Unit-6: Model-checking ω-regular properties

B. Srivathsan

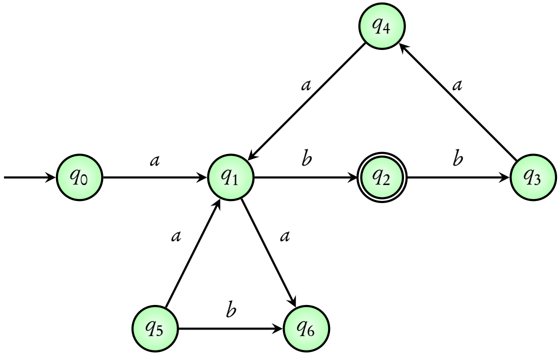
Chennai Mathematical Institute

NPTEL-course

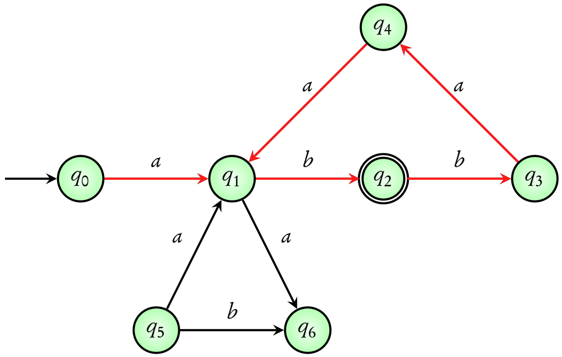
July - November 2015

Module 3:

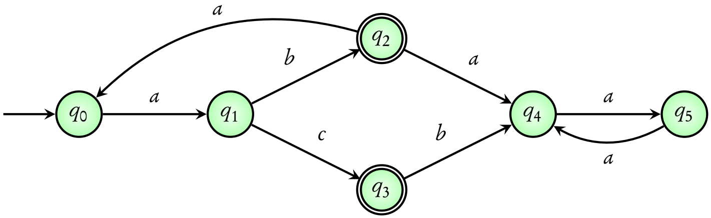
Checking emptiness of Büchi automata



Is the **language** of above NBA **empty**?



Is the **language** of above NBA **empty**? **No**



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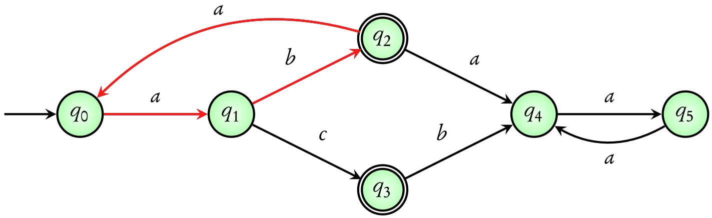

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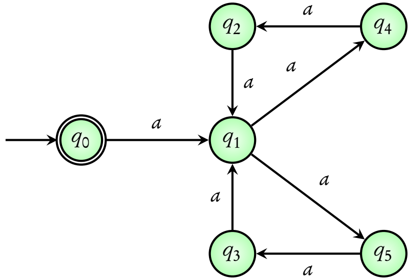

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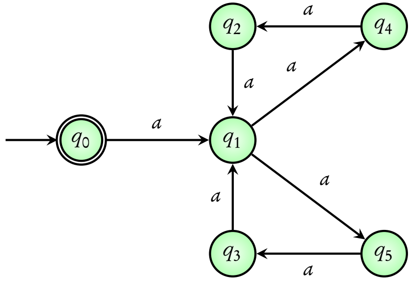
$$\begin{tikzcd} & q_2 \arrow[d, "a"] & & q_4 \arrow[ll, "a"'] \\ \arrow[r] q_0 \arrow[r, "a"] & q_1 \arrow[urr, "a"] \arrow[drr, "a"] & & \\ & q_3 \arrow[u, "a"] & & q_5 \arrow[ll, "a"] \end{tikzcd}$$

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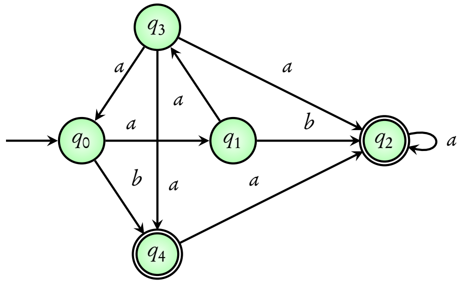

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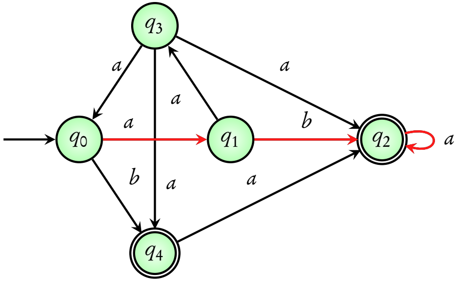

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Main idea of algorithm

Find a **reachable cycle** in the automaton that contains an **accepting state**

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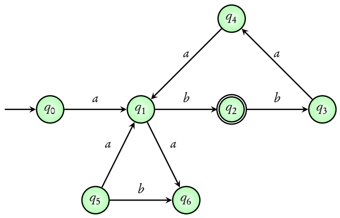
Find a **reachable cycle** in the automaton that contains an **accepting state**

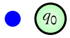
- Do a preliminary DFS to get all **reachable states**
- From every **accepting state**, do a secondary DFS to see if it can **come back to itself**

Coming next: A **nested-DFS** algorithm

Courcoubetis, Vardi, Wolper, Yannakakis. Memory-efficient algorithms for the verification of temporal properties

Formal Methods in System Design, 1992





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    call $dfs\_blue\ (s_0)$

procedure $dfs\_blue(s)$
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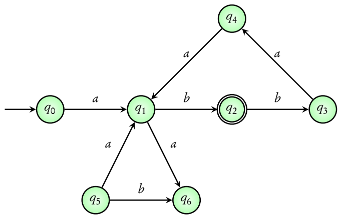

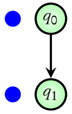

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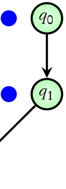

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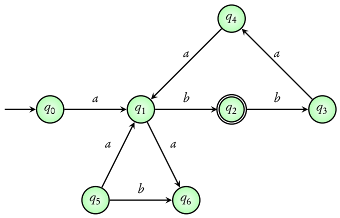

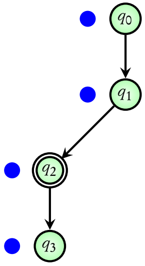

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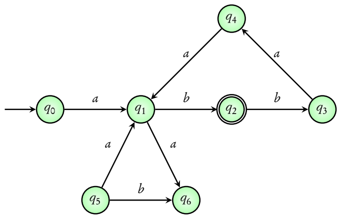

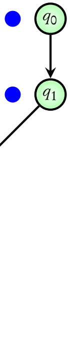

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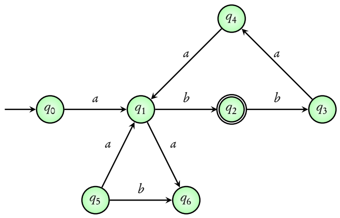

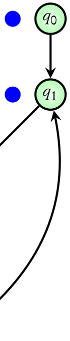

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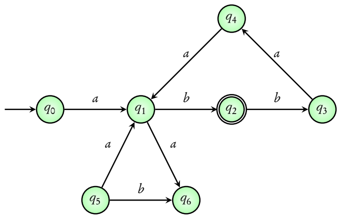

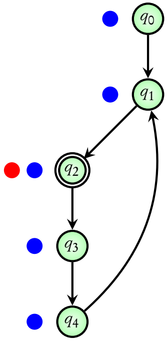

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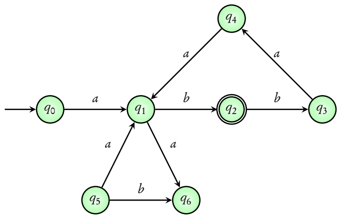

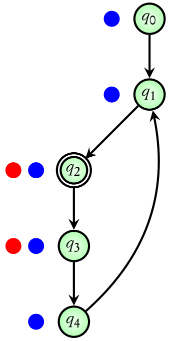

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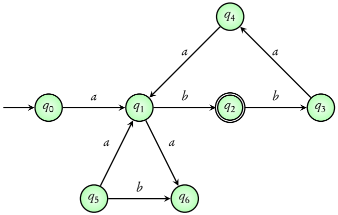

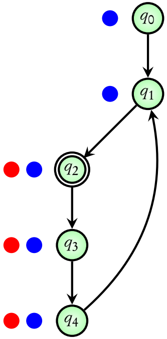

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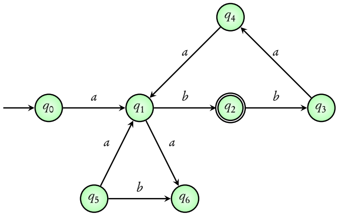

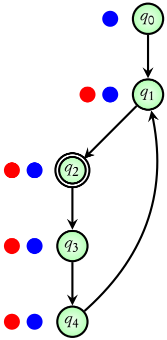

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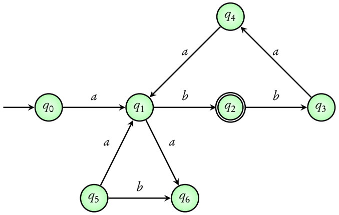

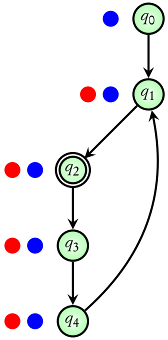

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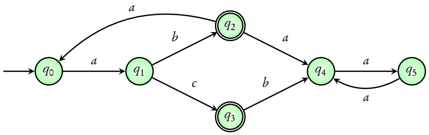

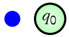

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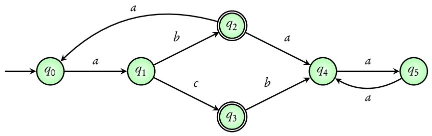

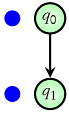

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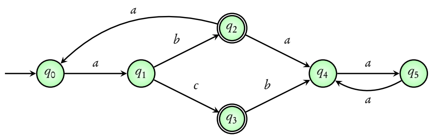

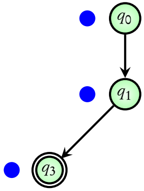

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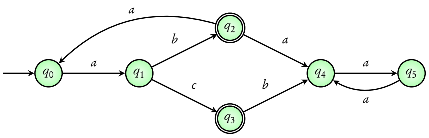

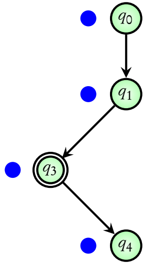

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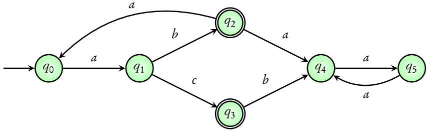

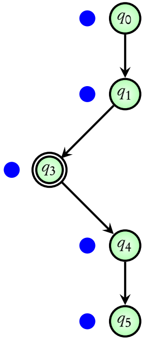

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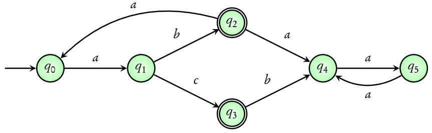

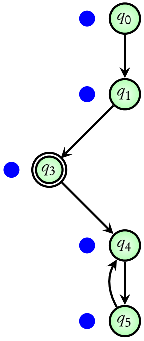

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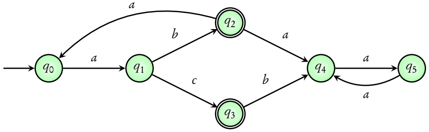
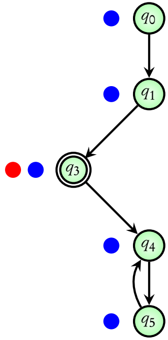
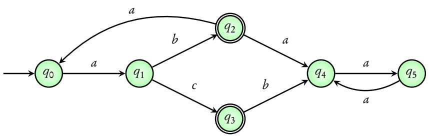


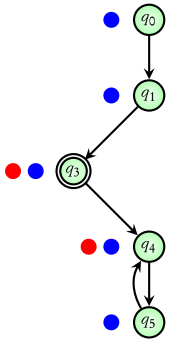


```
procedure $nested\_dfs()$
    call $dfs\_blue(s_0)$

procedure $dfs\_blue(s)$
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call $dfs\_blue(t)$
    if $s \in Accept$ then
        $seed := s$
        call $dfs\_red(s)$

procedure $dfs\_red(s)$
    $s.red$ := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call $dfs\_red(t)$
        else if $t = seed$
            report cycle
```





```
procedure $nested\_dfs()$
    call $dfs\_blue\,(s_0)$

procedure $dfs\_blue(s)$
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call $dfs\_blue\,(t)$
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    $s.red$ := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call $dfs\_red\,(t)$
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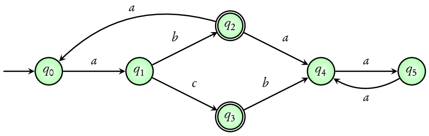
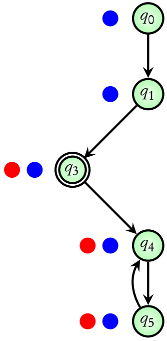
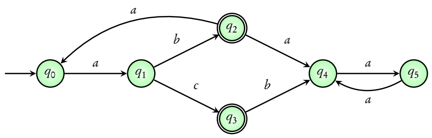


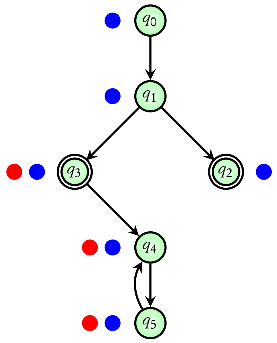


```
procedure $nested\_dfs()$
    call $dfs\_blue(s_0)$

procedure $dfs\_blue(s)$
    $s.blue := \textbf{true}$
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call $dfs\_blue(t)$
    if $s \in Accept$ then
        $seed := s$
        call $dfs\_red(s)$

procedure $dfs\_red(s)$
    $s.red := \textbf{true}$
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call $dfs\_red(t)$
        else if $t = seed$
            report cycle
```





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procedure $nested\_dfs()$
    call $dfs\_blue\,(s_0)$

procedure $dfs\_blue(s)$
    $s.blue := $ true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call $dfs\_blue\,(t)$
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    $s.red := $ true
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        if $\neg t.red$ then
            call $dfs\_red\,(t)$
        else if $t = seed$
            report cycle
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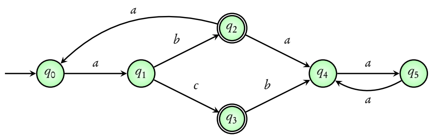
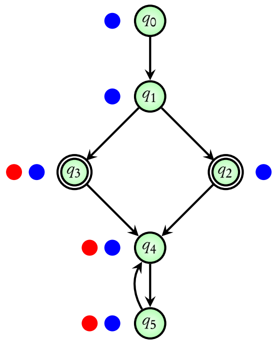
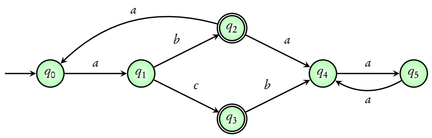


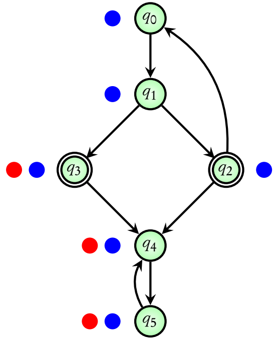


```
procedure nested_dfs()
    call dfs_blue ( $s_0$ )

procedure dfs_blue( s )
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call dfs_blue ( t )
    if $s \in Accept$ then
        $seed$ := $s$
        call dfs_red ( s )

procedure dfs_red ( s )
    $s.red$ := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call dfs_red ( t )
        else if $t = seed$
            report cycle
```





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procedure nested_dfs()
    call dfs_blue ( $s_0$ )

procedure dfs_blue( s )
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call dfs_blue ( t )
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    $s.red$ := true
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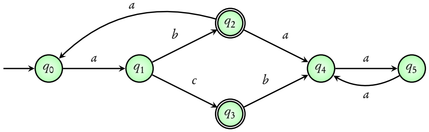
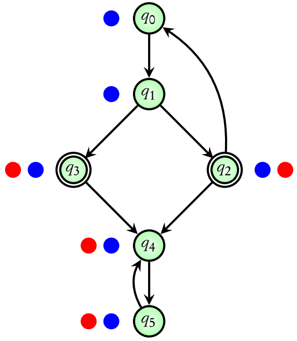
            call dfs_red ( t )
        else if $t = seed$
            report cycle
```





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    call $dfs\_blue\,(s_0)$

procedure $dfs\_blue(s)$
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call $dfs\_blue\,(t)$
    if $s \in Accept$ then
        $seed := s$
        call $dfs\_red\,(s)$

procedure $dfs\_red\,(s)$
    $s.red$ := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call $dfs\_red\,(t)$
        else if $t = seed$
            report cycle
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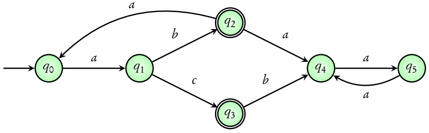

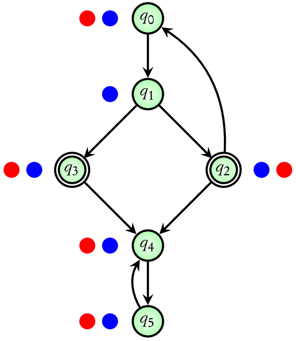

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procedure $nested\_dfs()$
    call $dfs\_blue\,(s_0)$

procedure $dfs\_blue(s)$
    $s.blue := $ true
    for all $t \in post(s)$ do
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    $s.red := $ true
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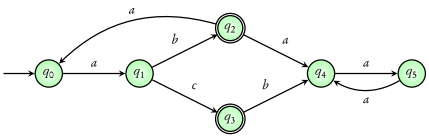
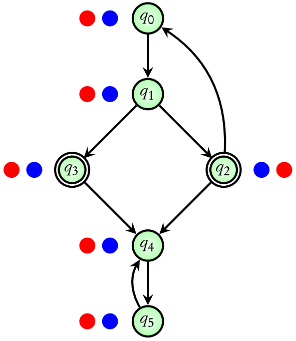
        if $\neg t.red$ then
            call $dfs\_red\,(t)$
        else if $t = seed$
            report cycle
```





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    call $dfs\_blue\,(s_0)$

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    $s.blue$ := true
    for all $t \in post(s)$ do
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    $s.red$ := true
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        if $\neg t.red$ then
            call $dfs\_red\,(t)$
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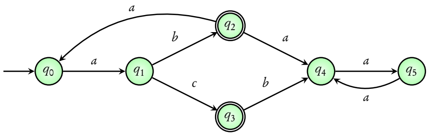
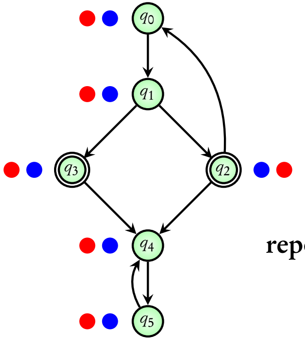
            report cycle
```



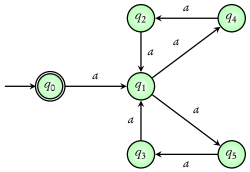


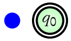
report cycle!

```
procedure nested_dfs( )
    call dfs_blue ( $s_0$ )

procedure dfs_blue( s )
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call dfs_blue ( t )
    if $s \in Accept$ then
        $seed$ := $s$
        call dfs_red ( s )

procedure dfs_red ( s )
    $s.red$ := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call dfs_red ( t )
        else if $t = seed$
            report cycle
```





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    call $dfs\_blue\,(s_0)$

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    $s.blue$ := true
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        if $\neg t.red$ then
            call $dfs\_red\,(t)$
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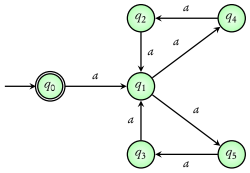

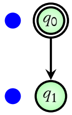

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procedure $nested\_dfs()$
    call $dfs\_blue(s_0)$

procedure $dfs\_blue(s)$
    $s.blue$ := true
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    for all $t \in post(s)$ do
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            call $dfs\_red(t)$
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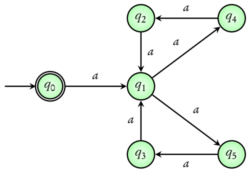

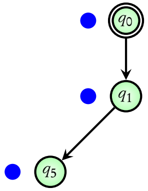

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procedure nested_dfs()
    call dfs_blue ( $s_0$ )

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    $s.blue$ := true
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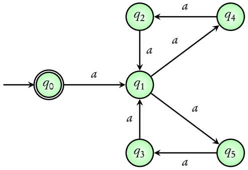

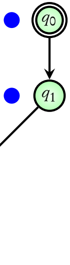

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procedure nested_dfs()
    call dfs_blue ( $s_0$ )

procedure dfs_blue( $s$ )
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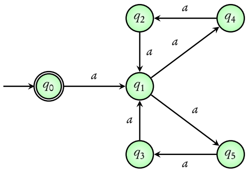

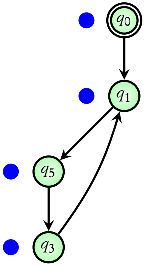

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procedure $nested\_dfs()$
    call $dfs\_blue\,(s_0)$

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            report cycle
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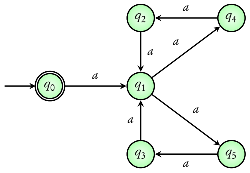

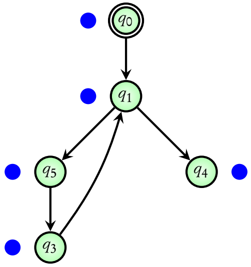

```
procedure nested_dfs()
    call dfs_blue ($s_0$)

procedure dfs_blue(s)
    $s.blue$ := true
    for all $t \in post(s)$ do
        if $\neg t.blue$ then
            call dfs_blue (t)
    if $s \in Accept$ then
        $seed$ := $s$
        call dfs_red (s)

procedure dfs_red (s)
    $s.red$ := true
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            call dfs_red (t)
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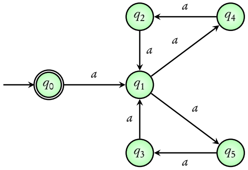

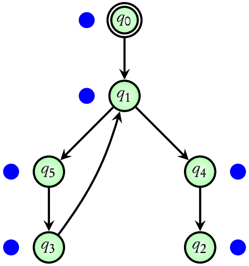

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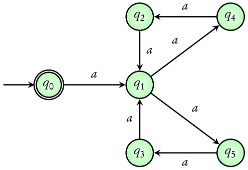

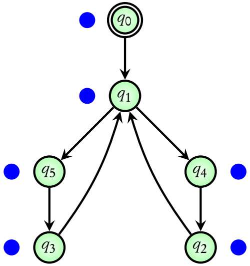

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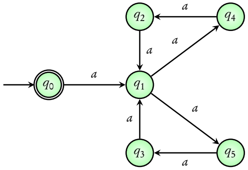

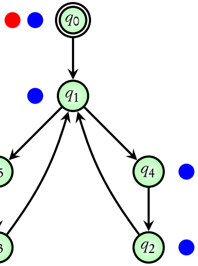

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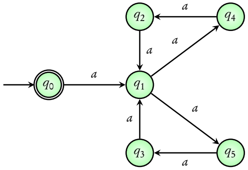

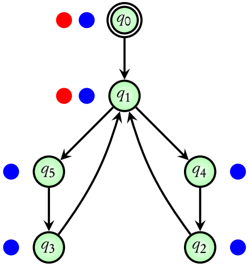

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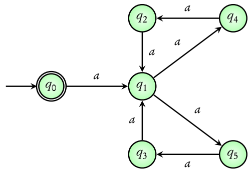
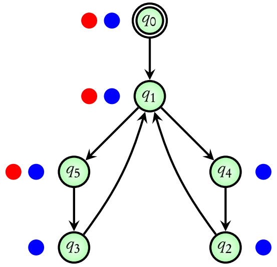
```

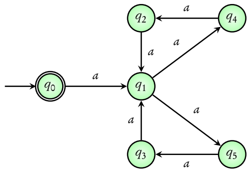


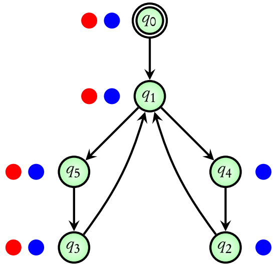


```
procedure nested_dfs()
    call dfs_blue ( $s_0$ )

procedure dfs_blue( s )
    s.blue := true
    for all $t \in post( s )$ do
        if $\neg t.blue$ then
            call dfs_blue ( t )
    if $s \in Accept$ then
        seed := s
        call dfs_red ( s )

procedure dfs_red ( s )
    s.red := true
    for all $t \in post(s)$ do
        if $\neg t.red$ then
            call dfs_red ( t )
        else if $t = seed$
            report cycle
```





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procedure $nested\_dfs()$
    call $dfs\_blue\,(s_0)$

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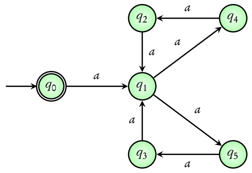

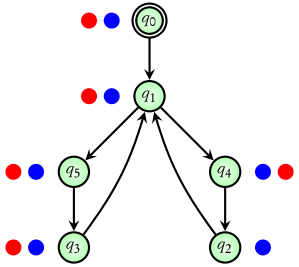

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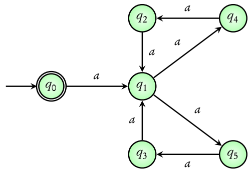

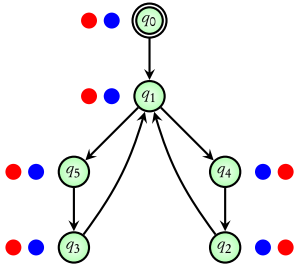

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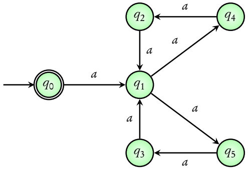

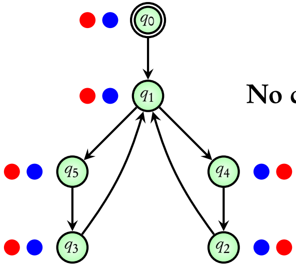

No cycle!

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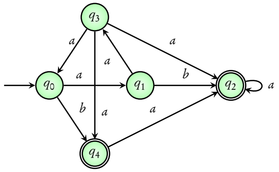

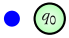

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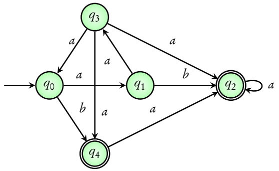

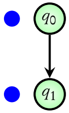

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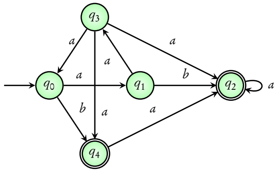

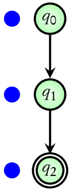

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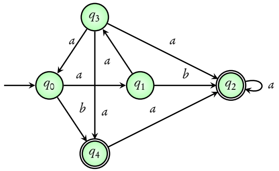

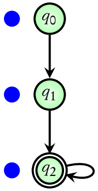

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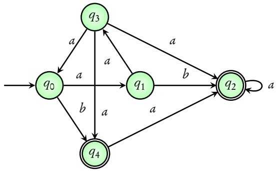

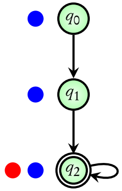

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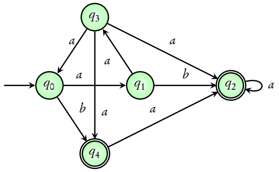

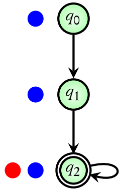

report cycle!

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```

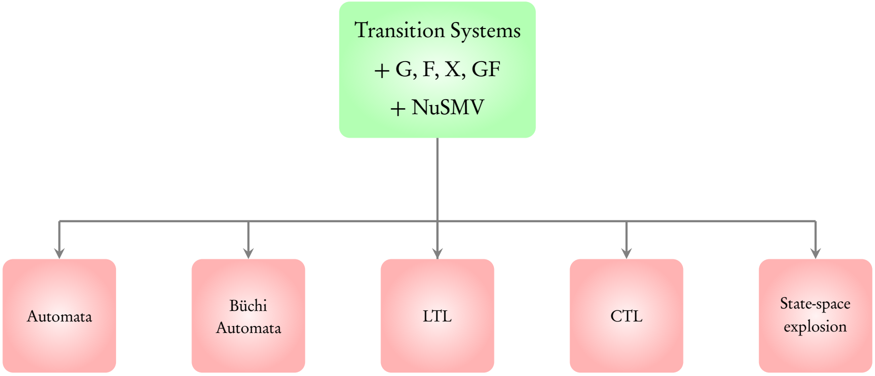
$$\begin{tikzcd}
\text{Does} & \text{Transition system} \arrow[dd] & \text{satisfy} & \omega\text{-regular property ?} \\
 & & & \omega\text{-regular expression } \phi \arrow[d] \\
 & \text{NBA } \mathscr{A}_{T.S} & & \text{NBA } \mathscr{A}_{\phi}
\end{tikzcd}$$

$$\boxed{L(\mathscr{A}_{T.S.}) \subseteq L(\mathscr{A}_{\phi})\ ?}$$

Is $L(\mathscr{A}_{T.S.}) \cap \overline{L(\mathscr{A}_{\phi})}$ empty ?

Is $L(\mathscr{A}_{T.S.}) \cap L(\overline{\mathscr{A}_{\phi}})$ empty ?

Is $L(\mathscr{A}_{T.S.} \times \overline{\mathscr{A}_{\phi}})$ empty ?



Unit: 4 **Unit: 5,6** **Unit: 7,8** **Unit: 9** **Unit: 10**