

# Unit-7: Linear Temporal Logic

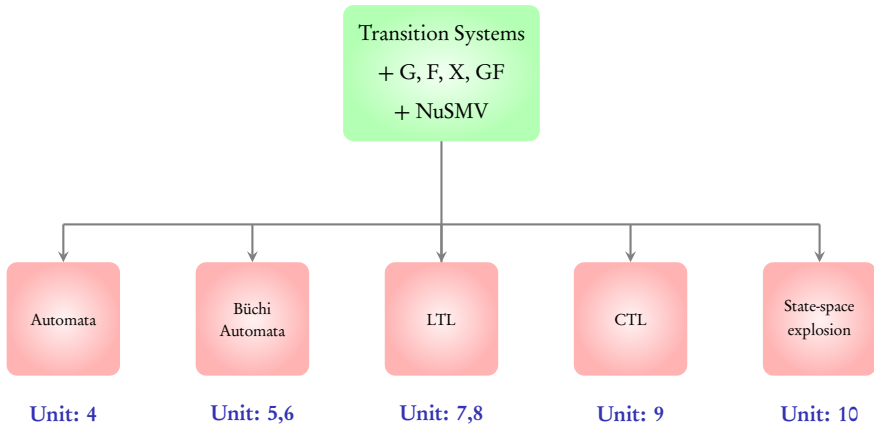
B. Srivathsan

Chennai Mathematical Institute

*NPTEL-course*

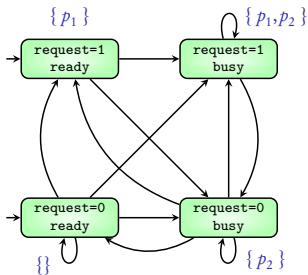
July - November 2015

Module 1:  
**Introduction to LTL**



$$AP = \{ p_1, p_2 \}$$

## Transition System



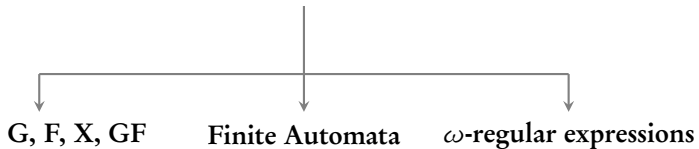
## Property

$P$

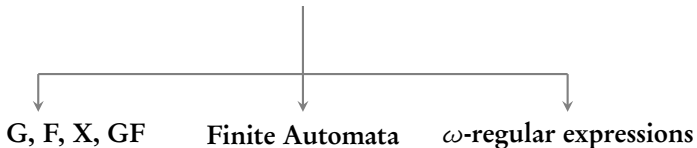
Transition system  $TS$  satisfies property  $P$  if

$$\text{Traces}(TS) \subseteq P$$

## Specifying properties



## Specifying properties



**Here:** Another formalism - **Linear Temporal Logic**

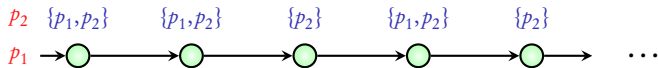


$\phi :=$



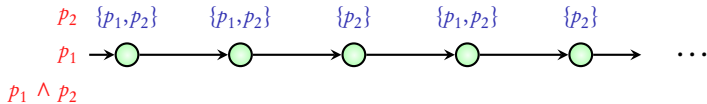
$\phi := \text{true} \mid$





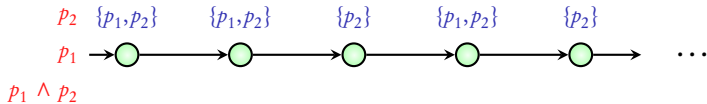
$$\phi := \text{true} \mid p_i \mid$$

$$p_i \in AP$$



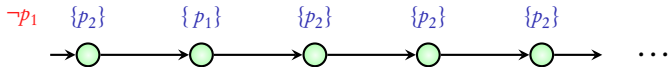
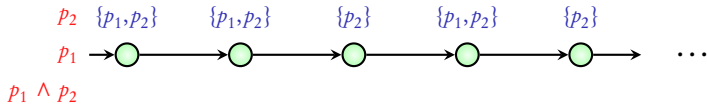
$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid$

$p_i \in AP$        $\phi_1, \phi_2$ : LTL formulas



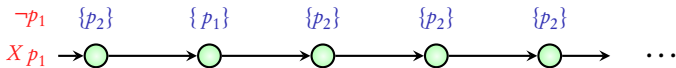
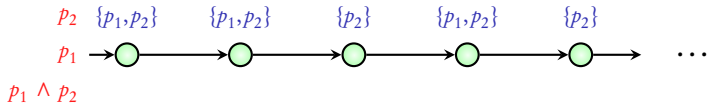
$$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg \phi_1 \mid$$

$$p_i \in AP \quad \phi_1, \phi_2 : \text{LTL formulas}$$



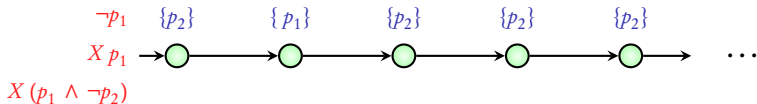
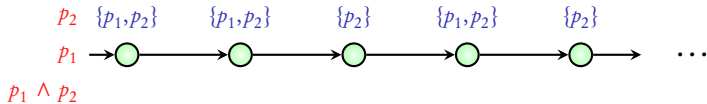
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$p_i \in AP$        $\phi_1, \phi_2$ : LTL formulas



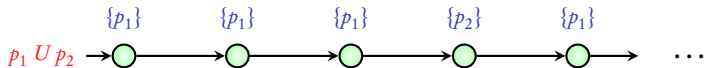
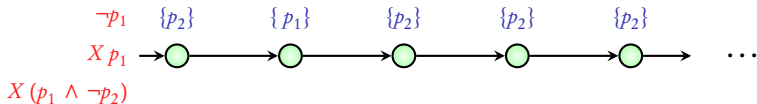
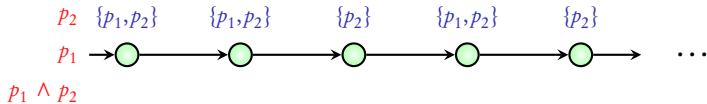
$$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg \phi_1 \mid X \phi \mid$$

$p_i \in AP$        $\phi_1, \phi_2$ : LTL formulas



$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg \phi_1 \mid X \phi \mid$

$p_i \in AP$        $\phi_1, \phi_2$ : LTL formulas



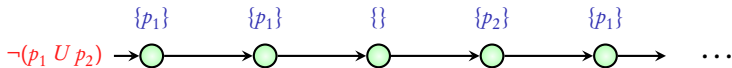
$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg \phi_1 \mid X \phi \mid \phi_1 U \phi_2$

$p_i \in AP$        $\phi_1, \phi_2$  : LTL formulas

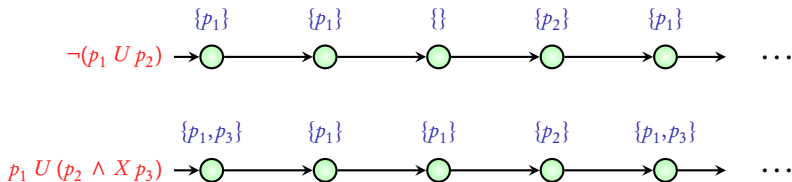
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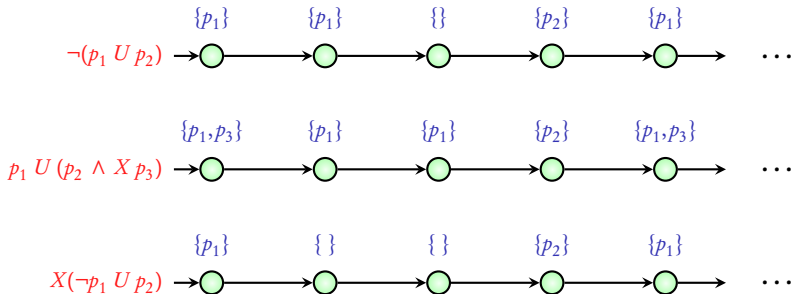
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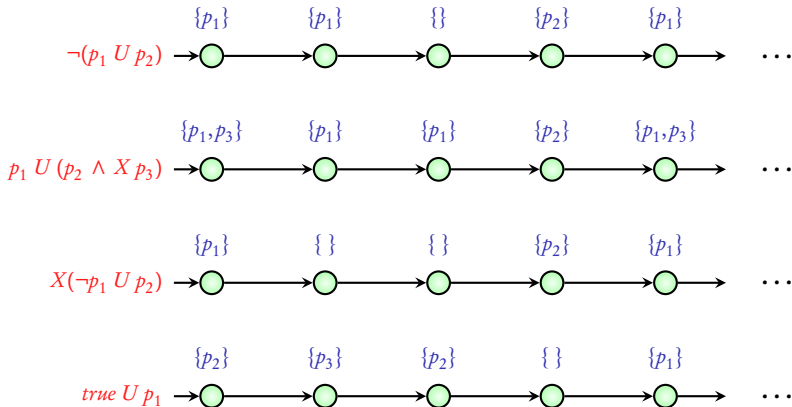
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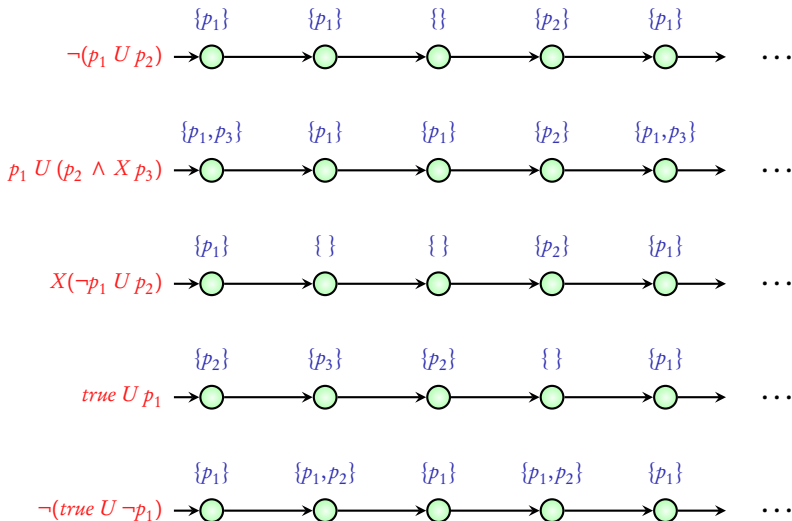
$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid X\phi \mid \phi_1 U \phi_2$



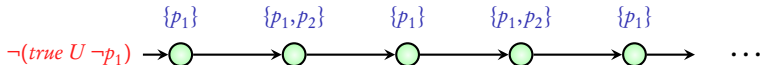
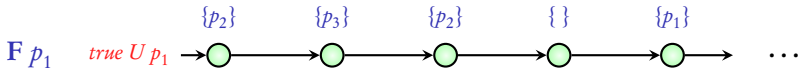
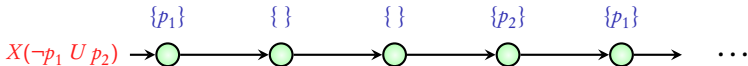
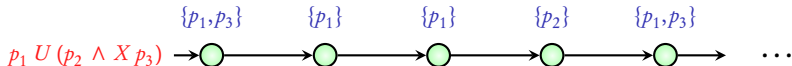
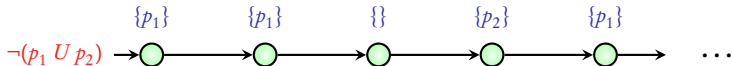
$$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid X\phi \mid \phi_1 U \phi_2$$



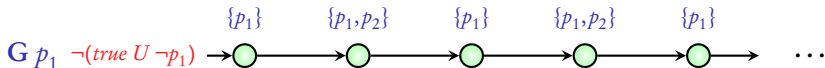
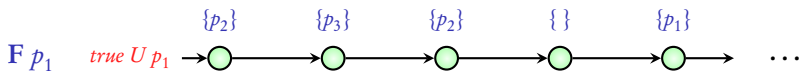
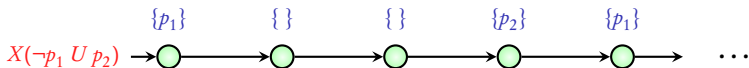
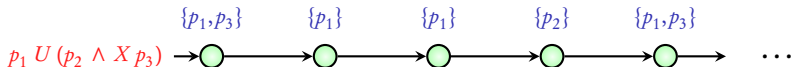
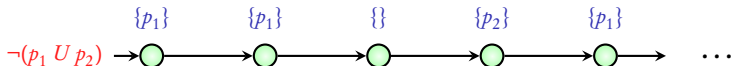
$$\phi ::= \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid X\phi \mid \phi_1 U \phi_2$$



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# Derived operators

▶  $\phi_1 \vee \phi_2: \neg(\neg\phi_1 \wedge \neg\phi_2)$  (Or)

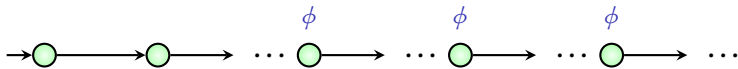
▶  $\phi_1 \rightarrow \phi_2: \neg\phi_1 \vee \phi_2$  (Implies)

▶  $F \phi: true U \phi$  (Eventually)

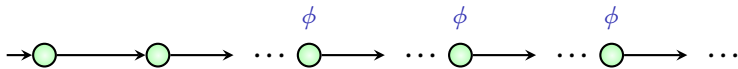
▶  $G \phi: \neg F \neg\phi$  (Always)



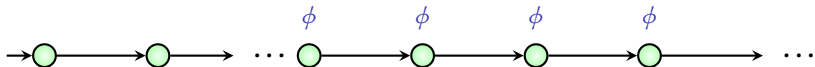
**G F  $\phi$  (Infinitely often)**



$G F \phi$  (Infinitely often)



$F G \phi$  (Eventually forever)



**Coming next:** More examples

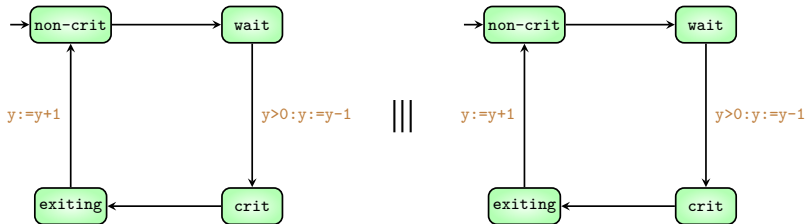
Atomic propositions  $AP = \{ crit_1, wait_1, crit_2, wait_2 \}$

$crit_1$ : `pr1.location=crit`

$wait_1$ : `pr1.location=wait`

$crit_2$ : `pr2.location=crit`

$wait_2$ : `pr2.location=wait`



- ▶ **Safety:** both processes cannot be in critical section simultaneously

$$G (\neg crit_1 \vee \neg crit_2)$$

- ▶ **Liveness:** each process visits critical section infinitely often

$$G F crit_1 \wedge G F crit_2$$

# Summary

$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid X\phi \mid \phi_1 U \phi_2$

$F\phi: \text{true} U \phi$       **(Eventually)**       $G\phi: \neg F\neg\phi$       **(Always)**