf{U} \; p_2$

$\{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_2\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1, p_2\} \cdots$

$$p_1 \ \mathbf{U} \ p_2$$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\cdots$
p_1										
p_2										
$p_1 \; U \; p_2$										

$$p_1 \ \mathbf{U} \ p_2$$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1									
p_2									
$p_1 \ U \ p_2$									

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$									

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1

p_1									
p_2									
$p_1 \ U \ p_2$									

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1										
p_2										
$p_1 \ U \ p_2$										

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1\ U\ p_2$	1	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$ $\cdots$
p_1	1	0	1	0	1	0	1	0	1
p_2	0	0	0	0	0	0	0	0	0
$p_1\ U\ p_2$									

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1\ U\ p_2$	1	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$ $\cdots$
p_1	1	0	1	0	1	0	1	0	1
p_2	0	0	0	0	0	0	0	0	0
$p_1\ U\ p_2$	0	0	0	0	0	0	0	0	0

G F p_1

$\mathbf{G\ F}\ p_1$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\, \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\,\mathbf{U}\,\neg(\mathit{true}\,\mathbf{U}\,p_1)$$

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\;\mathrm{U}\;\neg\phi$

$$\neg\;\mathit{true}\;\mathbf{U}\;\neg(\mathit{true}\;\mathbf{U}\;p_1)$$

$\{\,\}$ $\{\,\}$ $\{p_1\}$ $\{\,\}$ $\{\,\}$ $\{p_1\}$ $\{\,\}$ $\{\,\}$ $\{p_1\}$

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1									
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = true\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ true\ \mathrm{U}\ \neg\phi$

$$\neg\ true\ \mathbf{U}\ \neg(true\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
$true$	1	1	1	1	1	1	1	1	1
$true\ \mathrm{U}\ p_1$									
$\neg\ true\ \mathrm{U}\ p_1$									
$true\ \mathrm{U}\ \neg\ (true\ \mathrm{U}\ p_1)$									
$\neg\ true\ \mathrm{U}\ \neg\ (true\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G\,F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1

G F p_1

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\, \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\,\mathbf{U}\,\neg(\mathit{true}\,\mathbf{U}\,p_1)$$

$\{p_1\}$ $\{p_1\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$

$\mathbf{G}\,\mathbf{F}\,p_1$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\;\mathrm{U}\;\neg\phi$

$$\neg\;\mathit{true}\;\mathbf{U}\;\neg(\mathit{true}\;\mathbf{U}\;p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1									
true									
$\mathit{true}\;\mathrm{U}\;p_1$									
$\neg\,\mathit{true}\;\mathrm{U}\;p_1$									
$\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									
$\neg\;\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
p_1	1	1	0	0	0	0	0	0	0
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\,\mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\, \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\, \mathbf{U}\, p_1))$$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

$\{\}$ $\{p_2\}$ $\{\}$ $\{\}$ $\{p_1\}$ $\{p_1, p_2\}$ $\{p_1, p_2\}$ $\cdots$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\dots$
p_1								
p_2								
$\neg p_1$								
$\neg p_2$								
$\neg p_2\;\mathrm{U}\;p_1$								
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	…
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$								
$\neg p_2$								
$\neg p_2\;\mathrm{U}\;p_1$								
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\dots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$								
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\mathit{true}\ \mathrm{U}\,(\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \;\mathrm{U}\; p_1$					1	1	1	
$\mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$			1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\dots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0							
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1						
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1					
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1				
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1			
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	1	1		
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\mathit{true}\ \mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \;\mathrm{U}\; p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0							
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\dots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1						
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1					
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1				
$\mathit{true}\ \mathrm{U}\,(\neg p_1 \wedge \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	1	1	0	
$\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	0			
$\mathit{true}\ \mathrm{U}\,(\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0		
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$	1	1	1	1				

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$	1	1	1	1	0	0	0	

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
p_1 U p_2	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
p_1 U p_2	0	0	0	0	0	0	0	0	0	

true **U** $(\neg p_1 \wedge$ **X** $(\neg p_2$ **U** $p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2$ U p_1	0	0	1	1	1	1	1	
X $(\neg p_2$ U $p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge$ X $(\neg p_2$ U $p_1)$	0	1	1	1	0	0	0	
true U $(\neg p_1 \wedge$ X $(\neg p_2$ U $p_1))$	1	1	1	1	1	1	0	

$\neg$ *true* **U** $\neg$(*true* **U** p_1)

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
true U p_1	1	1	1	1	1	1	1	1
$\neg$ *true* U p_1	0	0	0	0	0	0	0	0
true U $\neg$ (*true* U p_1)	0	0	0	0	0	0	0	0
$\neg$ *true* U $\neg$ (*true* U p_1)	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
true U p_1	1	1	0	0	0	0	0	0	0
$\neg$ *true* U p_1	0	0	1	1	1	1	1	1	1
true U $\neg$ (*true* U p_1)	1	1	1	1	1	1	1	1	1
$\neg$ *true* U $\neg$ (*true* U p_1)	0	0	0	0	0	0	0	0	0

Formula expansions

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \mathrm{U} p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \mathrm{U} p_2$	0	0	0	0	0	0	0	0	0	

$\mathit{true}\ \mathbf{U}\ (\neg p_1 \ \wedge \ \mathbf{X}\,(\neg\, p_2 \ \mathbf{U}\ p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \mathrm{U} p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \mathrm{U} p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X}\,(\neg p_2 \mathrm{U} p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\ \mathrm{U}\,(\neg p_1 \wedge \mathrm{X}\,(\neg p_2 \mathrm{U} p_1))$	1	1	1	1	1	1	0	

$\neg \ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1

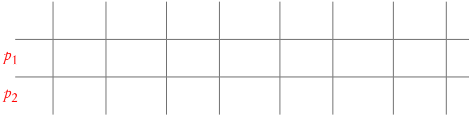
	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0

Key idea: Construct automata whose **states are columns of the formula expansion**

Key idea: Construct automata whose **states are columns of the formula expansion**

Next in this module: **understand properties** of formula expansions

Word compatibility



Word compatibility

	$\{\}$							
p_1	0							
p_2	0							

Word compatibility

	$\{\}$		$\{p_1\}$					
p_1	0		1					
p_2	0		0					

Word compatibility

	$\{\,\}$		$\{p_1\}$		$\{p_2\}$			
p_1	0		1		0			
p_2	0		0		1			

Word compatibility

	$\{\ \}$	$\{p_1\}$	$\{p_2\}$	$\{p_1, p_2\}$
p_1	0	1	0	1
p_2	0	0	1	1

AND-NOT-compatibility

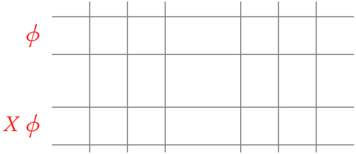
ϕ		0		1	
$\neg\phi$		1		0	

AND-NOT-compatibility

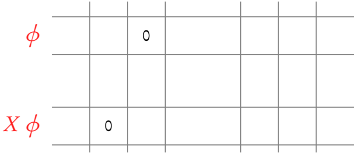
ϕ		0		1	
$\neg\phi$		1		0	

ϕ_1	1		0		1		0	
ϕ_2	1		1		0		0	
$\phi_1 \wedge \phi_2$	1		0		0		0	

X-compatibility



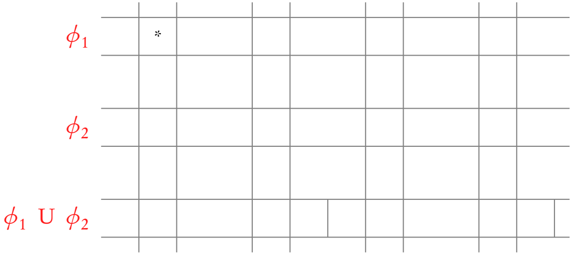
X-compatibility



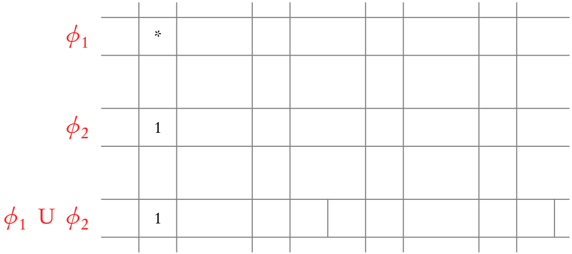
X-compatibility

ϕ			0			1	
$X\,\phi$		0			1		

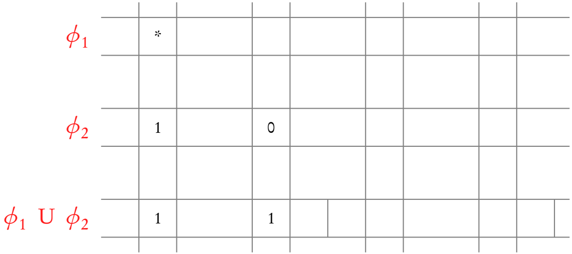
Until-compatibility



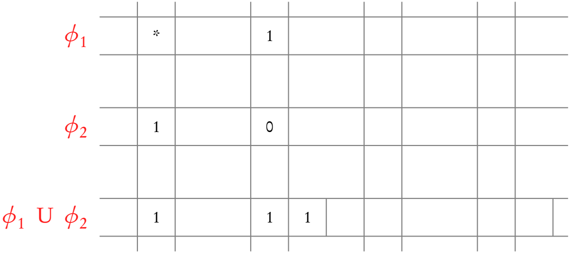
Until-compatibility



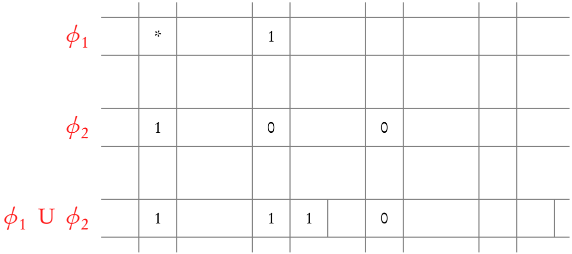
Until-compatibility



Until-compatibility



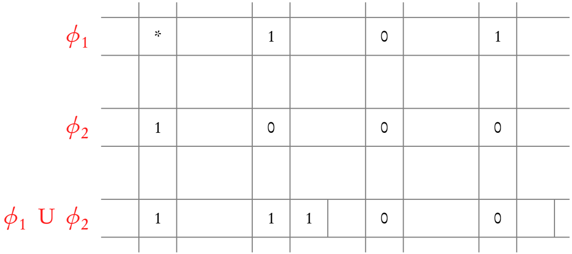
Until-compatibility



Until-compatibility

ϕ_1	*		1			0			
ϕ_2	1		0			0			
$\phi_1 \ \mathsf{U} \ \phi_2$	1		1	1		0			

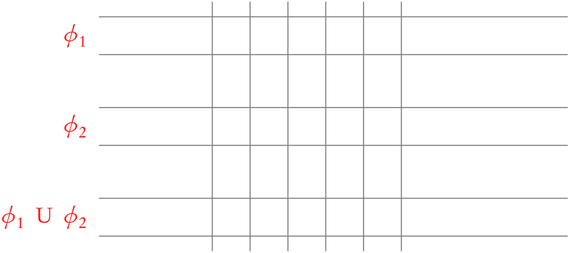
Until-compatibility



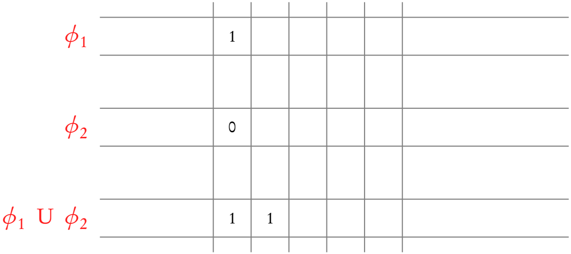
Until-compatibility

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

Until-compatibility: eventuality condition



Until-compatibility: eventuality condition



Until-compatibility: eventuality condition

ϕ_1		1	1				
ϕ_2		0	0				
$\phi_1 \ \mathrm{U} \ \phi_2$		1	1	1			

Until-compatibility: eventuality condition

ϕ_1		1	1	1			
ϕ_2		0	0	0			
$\phi_1 \ \mathrm{U} \ \phi_2$		1	1	1	1		

Until-compatibility: eventuality condition

ϕ_1	1	1	1	1	
ϕ_2	0	0	0	0	
$\phi_1 \ \mathsf{U} \ \phi_2$	1	1	1	1	1

Until-compatibility: eventuality condition

ϕ_1		1	1	1	1	1	
ϕ_2		0	0	0	0	0	$\cdots$
$\phi_1 \; \mathrm{U} \; \phi_2$		1	1	1	1	1	

Until-compatibility: eventuality condition

ϕ_1	1	1	1	1	1	
ϕ_2	0	0	0	0	0	$\cdots$
ϕ_1 U ϕ_2	1	1	1	1	1	

Cannot happen forever that $\phi_1 \mathrm{U} \phi_2 = 1$, $\phi_1 = 1$ but $\phi_2 = 0$

Accepting expansions

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \mathrm{U}\, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \mathrm{U}\, p_2$	0	0	0	0	0	0	0	0	0	

true **U** $(\neg p_1 \wedge$ **X** $(\neg p_2$ **U** $p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \mathrm{U}\, p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\, (\neg p_2 \mathrm{U}\, p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X}\, (\neg p_2 \mathrm{U}\, p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\ \mathrm{U}\, (\neg p_1 \wedge \mathrm{X}\, (\neg p_2 \mathrm{U}\, p_1))$	1	1	1	1	1	1	0	

$\neg$ *true* **U** $\neg($*true* **U** $p_1)$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\, p_1$	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\, p_1$	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U} \neg (\mathit{true}\ \mathrm{U}\, p_1)$	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U} \neg (\mathit{true}\ \mathrm{U}\, p_1)$	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\, p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\, p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U} \neg (\mathit{true}\ \mathrm{U}\, p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U} \neg (\mathit{true}\ \mathrm{U}\, p_1)$	0	0	0	0	0	0	0	0	0

Entry in **first column** of **last row** (corresponding to final formula) is **1**

Summary

LTL to NBA

Formula expansions

Properties

Columns as states of NBA