Unit-5: ω-regular properties

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Chennai Mathematical Institute

NPTEL-course

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Module 3:

Büchi automata

Goal

- Give some kind of an **automaton** for ω-regular expressions
- Take **synchronous product** with the transition system of the model
- Check **emptiness** of this automaton

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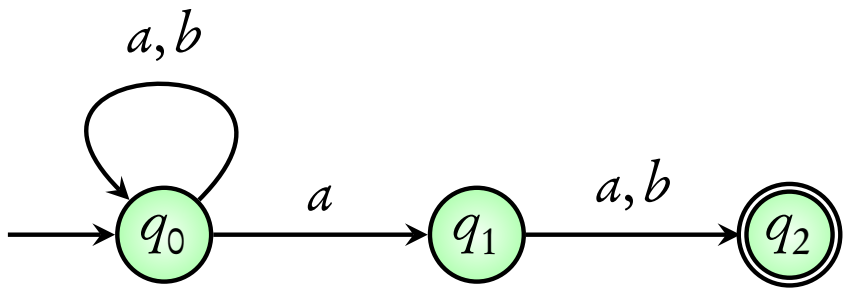
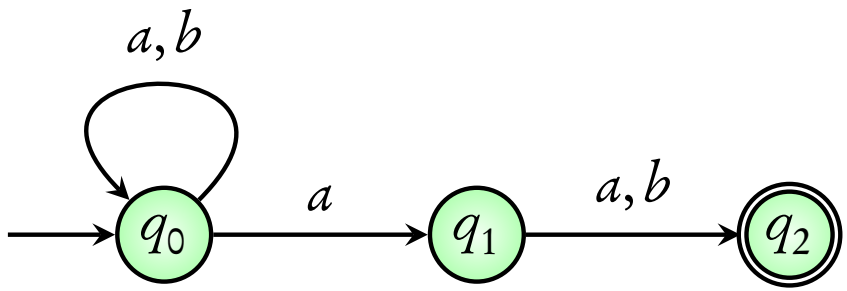
Coming next: A short recap of **finite automata**

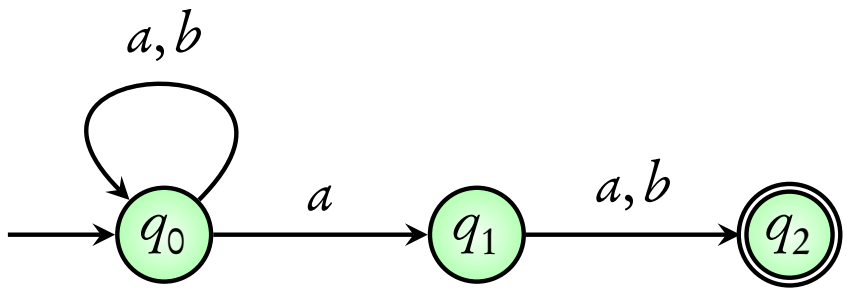

a, b
a
a, b
q_0
q_1
q_2

$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$



$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$

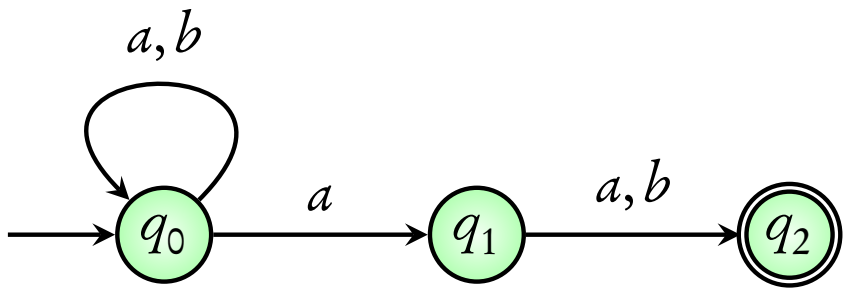
Runs:



$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$

Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$$

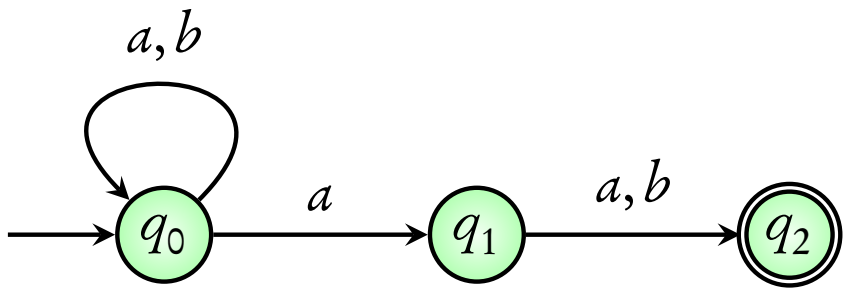


$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$

Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$$

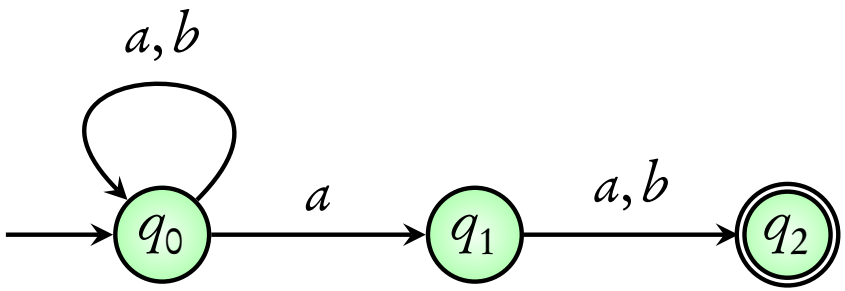


$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$

Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$$ accepting run

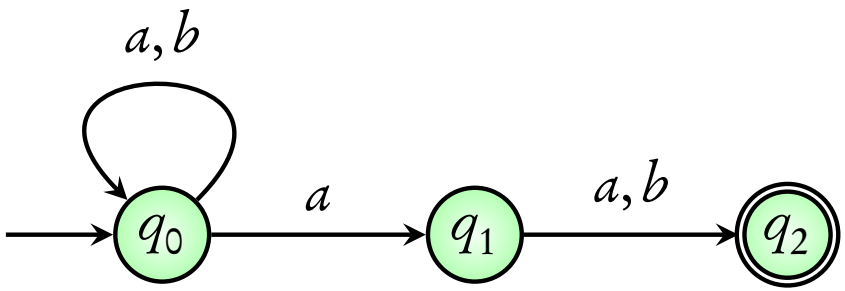


$$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$$

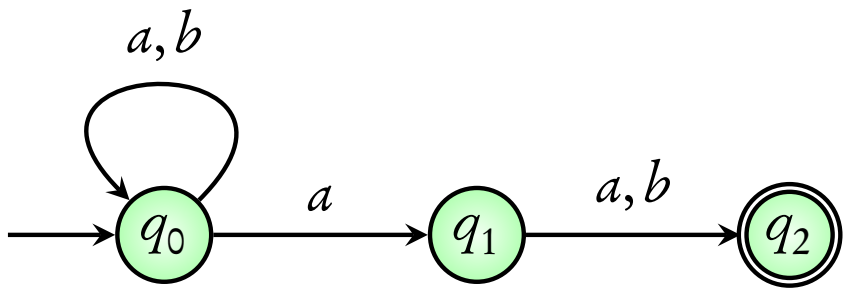
Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$$ accepting run

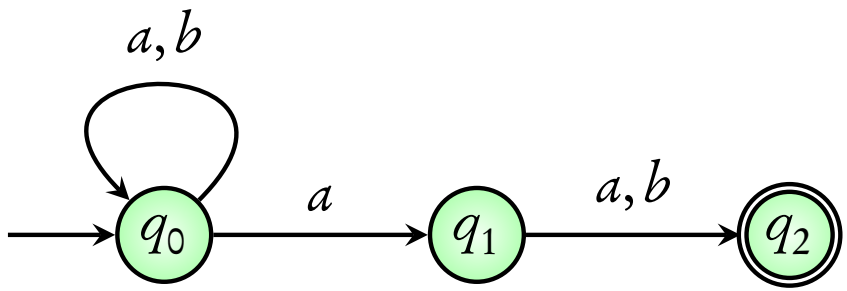


Language: set of words for which **there exists** an accepting run



Language: set of words for which **there exists** an accepting run

$a \quad b \quad b \quad b \quad a$



Language: set of words for which **there exists** an accepting run

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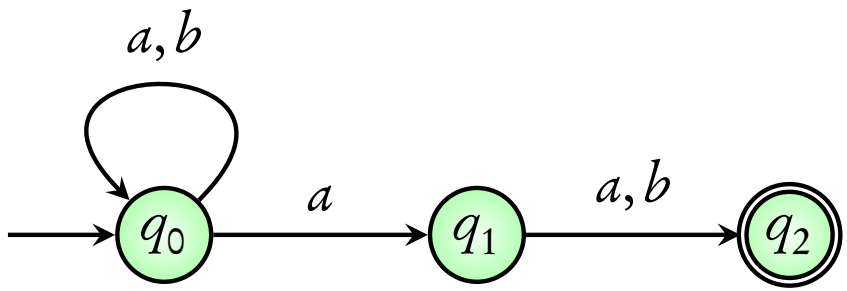
Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0$$



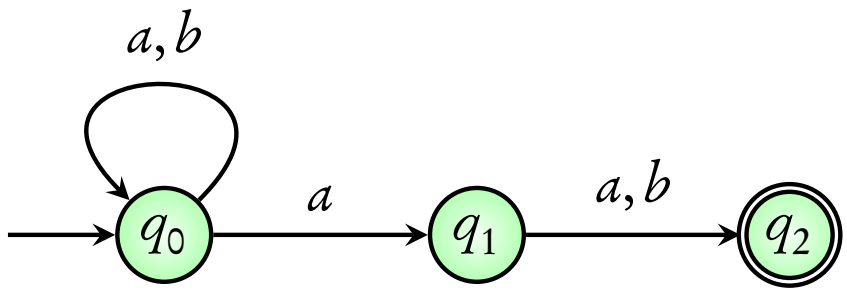
Language: set of words for which **there exists** an accepting run

$$a \quad b \quad b \quad b \quad a$$

Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0$$

Not accepted



Language: set of words for which **there exists** an accepting run

In finite words, there is an **end**

A run is accepting if it **ends in an accepting state**

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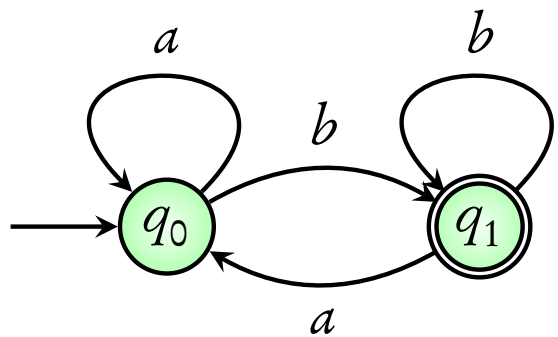
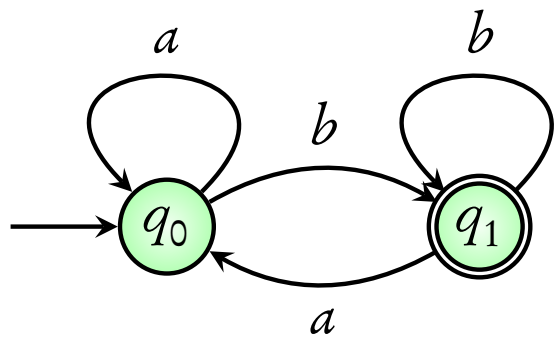
A run is accepting if it **ends in an accepting state**

How do we define **accepting runs** for **infinite words**?

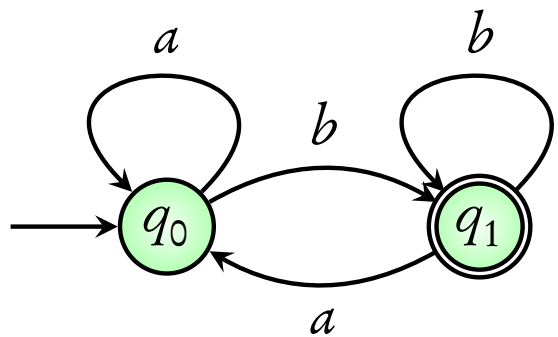

a
b
b
q_0
q_1
a

$a\ b\ a\ b\ a\ a\ b\ b\ b\ b\ b\ b\ \ldots$



$$a\ b\ a\ b\ a\ a\ b\ b\ b\ b\ b\ b\ \ldots$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \cdots$$



$$a\ b\ a\ b\ a\ a\ b\ b\ b\ b\ b\ b\ \ldots$$

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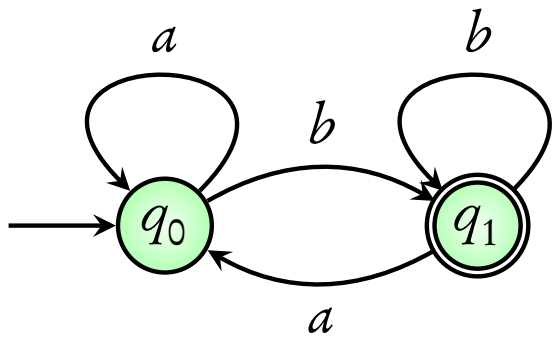
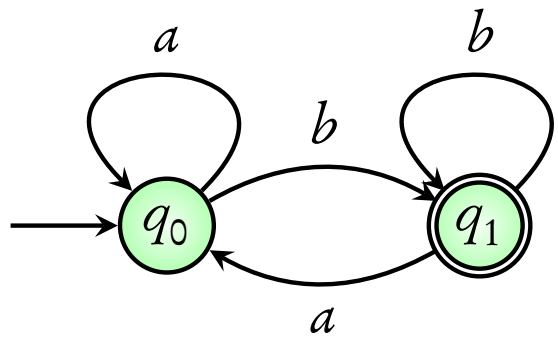


Run is accepting if **some accepting state occurs infinitely often**

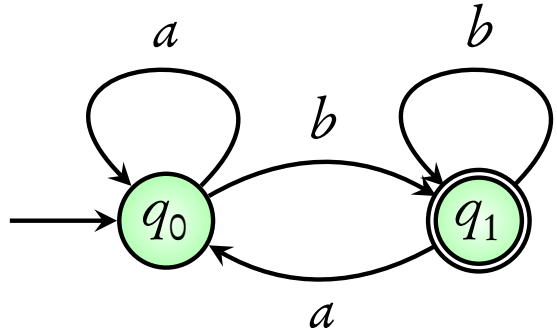
$$a\ b\ a\ b\ a\ a\ b\ b\ b\ b\ b\ b\ \ldots$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \cdots$$

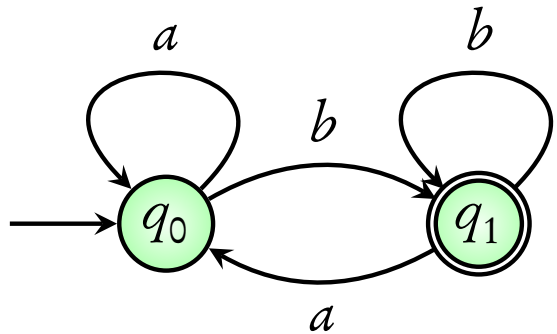


Above word is accepted by this automaton

Run is accepting if **some accepting state occurs infinitely often**


a
b
b
q_0
q_1
a

$a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ a\ b\ \ldots$



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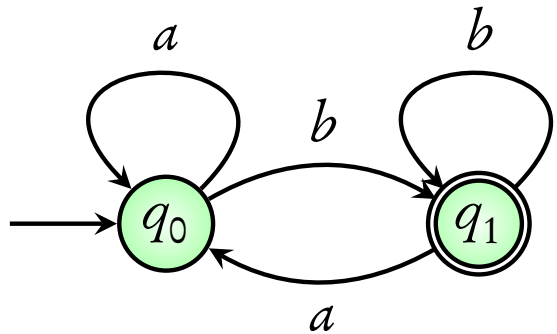
$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \cdots$$



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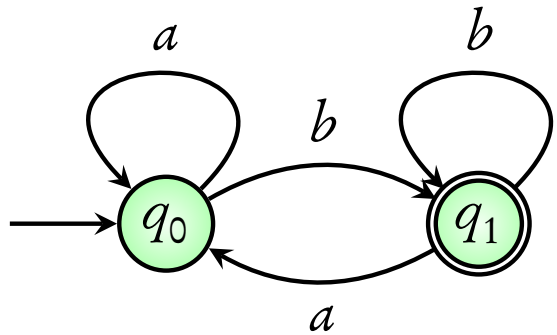

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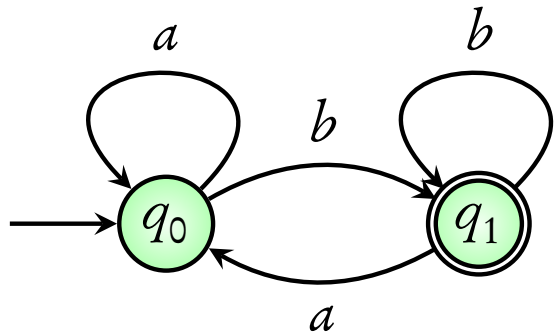
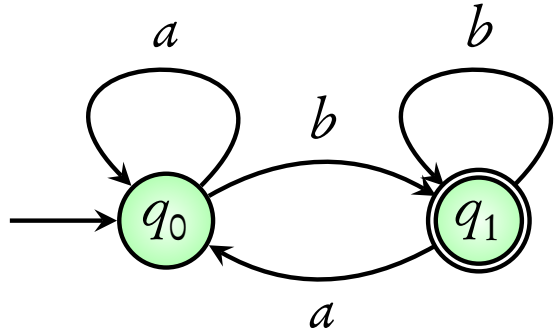
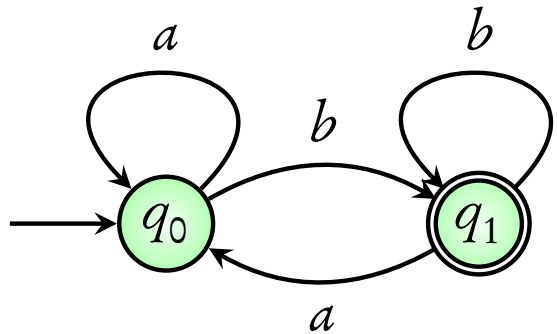


Above word is accepted by this automaton

Run is accepting if **some accepting state occurs infinitely often**


a
b
b
q_0
q_1
a

$a\ b\ a\ b\ a\ a\ a\ a\ a\ a\ a\ a\ \ldots$



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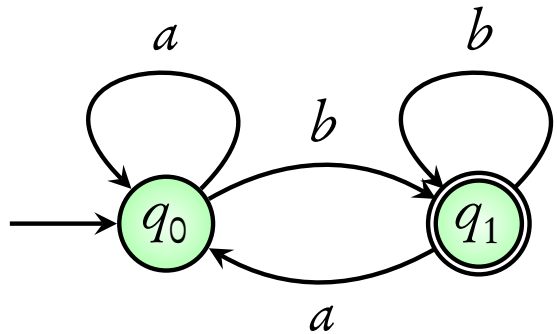
$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \cdots$$



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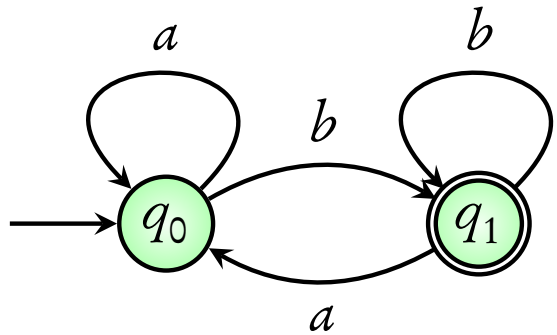

Run is accepting if **some accepting state occurs infinitely often**

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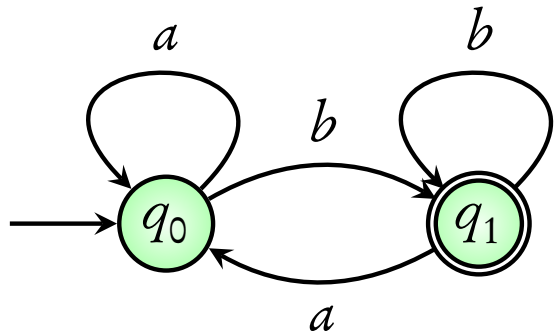
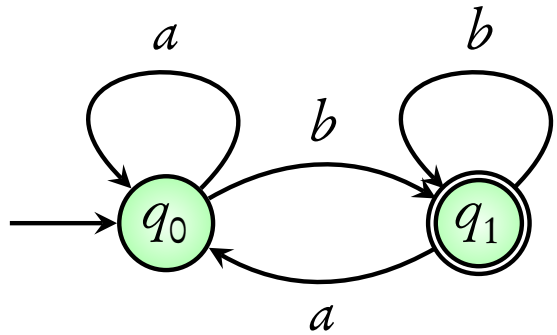


Above word is **not accepted** by this automaton

Run is accepting if **some accepting state occurs infinitely often**



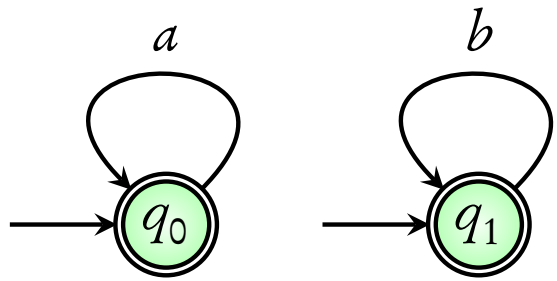
Language: set of infinite words which contain **infinitely many** *b*-s

Non-deterministic Büchi Automata

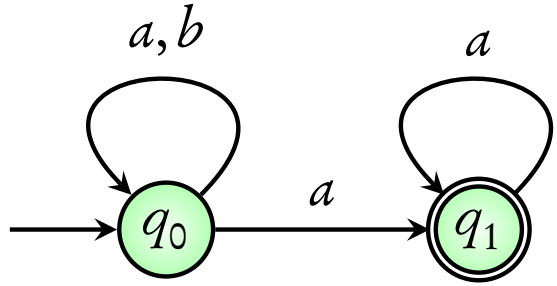
- States, transitions, initial and accepting states like an NFA
- Difference in accepting condition

Word is accepted if it has a run in which **some accepting state occurs infinitely often**

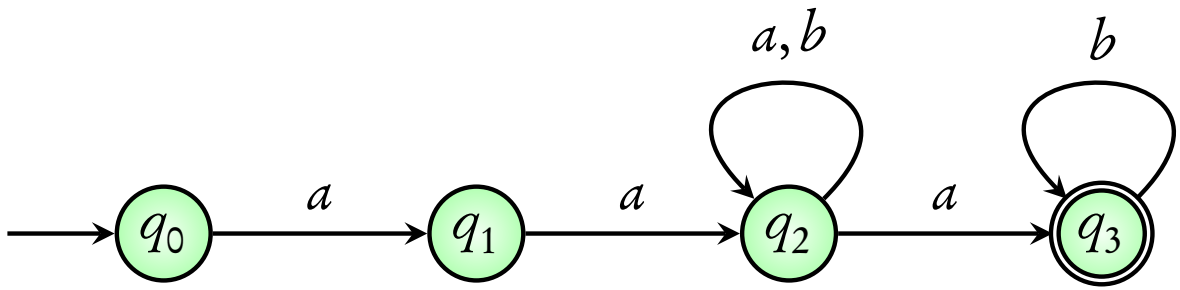
Example: $a^\omega + b^\omega$



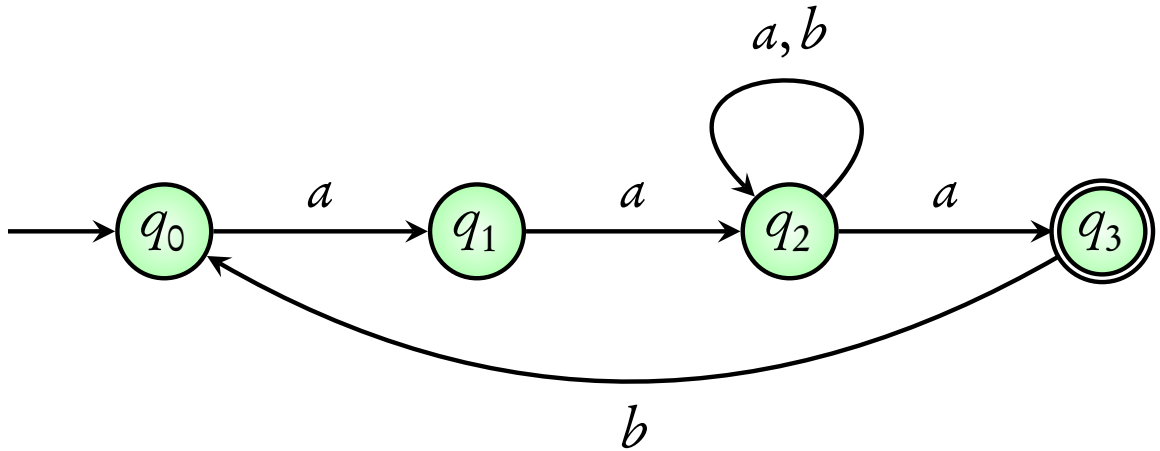
Example: $(a + b)^* a^\omega$



Example: $aa(a+b)^*ab^\omega$



Example: $(aa(a+b)^*ab)^\omega$



Non-deterministic Büchi Automaton

Accepting state occurs infinitely often