

Unit-6: Model-checking ω -regular properties

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NPTEL-course

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Module 2:
 ω -regular expressions to NBA

$$\Sigma = \{ a, b \}$$

Example 1: Infinite word consisting only of a a^ω

$$\{ aaaaaaaaaaaaaaaaaa \dots \}$$

Example 2: Infinite words containing only a or only b $a^\omega + b^\omega$

$$\{ aaaaaaaaaaaaaaaaaa \dots, bbbbbbbbbbbbbb \dots \}$$

Example 3: a word in $aa\Sigma^*aa$ followed by only b -s $aa\Sigma^*aa \cdot b^\omega$

$$\{ aaaabbbbbbb \dots, aababaabbbbbbb \dots, aabbbbaabbbbbbb \dots, \dots \}$$

Example 4: Infinite words where b occurs **only finitely often** $(a + b)^* \cdot a^\omega$

$$\{ aaaaaaaaaaaaaaaaaa \dots, baaaaaaaaaaaaa \dots, babbaaaaaaaaaaaaaa \dots, \dots \}$$

Example 5: Infinite words where b occurs **infinitely often** $(a^*b)^\omega$

$$\{ abababababab \dots, bbbabbbabbbabbbba \dots, bbbbbbbbbbbbbb \dots, \dots \}$$

ω -regular expressions

$$G = E_1 \cdot F_1^\omega + E_2 \cdot F_2^\omega + \dots + E_n \cdot F_n^\omega$$

$E_1, \dots, E_n, F_1, \dots, F_n$ are **regular expressions**
and $\epsilon \notin L(F_i)$ for all $1 \leq i \leq n$

$$L(F^\omega) = \{ \omega_1 \omega_2 \omega_3 \dots \mid \text{each } \omega_i \in L(F) \}$$

More examples

- ▶ $(a + b)^\omega$ set of **all infinite words**
- ▶ $a(a + b)^\omega$ infinite words **starting with an a**
- ▶ $(a + bc + c)^\omega$ words where every b is **immediately followed by c**
- ▶ $(a + b)^*c(a + b)^\omega$ words with a **single occurrence of c**
- ▶ $((a + b)^*c)^\omega$ words where c **occurs infinitely often**

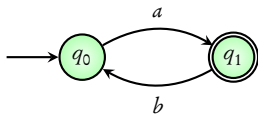
ω -regular expressions

$$G = E_1 \cdot F_1^\omega + E_2 \cdot F_2^\omega + \dots + E_n \cdot F_n^\omega$$

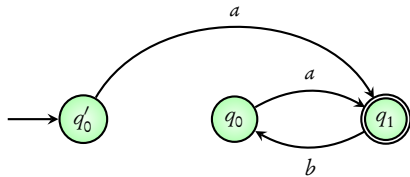
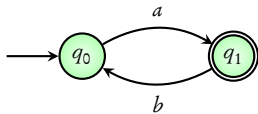
Goal: Convert ω -regular expression to NBA

Part 1: Given regular expression U , find NBA for U^ω

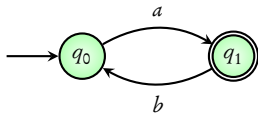
NFA for U



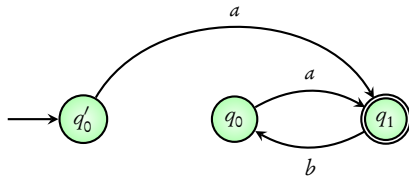
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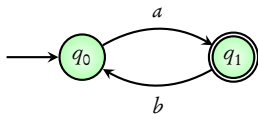
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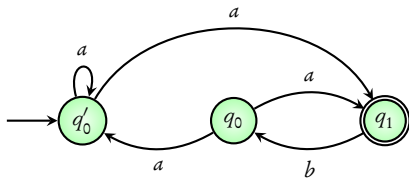
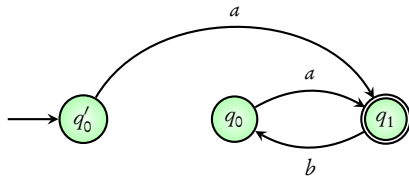
Standardized NFA



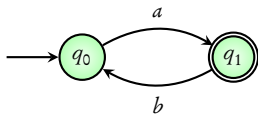
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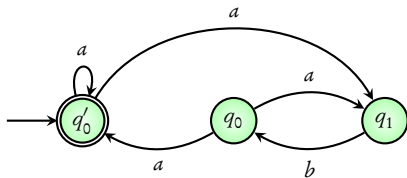
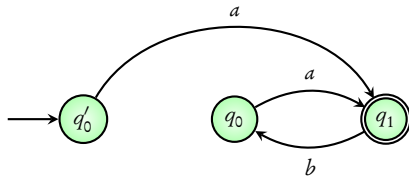
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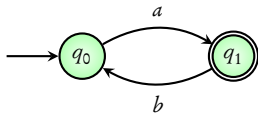
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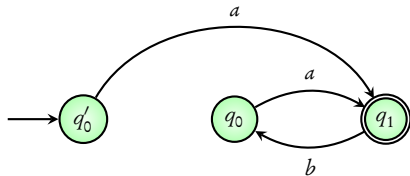
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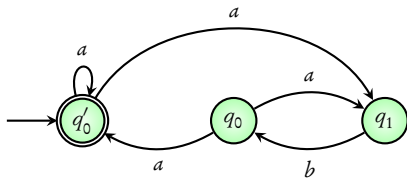
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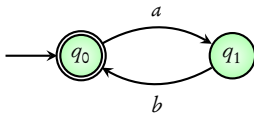
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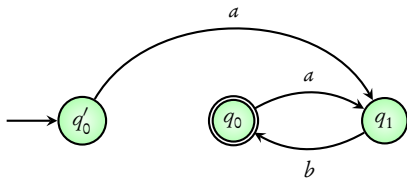
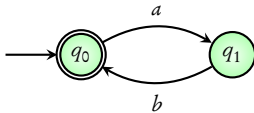
NBA for U^ω



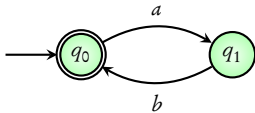
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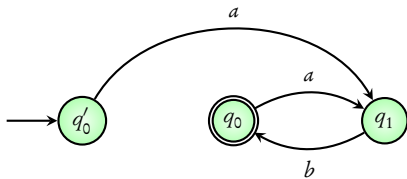
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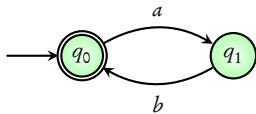
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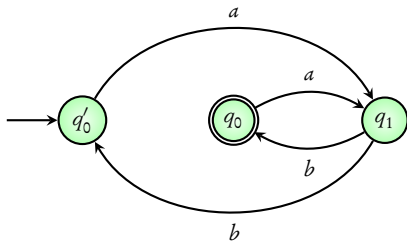
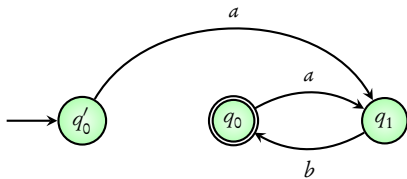
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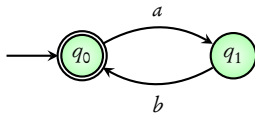
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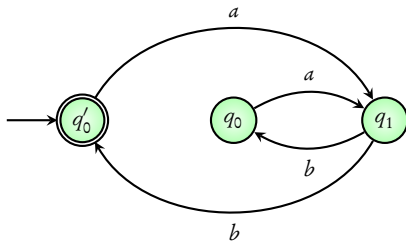
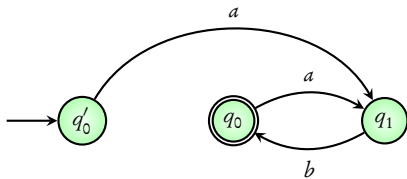
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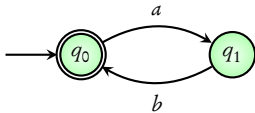
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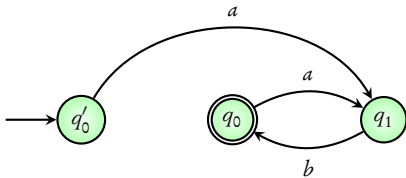
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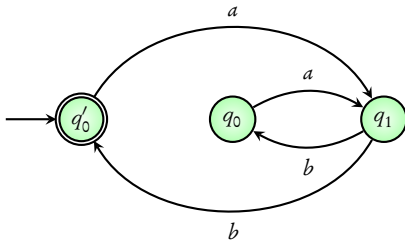
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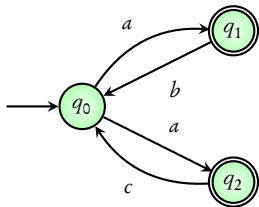
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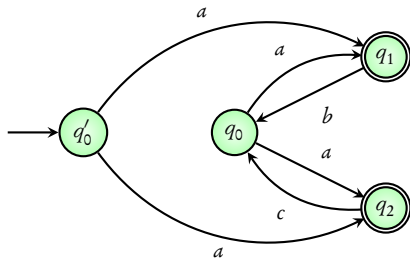
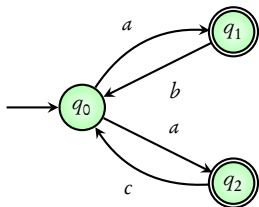
NBA for U^ω



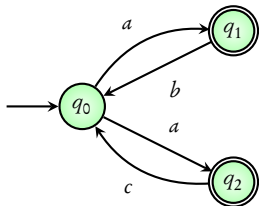
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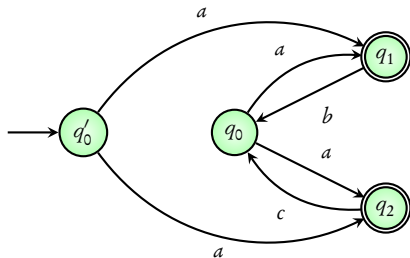
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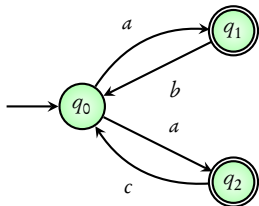
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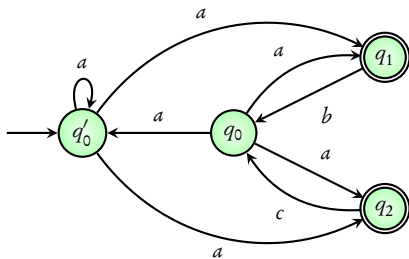
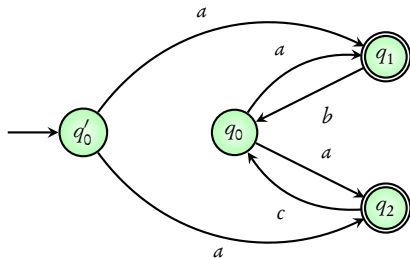
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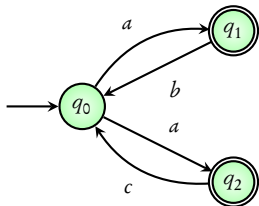
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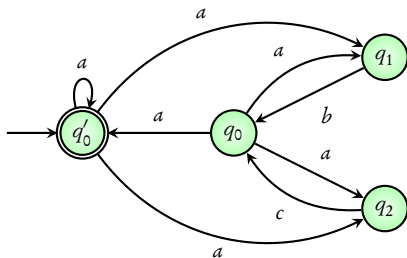
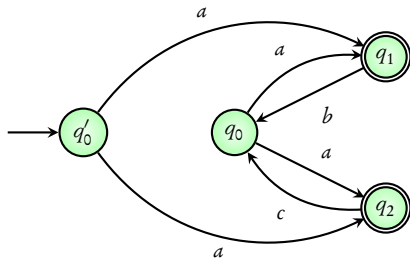
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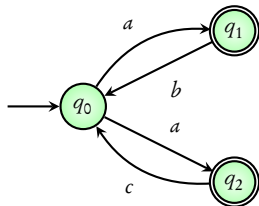
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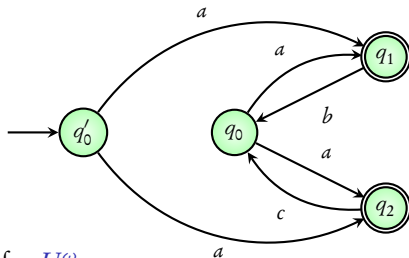
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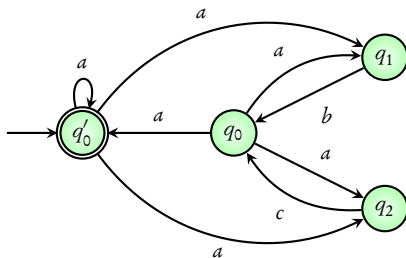
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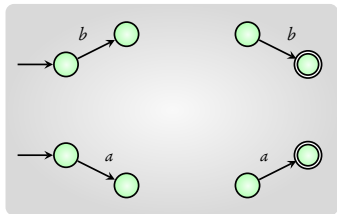
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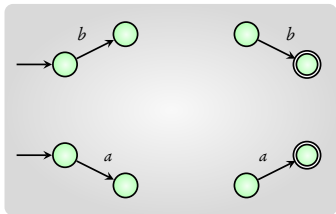
NBA for U^ω



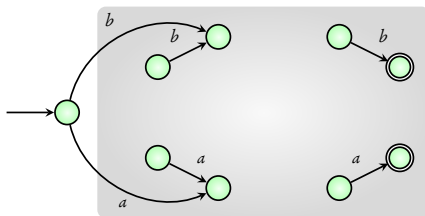
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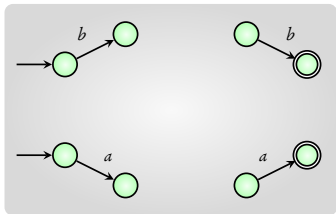
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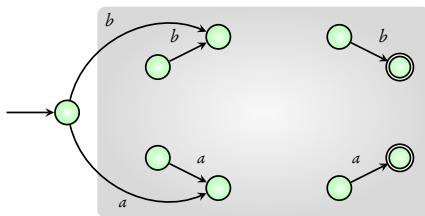
Standardized NFA for U



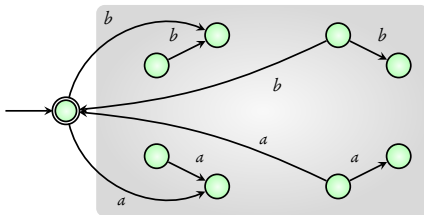
NFA for U



Standardized NFA for U



NBA for U^ω



ω -regular expressions

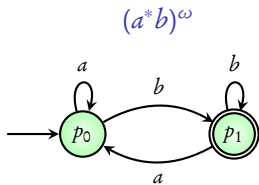
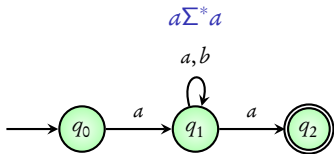
$$G = E_1 \cdot F_1^\omega + E_2 \cdot F_2^\omega + \dots + E_n \cdot F_n^\omega$$

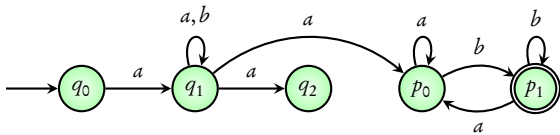
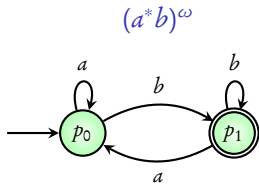
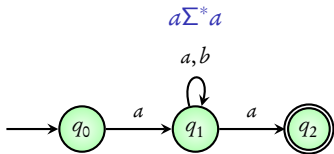
Goal: Convert ω -regular expression to NBA

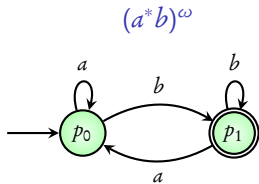
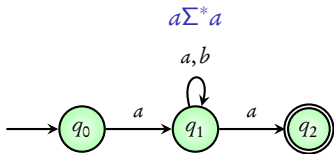
Part 1: Given regular expression U , find NBA for U^ω

Done!

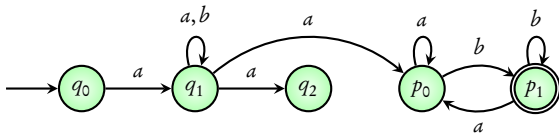
Part 2: Given regular expression U and NBA for V find NBA for $U \cdot V$



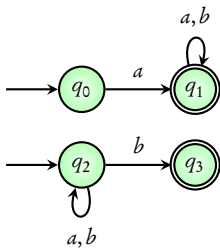




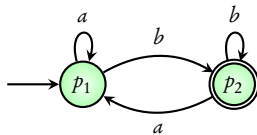
$a\Sigma^*a \cdot (a^*b)^\omega$



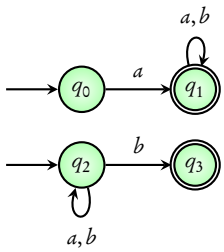
$$a\Sigma^* + \Sigma^*b$$



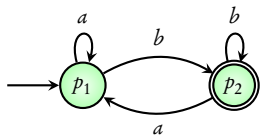
$$(a^*b)^\omega$$



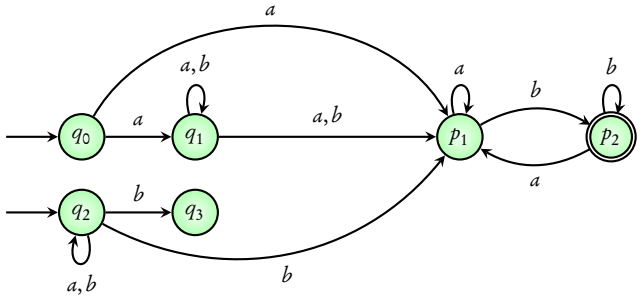
$$a\Sigma^* + \Sigma^*b$$



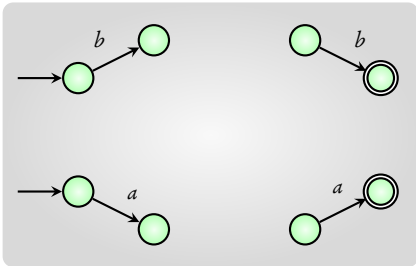
$$(a^*b)^\omega$$



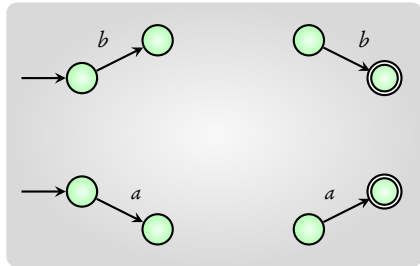
$$(a\Sigma^* + \Sigma^*b) \cdot (a^*b)^\omega$$



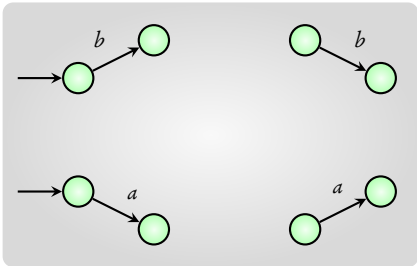
U



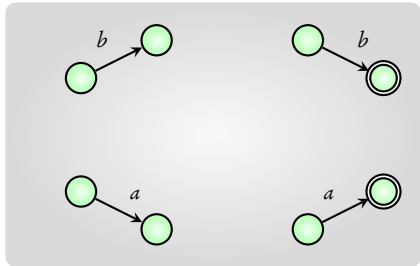
V

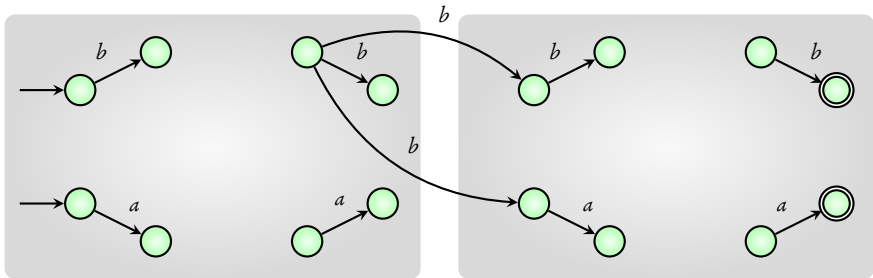


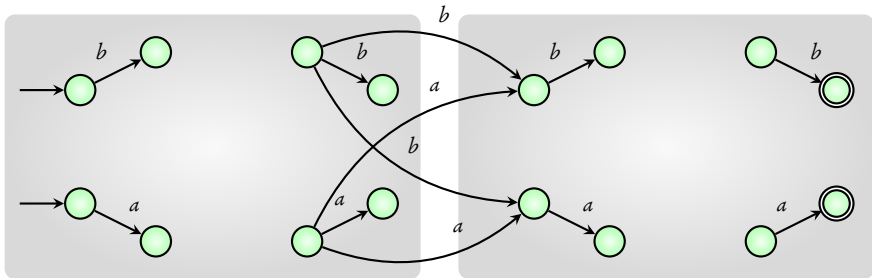
U



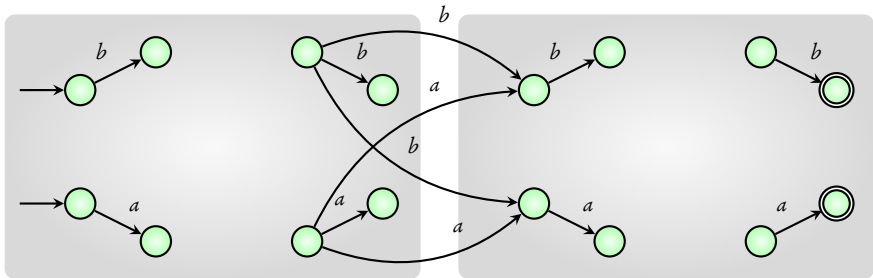
V



U V 

U V 

$U \cdot V$



ω -regular expressions

$$G = E_1 \cdot F_1^\omega + E_2 \cdot F_2^\omega + \dots + E_n \cdot F_n^\omega$$

Goal: Convert ω -regular expression to NBA

Part 1: Given regular expression U , find NBA for U^ω

Part 2: Given regular expression U and NBA for V find NBA for $U \cdot V$

Done!

Part 3: Given NBA for U and NBA for V find NBA for $U + V$

Part 3: Given NBA for U and NBA for V find NBA for $U + V$

Union of NBA already seen in Unit 5

Part 1: Given regular expression U , find NBA for U^ω

Part 2: Given regular expression U and NBA for V find NBA for $U \cdot V$

Part 3: Given NBA for U and NBA for V find NBA for $U + V$

Theorem

Every ω -regular expression can be **converted** to an NBA