Unit-4: Regular properties

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Chennai Mathematical Institute

NPTEL-course

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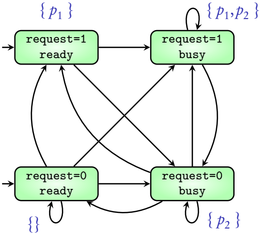
Module 2:

A gentle introduction to automata



AP = set of **atomic propositions**

AP-INF = set of **infinite words** over *PowerSet*(**AP**)

A property over AP is a **subset** of AP-INF

Goal: Need **finite descriptions** of properties

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Here: **Finite state automata** to describe sets of **words**

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Here: **Finite state automata** to describe sets of finite **words**

Alphabet: $\{ a, b \}$

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$$L_1 = \{ ab,\ abab,\ ababab,\ \ldots \}$$

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Design a TS with actions $\{ a, b \}$ and mark some states as **accepting** so that

the set of **all paths** from an initial state to an accepting state equals L_1

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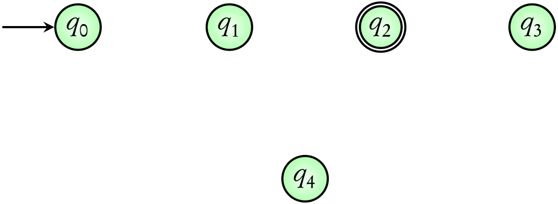

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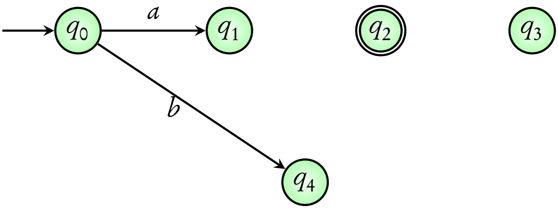

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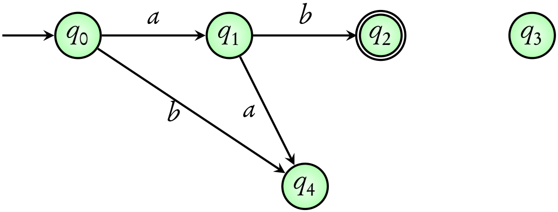

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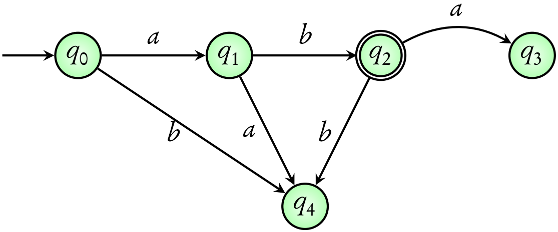

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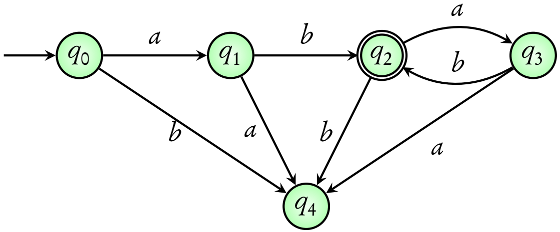

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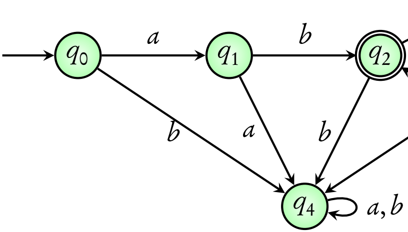

Alphabet: $\{ a, b \}$

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L_2 is the set of all words starting with a

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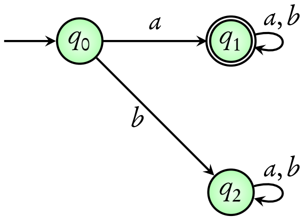

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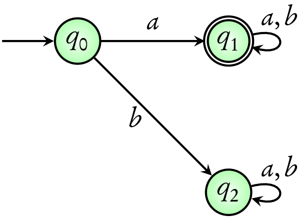

Finite Automaton

Coming next: Some terminology

Alphabet $\Sigma = \{ a, b \}$

$$\textbf{Alphabet} \quad \Sigma = \{ a, b \}$$

$$\Sigma \cdot \Sigma = \{ a, b \} \cdot \{ a, b \}$$

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Alphabet $\Sigma = \{ a, b \}$

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Σ^2 = words of length 2

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$\vdots$

Σ^k = words of length k

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$$\textbf{Alphabet} \quad \Sigma = \{ a, b \}$$

$$\Sigma \cdot \Sigma = \{ a, b \} \cdot \{ a, b \} = \{ aa, ab, ba, bb \}$$

$$aba \cdot \epsilon = aba$$

$$\epsilon \cdot bbb = bbb$$

$$w \cdot \epsilon = w$$

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$\Sigma^0 = \{ \epsilon \}$ (empty word, with length 0)

$\Sigma^1 =$ words of length 1

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$$\begin{aligned}aba \cdot \epsilon &= aba \\ \epsilon \cdot bbb &= bbb \\ w \cdot \epsilon &= w \\ \epsilon \cdot w &= w\end{aligned}$$

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$\Sigma^* = \bigcup_{i \geq 0} \Sigma^i$

$=$ **set of all finite length words**

$\Sigma^* \;=\;$ set of **all words** over Σ

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Any set of words is called a **language**

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words starting with an a

words starting with a b

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$\{\, \epsilon,\, bbb,\, bbbbbb,\, (bbb)^3,\, \dots \}$

words starting and ending with an a

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$a\Sigma^*a$ words starting and ending with an a

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In this module...

Task: Design **Finite Automata** for some languages

Words

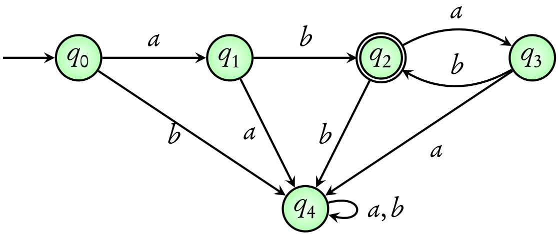
Languages

Finite Automata

Alphabet: $\{ a, b \}$

$$L_1 = \{ ab, abab, ababab, \ldots \}$$

Design a Finite automaton for L_1



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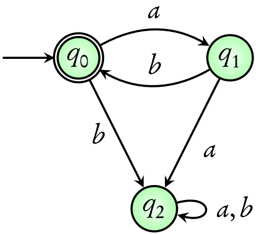
$$L_3 = \{ \epsilon, ab, abab, ababab, \dots \}$$

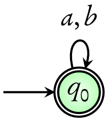
Design a Finite automaton for L_3



Alphabet: $\{ a, b \}$

$$\Sigma^* = \{ \epsilon, a, b, aa, ab, ba, bb \ldots \}$$

Design a Finite automaton for Σ^*



Alphabet: $\{ a, b \}$

$$a^* = \{ \epsilon, a, aa, aaa, aaaa, a^5, \ldots \}$$

a^* is the set of all words having only a

Design a Finite automaton for a^*

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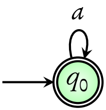

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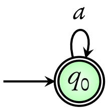


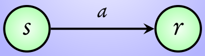
Non-deterministic automaton

Transition Systems

Deterministic

Single initial state

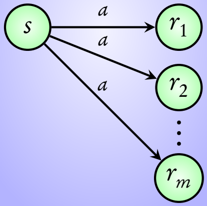
and



Non-deterministic

Multiple initial states

or



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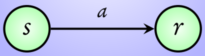

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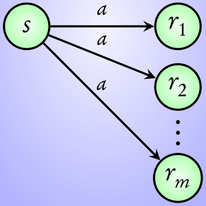

Same applies in the case of Finite Automata

Alphabet: $\{ a, b \}$

$$ab^* = \{ a, ab, ab^2, ab^3, ab^4, \ldots \}$$

Design a Finite automaton for ab^*

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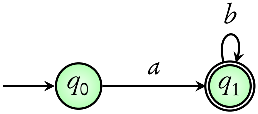

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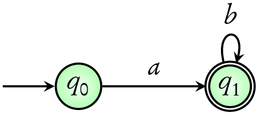

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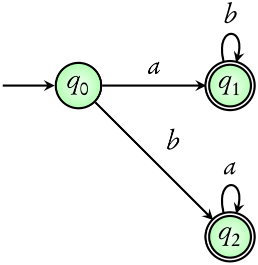

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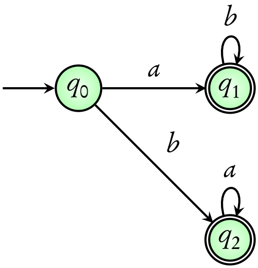

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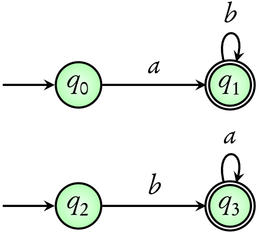

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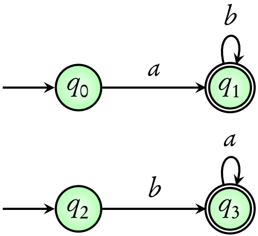
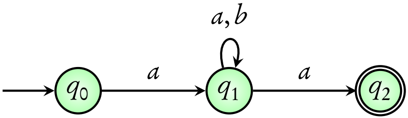


Multiple initial states: **non-deterministic** automaton

What is the language of the following automaton?



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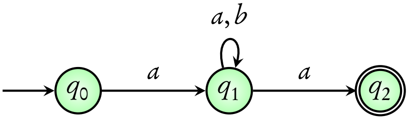

Answer: $a\,\Sigma^* a$

words starting and ending with a

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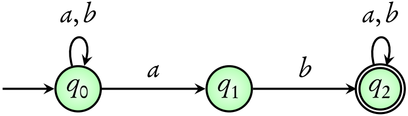

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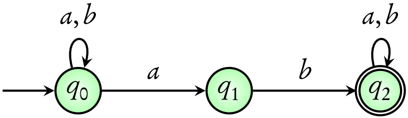

Answer: $\Sigma^* ab \, \Sigma^*$

words **containing** *ab*

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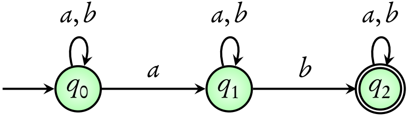

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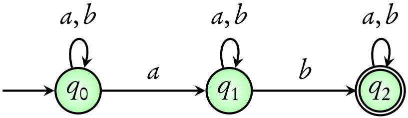

Answer: $\Sigma^* a \, \Sigma^* \, b \, \Sigma^*$

words where there exists an **a** followed by a **b** after sometime

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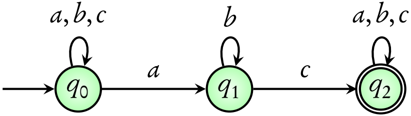

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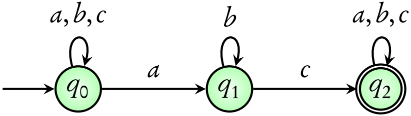

Answer: $\Sigma^* a\, b^*\, c\, \Sigma^* \quad (\Sigma = \{ a, b, c \})$

words where there exists an **a** followed by only **b**'s and after sometime a **c** occurs

Alphabet: $\{ a, b \}$

$$L = \{ \epsilon, ab, aabb, aaabbb, \ldots, a^i b^i, \ldots \}$$

Can we design a Finite automaton for L?

Alphabet: $\{ a, b \}$

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Can we design a Finite automaton for L?

Need **infinitely many states** to remember the number of a's

Alphabet: $\{ a, b \}$

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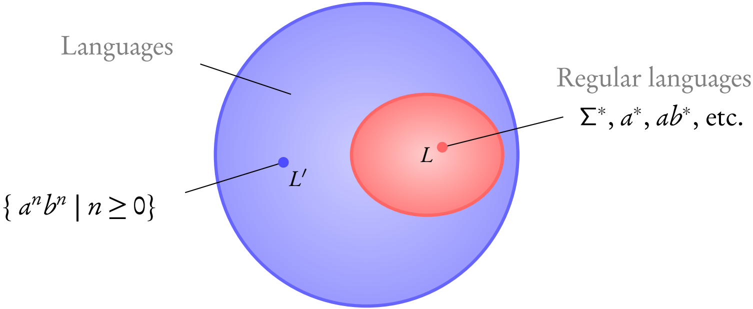
Can we design a Finite automaton for L?

Need **infinitely many states** to remember the number of a's

Cannot construct finite automaton for this language

Regular languages



Definition

A language is called **regular** if it can be **accepted** by a finite automaton

Words

Languages

Finite Automata

Deterministic (DFA)

Non-deterministic (NFA)

Regular languages

Words

Languages

Finite Automata

Deterministic (DFA)

Non-deterministic (NFA)

Regular languages

Next module: Are DFA and NFA **equivalent**?