Unit-11: Binary Decision Diagrams (BDDs)

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NPTEL-course

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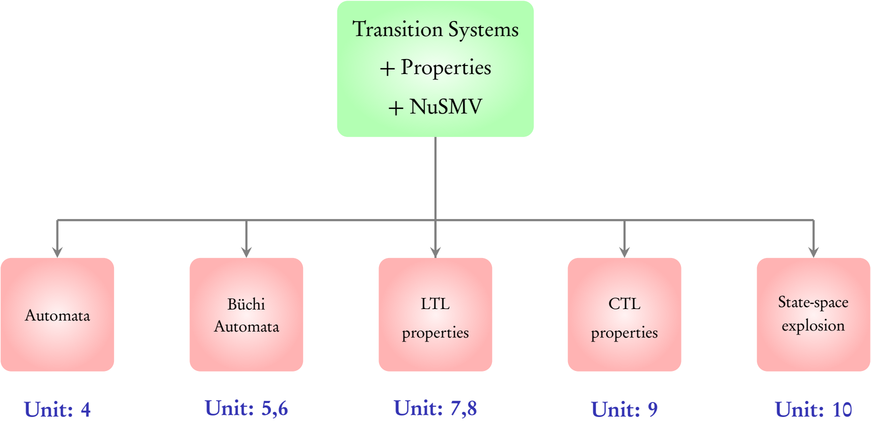
Module 1:

Introduction to BDDs

Model-checking



In this unit: An **efficient data structure** for representing **transition systems**

- **Module 1:** Introduction to the data structure: **Binary Decision Diagrams (BDDs)**
- **Module 2:** Operations on BDDs
- **Module 3:** Using BDDs in the model-checking process

In this unit: An **efficient data structure** for representing **transition systems**

- **Module 1:** Introduction to the data structure: **Binary Decision Diagrams (BDDs)**
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Reference: Logic in Computer Science, 2^{nd} edition, by *Huth and Ryan*, Section 6.1 - 6.3

Boolean functions

x, y : ***Boolean*** variables

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$$f(x,y) = x + y \qquad \textbf{\textit{Boolean function}}$$

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$$f(x,y) = x + y \qquad \textbf{\textit{Boolean function}}$$

$$\begin{aligned} 0 + 0 &= 0 \\ 0 + 1 &= 1 \\ 1 + 0 &= 1 \\ 1 + 1 &= 1 \end{aligned}$$

x, y : ***Boolean*** variables

$$f(x,y) = x + y$$ ***Boolean function***

$$\begin{aligned} 0 + 0 &= 0 \\ 0 + 1 &= 1 \\ 1 + 0 &= 1 \\ 1 + 1 &= 1 \end{aligned}$$

Logical OR

x, y : ***Boolean*** variables

$f(x,y) = x + y$ ***Boolean function***

$$\begin{aligned} 0+0 &= 0 \\ 0+1 &= 1 \\ 1+0 &= 1 \\ 1+1 &= 1 \end{aligned}$$

Logical OR

x	y	$f(x,y)$
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

x, y : ***Boolean*** variables

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$$f(x, y) = x \cdot y \qquad \textbf{\textit{Boolean function}}$$

x, y : ***Boolean*** variables

$f(x,y) = x \cdot y$ ***Boolean function***

$$
\begin{aligned}
0 \cdot 0 &= 0 \\
0 \cdot 1 &= 0 \\
1 \cdot 0 &= 0 \\
1 \cdot 1 &= 1
\end{aligned}
$$

x, y : ***Boolean*** variables

$$f(x,y) = x \cdot y$$ ***Boolean function***

$$\begin{aligned} 0 \cdot 0 &= 0 \\ 0 \cdot 1 &= 0 \\ 1 \cdot 0 &= 0 \\ 1 \cdot 1 &= 1 \end{aligned}$$

Logical AND

x, y : ***Boolean*** variables

$f(x,y) = x \cdot y$ ***Boolean function***

$$\begin{aligned} 0 \cdot 0 &= 0 \\ 0 \cdot 1 &= 0 \\ 1 \cdot 0 &= 0 \\ 1 \cdot 1 &= 1 \end{aligned}$$

Logical AND

x	y	$f(x,y)$
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

x : ***Boolean*** variable

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$f(x) = \overline{x}$ ***Boolean function***

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$$\begin{aligned} \overline{0} &= 1 \\ \overline{1} &= 0 \end{aligned}$$

x : ***Boolean*** variable

$f(x) = \overline{x}$ ***Boolean function***

$$\begin{aligned} \overline{0} &= 1 \\ \overline{1} &= 0 \end{aligned}$$

Logical NOT

x : ***Boolean*** variable

$f(x) = \overline{x}$ ***Boolean function***

$$\begin{aligned} \overline{0} &= 1 \\ \overline{1} &= 0 \end{aligned}$$

Logical NOT

x	$f(x)$
0	1
1	0

Truth table

$x_1, x_2, \ldots, x_n$: ***Boolean*** variables

$$f : \{x_1, x_2, \ldots, x_n\} \mapsto \{0, 1\}$$

Boolean function

$+ \quad \cdot \quad -$

Boolean operations

$x_1, x_2, \ldots, x_n$: ***Boolean*** variables

$$f : \{x_1, x_2, \ldots, x_n\} \mapsto \{0, 1\}$$

Boolean function

$+ \quad \cdot \quad -$

Boolean operations

Examples: $f_1(x,y) = \overline{x} + y, \quad f_2(x,y,z) = x \cdot y + \overline{y} \cdot z, \quad f_3(x,y,z) = \overline{x + \overline{y} \cdot z}$

Representing boolean functions

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

x	y	z	f
0	0	0	0
0	0	1	1
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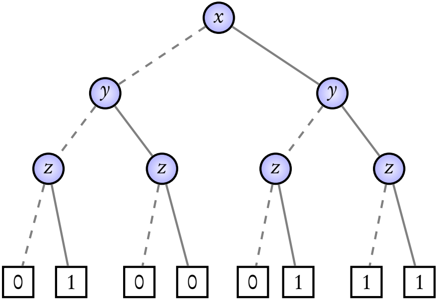
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

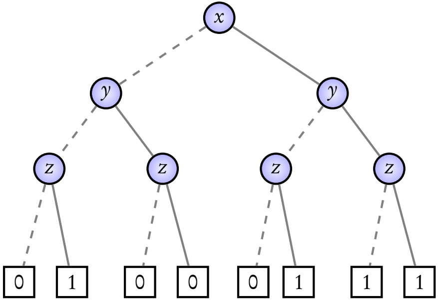


Truth table

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table



Binary Decision Tree

Operations on truth tables

$$g(x,y,z)=\overline{f(x,y,z)}=\overline{x\cdot y+\overline{y}\cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$g(x,y,z) = \overline{f(x,y,z)} = \overline{x \cdot y + \overline{y} \cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$g(x,y,z) = \overline{f(x,y,z)} = \overline{x \cdot y + \overline{y} \cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$$g(x,y,z) = \overline{f(x,y,z)} = \overline{x \cdot y + \overline{y} \cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	1
0	0	1	0
0	1	0	
0	1	1	
1	0	0	
1	0	1	
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x	y	z	g
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0	1	0	1
0	1	1	
1	0	0	
1	0	1	
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$$g(x,y,z) = \overline{f(x,y,z)} = \overline{x \cdot y + \overline{y} \cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
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1	0	0	0
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x	y	z	g
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x	y	z	f
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0	0	1	1
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0	1	1	0
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1	0	1	1
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x	y	z	g
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$$g(x,y,z) = \overline{x} \cdot \overline{y}$$

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$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

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$$h(x,y,z) = f(x,y,z) + g(x,y,z)$$

x	y	z	g
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

x	y	z	f
0	0	0	0
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1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

x	y	z	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
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1	0	0	0
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x	y	z	h
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
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x	y	z	f
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0	0	1	1
0	1	0	0
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0	0	1	1
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1	0	1	0
1	1	0	0
1	1	1	0

x	y	z	h
0	0	0	1
0	0	1	
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x	y	z	f
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x	y	z	f
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x	y	z	g
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0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

x	y	z	h
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	
1	0	0	
1	0	1	
1	1	0	
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x	y	z	f
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x	y	z	h
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x	y	z	f
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1	0	1	1
1	1	0	1
1	1	1	1

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0	0	1	1
0	1	0	0
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1	0	0	0
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1	1	0	0
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x	y	z	h
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0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
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1	1	0	1
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x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
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x	y	z	g
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1	0	0	0
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1	1	0	0
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x	y	z	h
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0	0	1	1
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x	y	z	f
0	0	0	0
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$$g(x,y,z) = x$$

x	y	z	g
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1	0	0	1
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1	1	0	1
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x	y	z	f
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x	y	z	f
0	0	0	0
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0	1	0	0
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0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

$$g(x,y,z) = x$$

$$h(x,y,z) = f(x,y,z) \cdot g(x,y,z)$$

x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
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0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

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x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
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0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
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0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
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x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
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x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
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x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
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x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table representation for boolean functions

- **Space:** For n variables, needs to store $2^n \cdot (n + 1)$ bits
- **Operations:** Visit each entry of truth table

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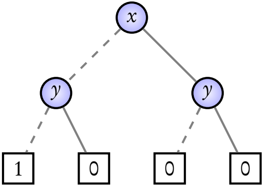
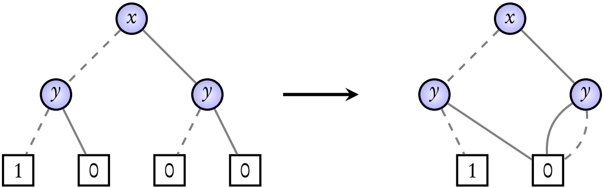
If boolean functions are represented using truth tables, a circuit with 100 variables needs **more than** 2^{100} **bits!**

Coming next: Efficient representation for Boolean formulas

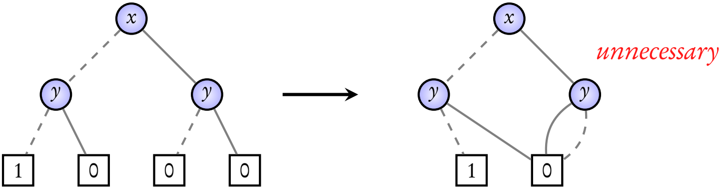
$$\overline{x} \cdot \overline{y}$$



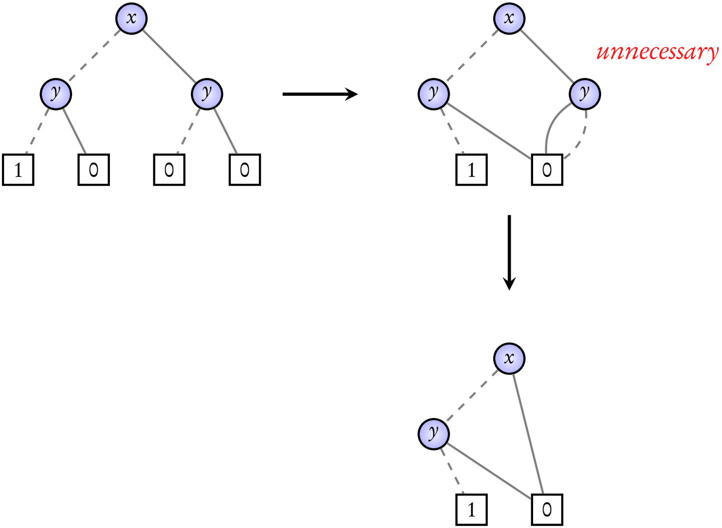
$\overline{x} \cdot \overline{y}$



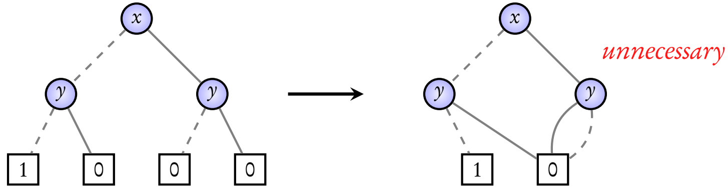
$\overline{x} \cdot \overline{y}$

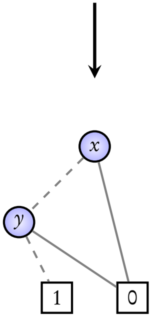


$\overline{x} \cdot \overline{y}$

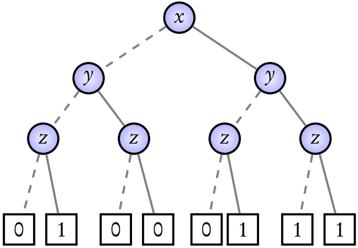


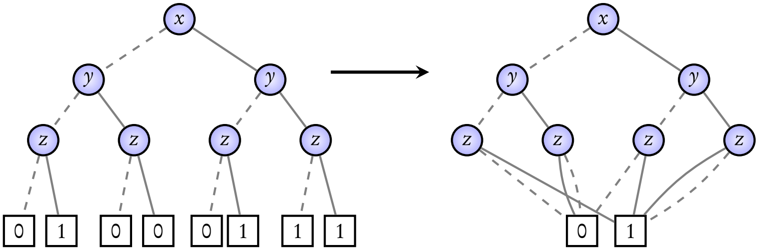
$\overline{x} \cdot \overline{y}$

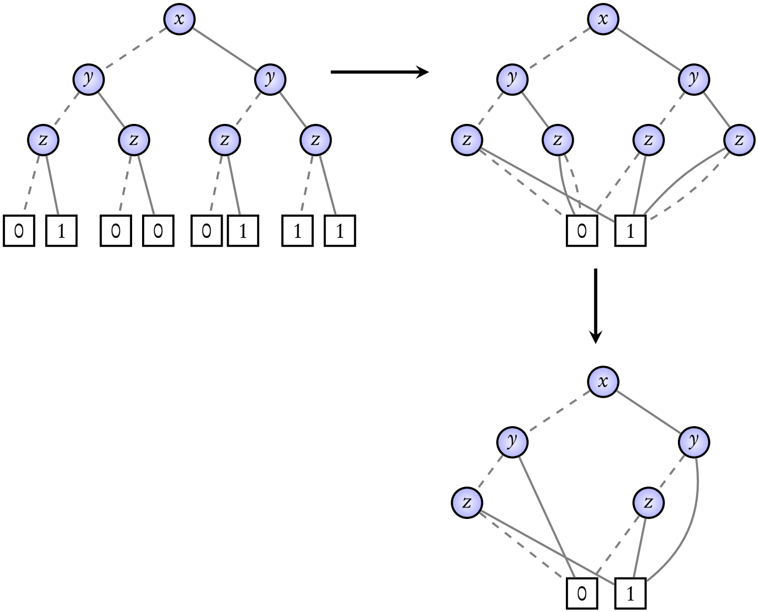




Binary Decision Diagram


x
y
y
z
z
z
z
0
1
0
0
0
1
1
1


x
y
y
z
z
z
z
0
1
0
0
0
1
1
1
x
y
y
z
z
z
z
0
1


x
y
y
z
z
z
z
0
1
0
0
0
1
1
1
x
y
y
z
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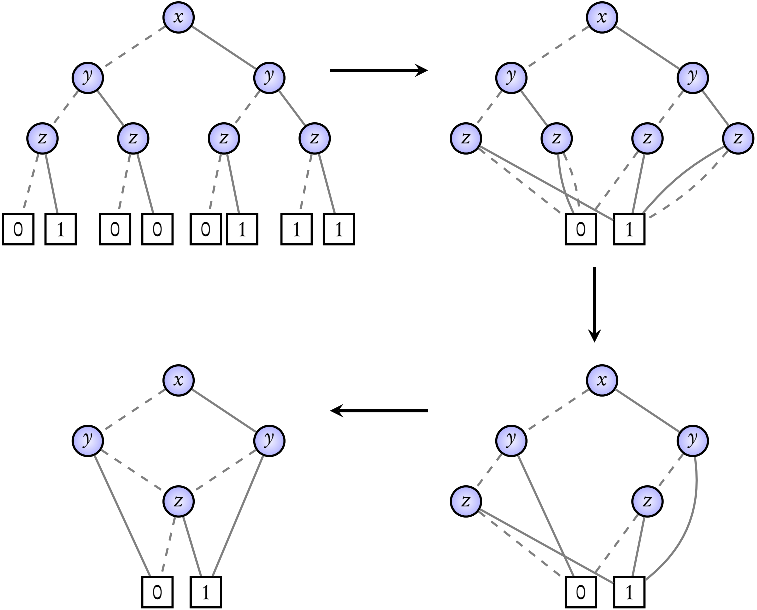
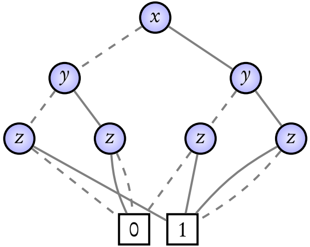
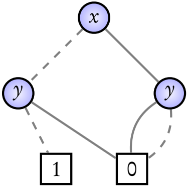
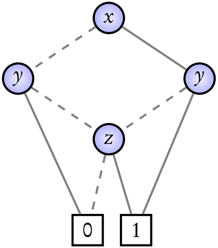
z
z
0
1
x
y
y
z
z
0
1

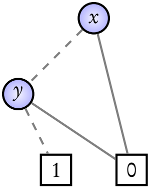

x
y y
z z z z
0 1 0 0 0 1 1 1
x
y y
z z z z
0 1
x
y y
z z
0 1
x
y y
z
0 1

Binary Decision Diagrams

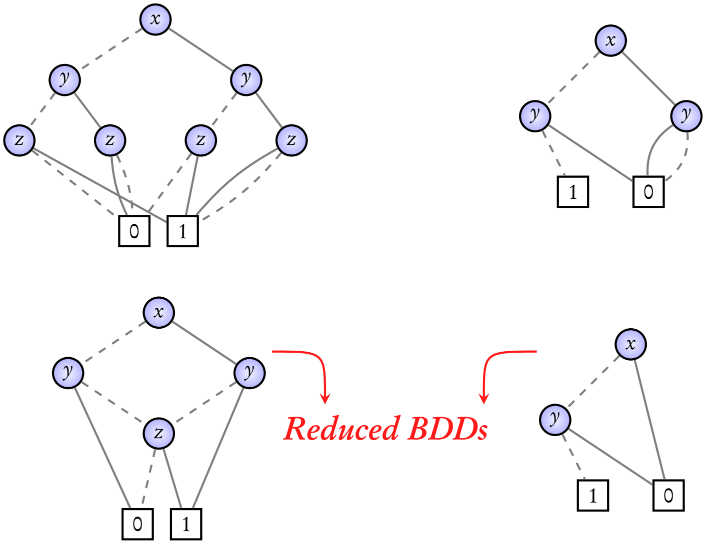








Binary Decision Diagrams

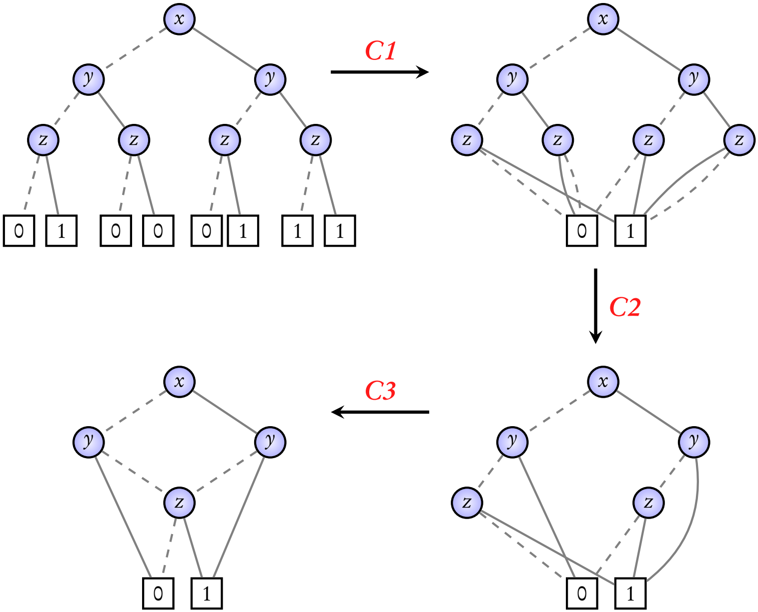


Reduction rules for BDDs

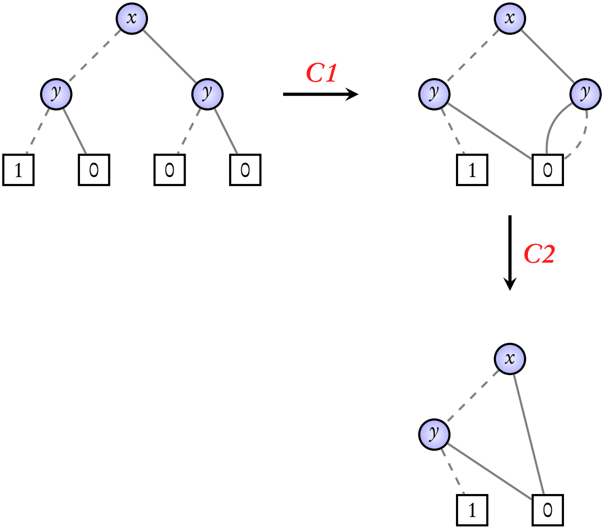
- **C1:** Removal of **duplicate leaves**

- **C2:** Removal of **redundant tests**

- **C3:** Removal of **duplicate sub-trees**


C1
C2
C3
x
y
z
0
1

$\overline{x} \cdot \overline{y}$



Representing boolean functions

BDDs

Reduced BDDs