Unit-5: ω-regular properties

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NPTEL-course

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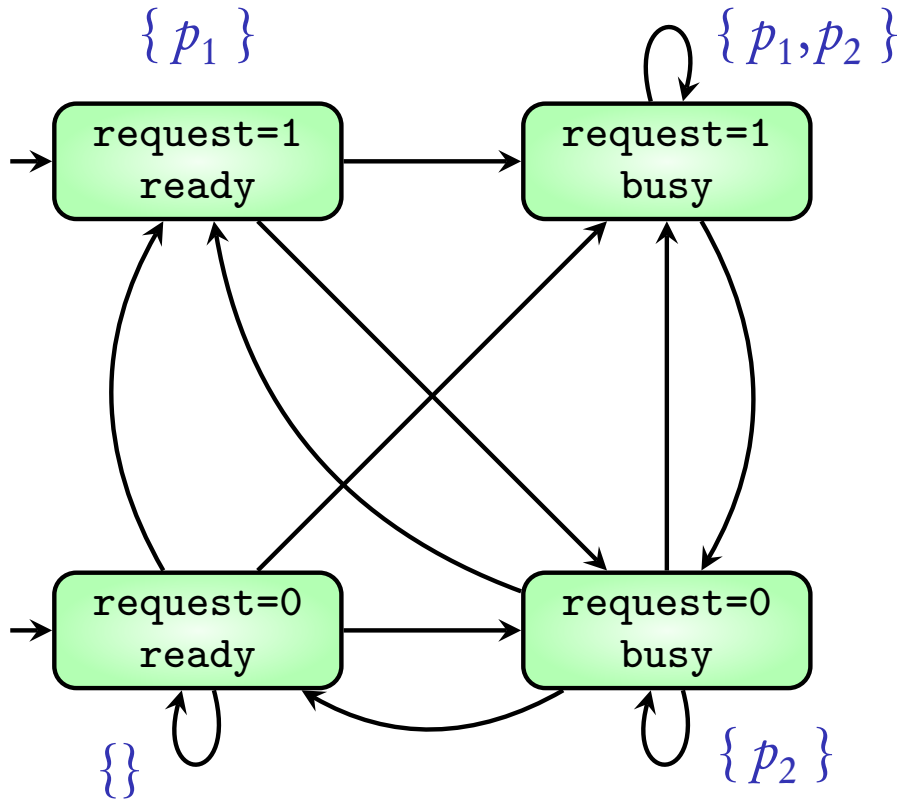
Module 1:
Specifying properties



Traces:

$\{\}\{\}\{\}\{\}\{\}\{\}\{\}\{\}\cdots$

$\{\}\{p_2\}\{p_2\}\{p_2\}\{p_2\}\{p_2\}\{p_2\}\cdots$

$\{p_1\}\{p_1,p_2\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_1,p_2\}\cdots$

$\{\}\{p_1,p_2\}\{p_1,p_2\}\{p_1,p_2\}\{p_1,p_2\}\{p_1,p_2\}\cdots$

$\vdots$

$$
\begin{aligned}
\mathbf{AP} = \quad & \{\ p_1, p_2, \ \cdots, p_k\ \} \\
\mathit{PowerSet}(\mathbf{AP}) = \quad & \{\ \{\ \},\ \{p_1\},\ \cdots,\ \{p_k\}, \\
& \quad \{\ p_1, p_2\ \},\ \{\ p_1, p_3\ \},\ \cdots,\ \{\ p_{k-1}, p_k\ \}, \\
& \quad \cdots \\
& \quad \{\ p_1, p_2, \ \cdots, \ p_k\ \}\ \}
\end{aligned}
$$

Trace(Execution) is an **infinite word** over *PowerSet*(**AP**)

Traces(TS) is the $\{$ Trace(σ) $\mid$ σ is an execution of the TS $\}$

AP-INF = set of **infinite words** over *PowerSet*(**AP**)

Property 1: p_1 is always true

$$\{\, A_0A_1A_2\cdots \in \text{AP-INF} \mid \textbf{each } A_i \text{ contains } p_1 \,\}$$

$$\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\{p_1\}\cdots$$
$$\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1,p_2\}\cdots$$
$$\vdots$$

Property 2: p_1 is true at least once and p_2 is always true

$$\{\, A_0A_1A_2\cdots \in \text{AP-INF} \mid \textbf{exists } A_i \text{ containing } p_1 \text{ and } \textbf{every } A_j \text{ contains } p_2 \,\}$$

$$\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\{p_1,p_2\}\{p_2\}\cdots$$
$$\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\{p_2\}\{p_2\}\cdots$$
$$\vdots$$

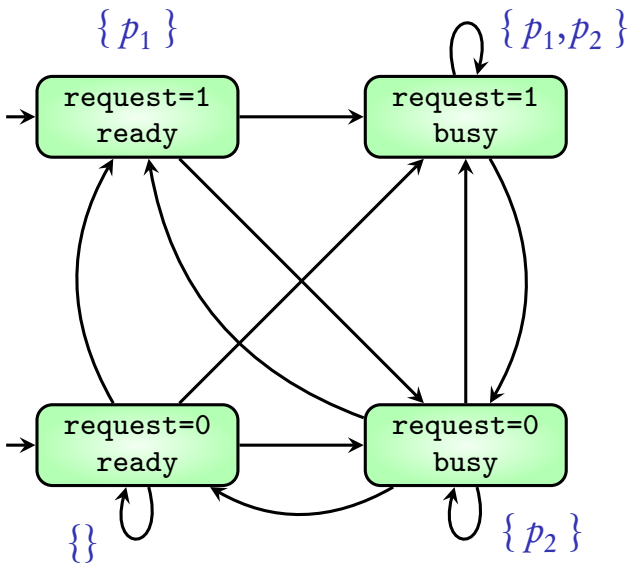
AP-INF = set of **infinite words** over *PowerSet*(**AP**)

A property over AP is a **subset** of AP-INF

$$\text{AP} = \{\ p_1, p_2\ \}$$

Transition System

Property

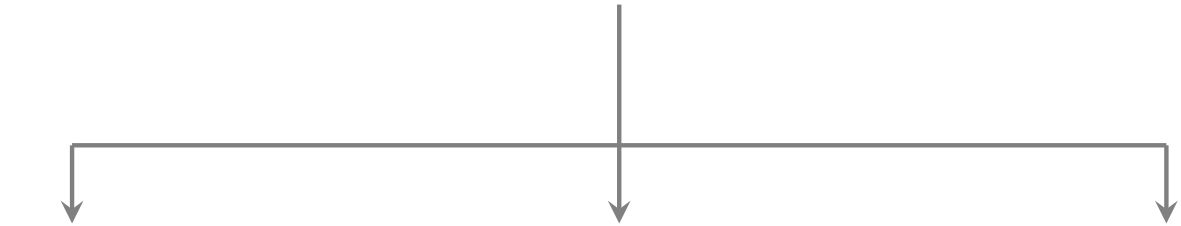


P

Transition system TS satisfies property P if

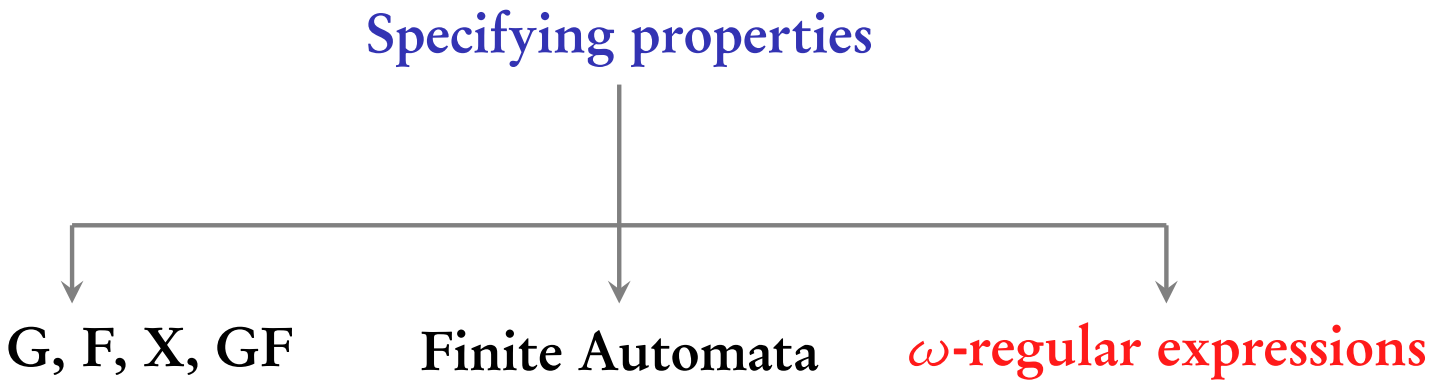
Traces$(TS) \subseteq P$

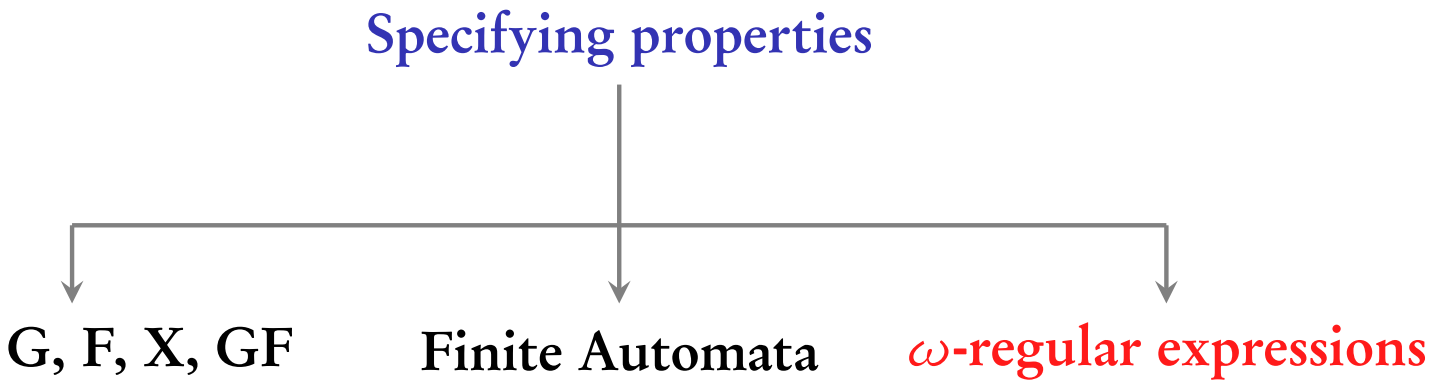
Specifying properties



G, F, X, GF

Finite Automata


Specifying properties
G, F, X, GF
Finite Automata
ω-regular expressions



- Use ω-regular expressions to specify properties
- An algorithm for model-checking ω-regular expressions on transition systems