Unit-11: Binary Decision Diagrams (BDDs)

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Chennai Mathematical Institute

NPTEL-course

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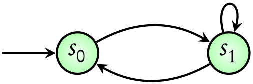
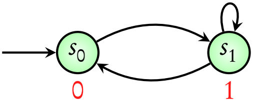
Module 3:

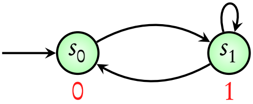
Representing transition systems using OBDDs



s_0
s_1



s_0
s_1
0
1

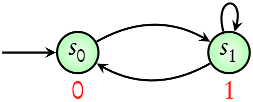


Transitions:

$0 \longrightarrow 1$

$1 \longrightarrow 1$

$1 \longrightarrow 0$



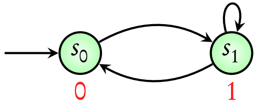
Transitions:

0 $\longrightarrow$ 1

1 $\longrightarrow$ 1

1 $\longrightarrow$ 0

x	x'	
0	0	0
0	1	1
1	0	1
1	1	1



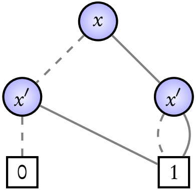
Transitions:

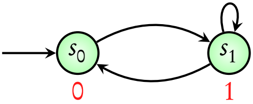
$0 \longrightarrow 1$

$1 \longrightarrow 1$

$1 \longrightarrow 0$

x	x'	
0	0	0
0	1	1
1	0	1
1	1	1





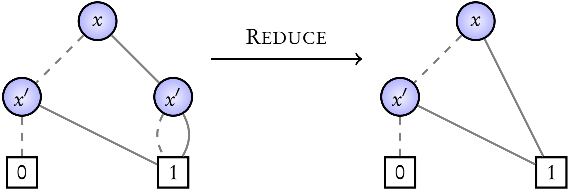
Transitions:

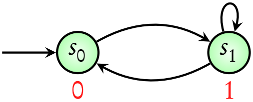
0 ⟶ 1

1 ⟶ 1

1 ⟶ 0

x	x'	
0	0	0
0	1	1
1	0	1
1	1	1

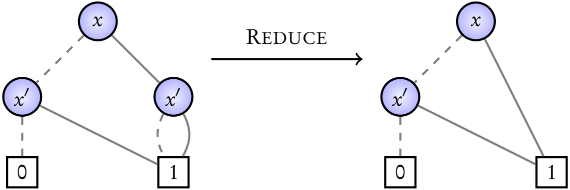




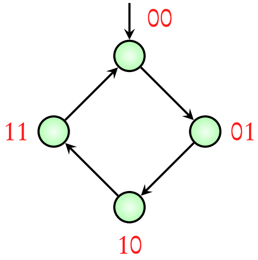
Transitions:

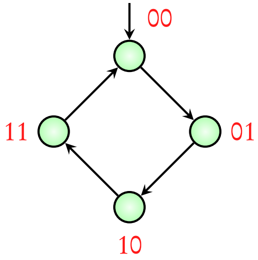
0 ⟶ 1
1 ⟶ 1
1 ⟶ 0

x	x'	
0	0	0
0	1	1
1	0	1
1	1	1



ROBDD representation of a transition system


00
11
01
10



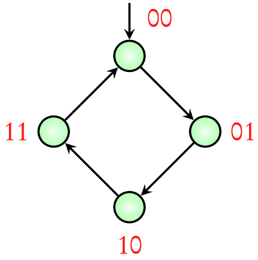
Transitions:

00 ⟶ 01

01 ⟶ 10

10 ⟶ 11

11 ⟶ 00



Transitions:

00 ⟶ 01

01 ⟶ 10

10 ⟶ 11

11 ⟶ 00

x_2	x_1	x'_2	x'_1	
0	0	0	1	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	1
		Rest		0

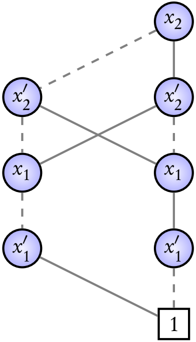
x_2	x_1	x_2'	x_1'	
0	0	0	1	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	1

x_2	x_1	x_2'	x_1'	
0	0	0	1	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	1

x_2	x_2'	x_1	x_1'	
0	0	0	1	1
0	1	1	0	1
1	1	0	1	1
1	0	1	0	1

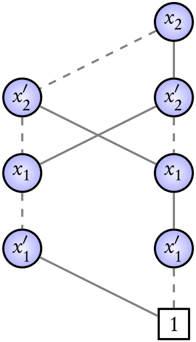
x_2	x_1	x_2'	x_1'	
0	0	0	1	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	1

x_2	x_2'	x_1	x_1'	
0	0	0	1	1
0	1	1	0	1
1	1	0	1	1
1	0	1	0	1

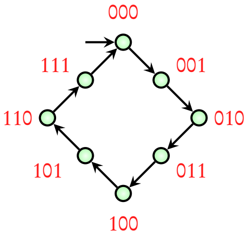


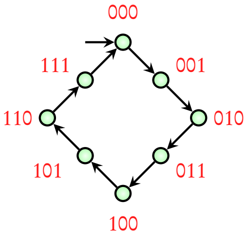
x_2	x_1	x'_2	x'_1	
0	0	0	1	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	1

x_2	x'_2	x_1	x'_1	
0	0	0	1	1
0	1	1	0	1
1	1	0	1	1
1	0	1	0	1

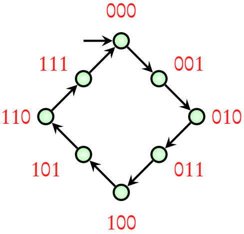


ROBDD with ordering $[x_2, x'_2, x_1, x'_1]$ for ring with 4 **nodes**

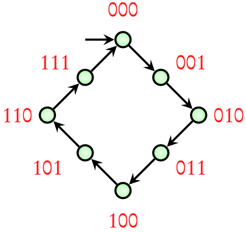

000
111
001
110
010
101
011
100



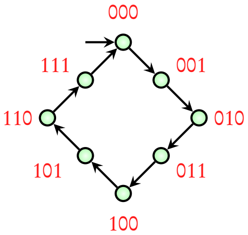
$x_3 \quad x_3' \quad x_2 \quad x_2' \quad x_1 \quad x_1'$



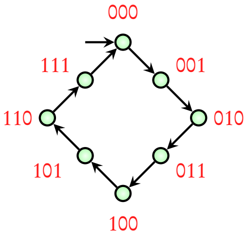
x_3	x_3'	x_2	x_2'	x_1	x_1'
0		0		0	



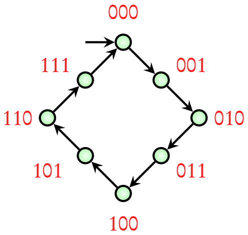
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1



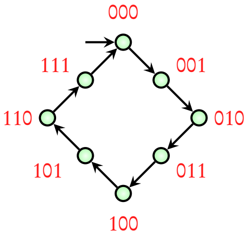
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0		0		1	



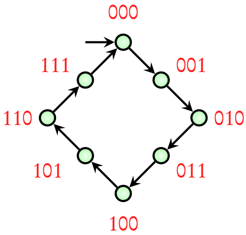
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0



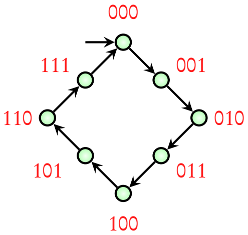
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0		1		0	



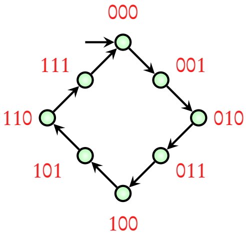
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1



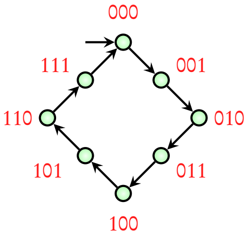
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0		1		1	



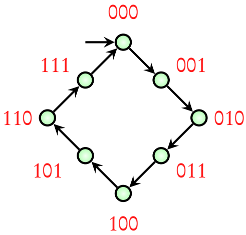
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0



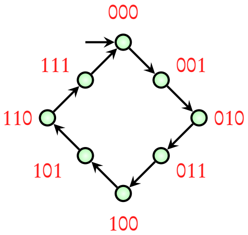
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1		0		0	



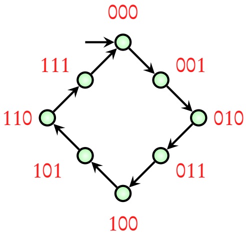
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1



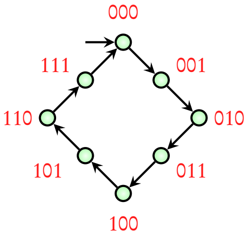
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1		0		1	



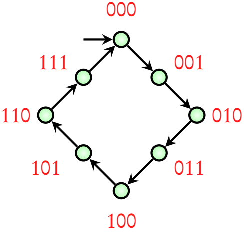
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0



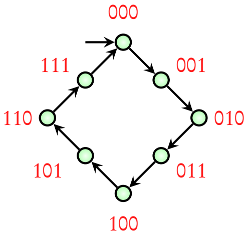
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1		1		0	



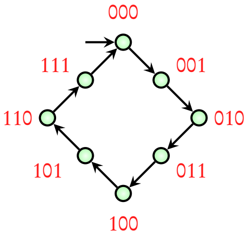
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1



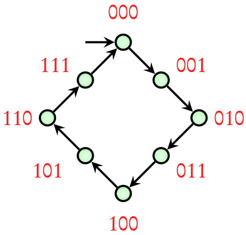
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1		1		1	



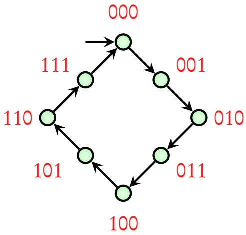
x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0



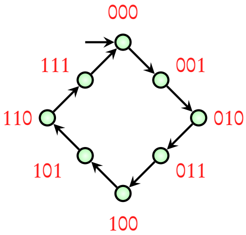
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0



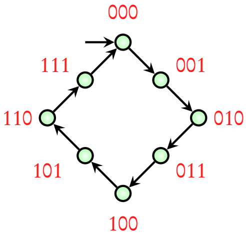
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0



x_3	x_3'	x_2	x_2'	x_1	x_1'
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0

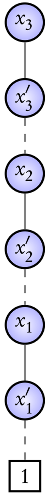


x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0

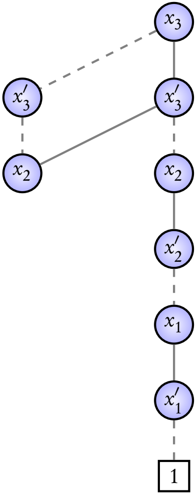


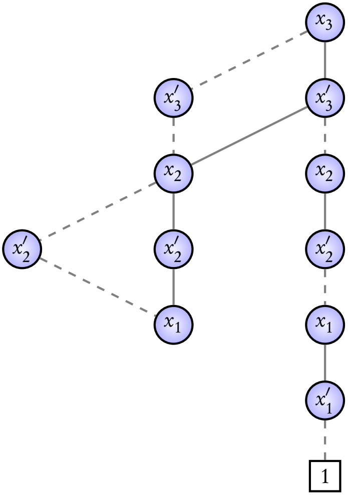
x_3	x'_3	x_2	x'_2	x_1	x'_1
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	1	0	1
0	1	1	0	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	1	0	1
1	0	1	0	1	0

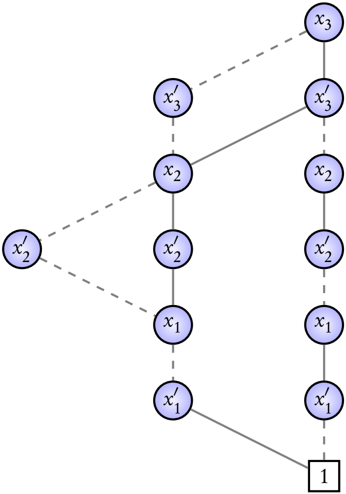
- Either fully 10 – 10 – 10
- Or 01 occurs:
  - after 01 only a sequence of 10
  - before 01 only sequences of 00 or 11

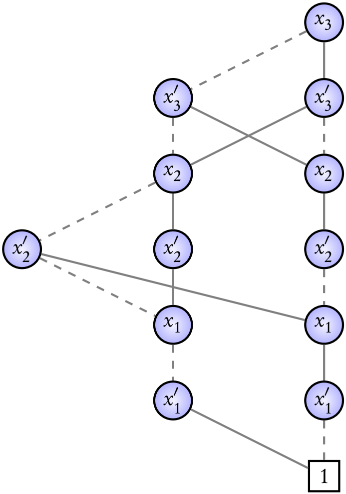


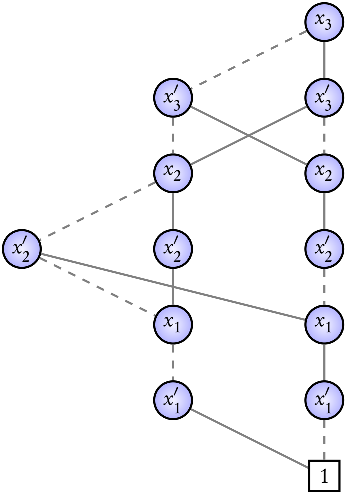
x_3
x_3'
x_2
x_2'
x_1
x_1'
1


x_3
x_3'
x_3'
x_2
x_2
x_2'
x_1
x_1'
1

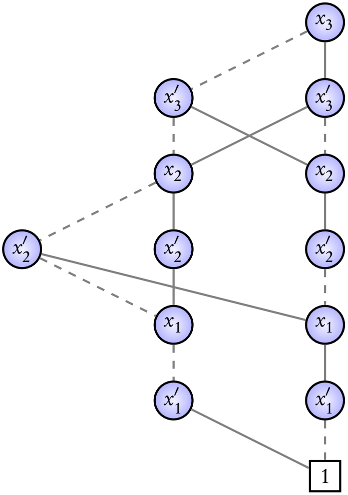

x_3
x_3'
x_3'
x_2
x_2
x_2'
x_2'
x_2'
x_1
x_1
x_1'
1


x_3
x_3'
x_3'
x_2
x_2
x_2'
x_2'
x_2'
x_1
x_1
x_1'
x_1'
1


x_3
x_3'
x_3'
x_2
x_2
x_2'
x_2'
x_2'
x_1
x_1
x_1'
x_1'
1

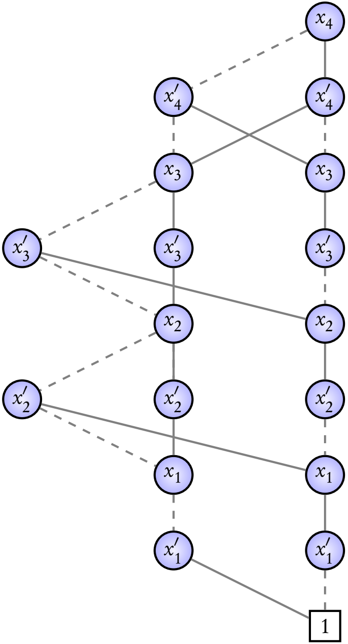


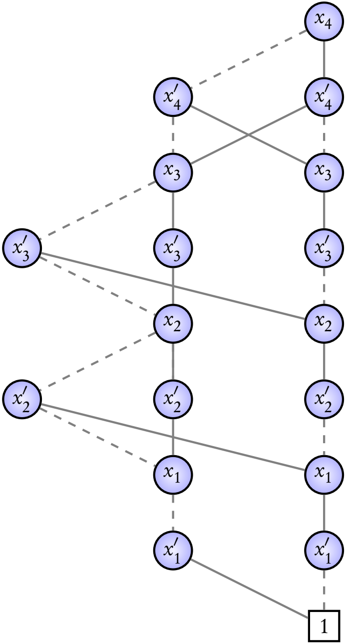
ROBDD for ring with 2^3 states



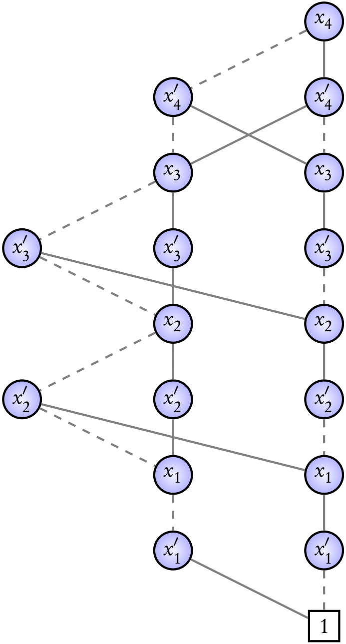
ROBDD for ring with 2^3 states

has less than $2 \cdot 3 \cdot 3$ nodes


x_4
x'_4
x'_4
x_3
x_3
x'_3
x'_3
x'_3
x_2
x_2
x'_2
x'_2
x'_2
x_1
x_1
x'_1
x'_1
1



ROBDD for ring with 2^4 states



ROBDD for ring with 2^4 states

has less than $2 \cdot 4 \cdot 3$ nodes

ROBDD for ring with 2^n states will have less than $6 \cdot n$ nodes

ROBDD for ring with 2^n states will have less than $6 \cdot n$ nodes

ROBDDs can **efficiently represent** transition systems

ROBDD for ring with 2^n states will have less than $6 \cdot n$ nodes

ROBDDs can **efficiently represent** transition systems

LTL and CTL model-checking can be efficiently done using **operations on ROBDDs**