

Week-1: Introduction to model checking

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Chennai Mathematical Institute

NPTEL-course

July - November 2015

Module 2:
Modeling hardware circuits



| x_1 | x_2 | y |
|-------|-------|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |



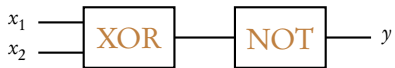
| x_1 | x_2 | y |
|-------|-------|-----|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



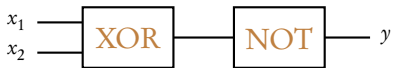
| x | y |
|-----|-----|
| 0 | 1 |
| 1 | 0 |



| x_1 | x_2 | y |
|-------|-------|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



$$y = \text{NOT} (\text{XOR} (x_1, x_2))$$

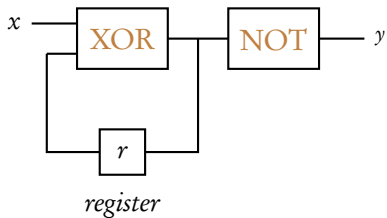


$$y = \text{NOT} (\text{XOR} (x_1, x_2))$$



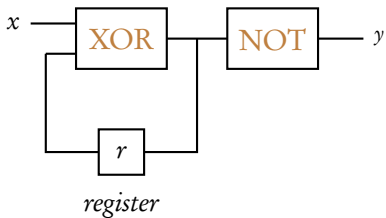
| x_1 | x_2 | y |
|-------|-------|-----|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$$y = \text{NOT}(\text{XOR}(x, r))$$



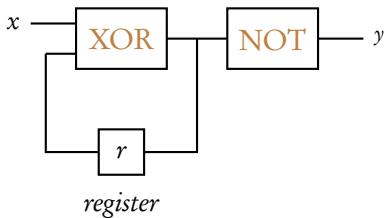
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$$r_{\text{next}} = \text{XOR} (x, r)$$



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x 1

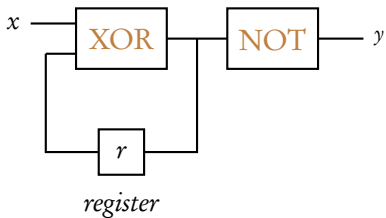
r 0

y 0



$$y = \text{NOT} (\text{XOR} (x, r))$$

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x 1

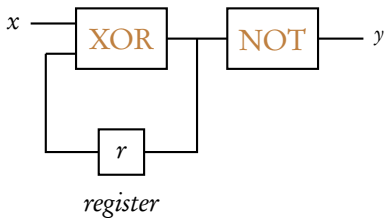
r 0 1

y 0



$$y = \text{NOT} (\text{XOR} (x, r))$$

$$r_{\text{next}} = \text{XOR} (x, r)$$

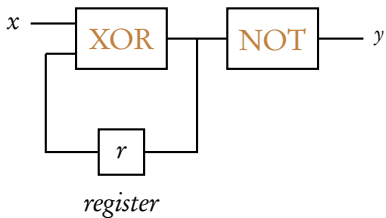


| | | |
|-----|---|---|
| x | 1 | 1 |
| r | 0 | 1 |
| y | 0 | 1 |



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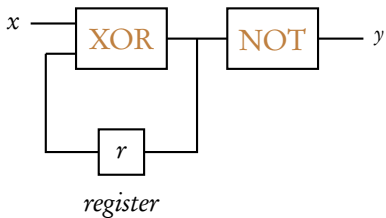


| | | | |
|-----|---|---|---|
| x | 1 | 1 | |
| r | 0 | 1 | 0 |
| y | 0 | 1 | |

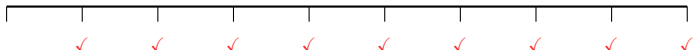


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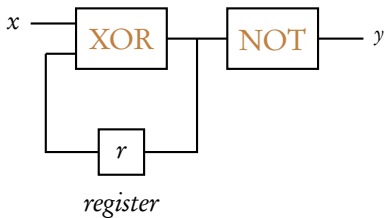


| | | | |
|-----|---|---|---|
| x | 1 | 1 | 0 |
| r | 0 | 1 | 0 |
| y | 0 | 1 | 1 |

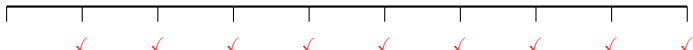


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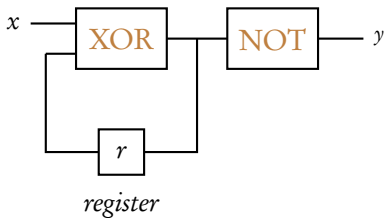


| | | | | |
|-----|---|---|---|---|
| x | 1 | 1 | 0 | |
| r | 0 | 1 | 0 | 0 |
| y | 0 | 1 | 1 | |



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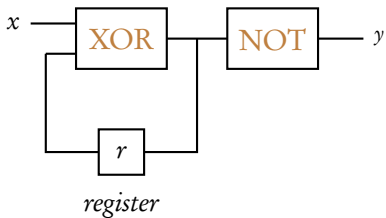


| | | | | |
|-----|---|---|---|---|
| x | 1 | 1 | 0 | 1 |
| r | 0 | 1 | 0 | 0 |
| y | 0 | 1 | 1 | 0 |

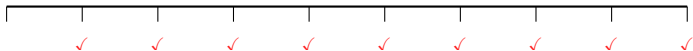


$$y = \text{NOT}(\text{XOR}(x, r))$$

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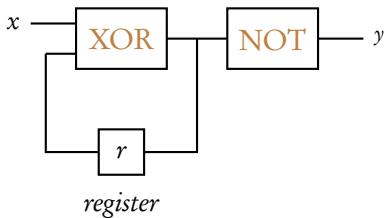


| | | | | | |
|-----|---|---|---|---|---|
| x | 1 | 1 | 0 | 1 | |
| r | 0 | 1 | 0 | 0 | 1 |
| y | 0 | 1 | 1 | 0 | |



$$y = \text{NOT} (\text{XOR} (x, r))$$

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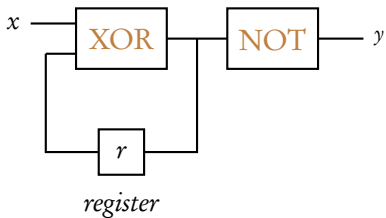


| | | | | | |
|-----|---|---|---|---|---|
| x | 1 | 1 | 0 | 1 | 1 |
| r | 0 | 1 | 0 | 0 | 1 |
| y | 0 | 1 | 1 | 0 | 1 |



$$y = \text{NOT}(\text{XOR}(x, r))$$

$$r_{\text{next}} = \text{XOR}(x, r)$$

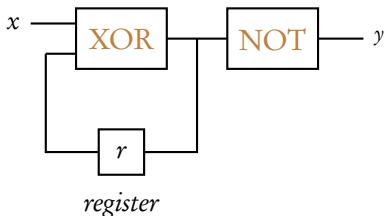


| | | | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|---|---|-----|
| x | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| r | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| y | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |



$$y = \text{NOT}(\text{XOR}(x, r))$$

$$r_{\text{next}} = \text{XOR}(x, r)$$



$$x = 0, r = 0, y = 1$$

$$x = 1, r = 0, y = 0$$

$$x = 0, r = 1, y = 0$$

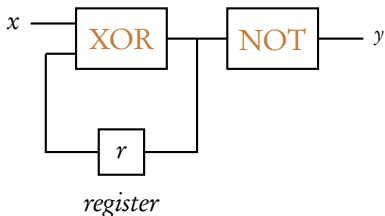
$$x = 1, r = 1, y = 1$$

| | | | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|---|---|-----|
| x | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| r | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| y | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |



$$y = \text{NOT}(\text{XOR}(x, r))$$

$$r_{\text{next}} = \text{XOR}(x, r)$$



↓

x = 0, r = 0, y = 1

↓

x = 1, r = 0, y = 0

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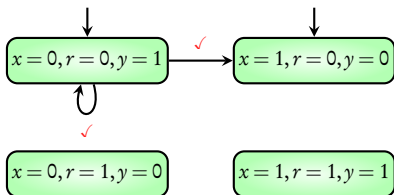
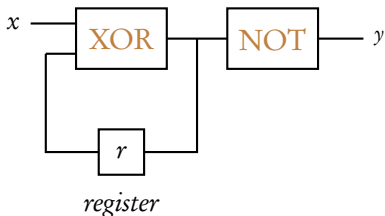
x = 1, r = 1, y = 1

| | | | | | | | | | | | |
|----------|---|---|---|---|---|---|---|---|---|---|-----|
| <i>x</i> | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| <i>r</i> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| <i>y</i> | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |

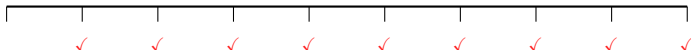


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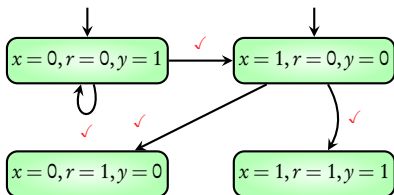
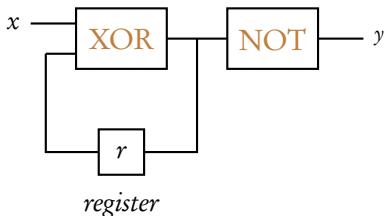


| | | | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|---|---|-----|
| x | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| r | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| y | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |



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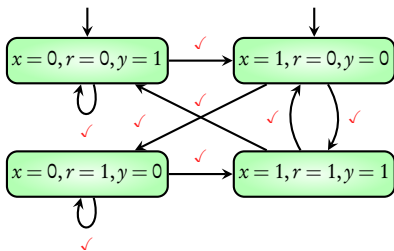
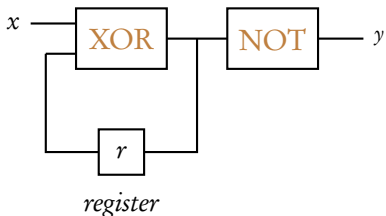


| | | | | | | | | | | | |
|----------|---|---|---|---|---|---|---|---|---|---|-----|
| <i>x</i> | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| <i>r</i> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| <i>y</i> | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |

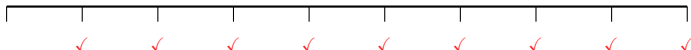


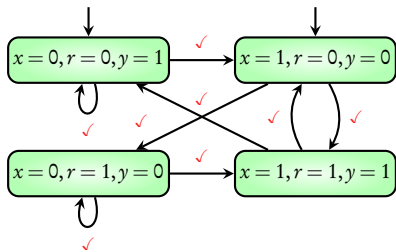
$$y = \text{NOT}(\text{XOR}(x, r))$$

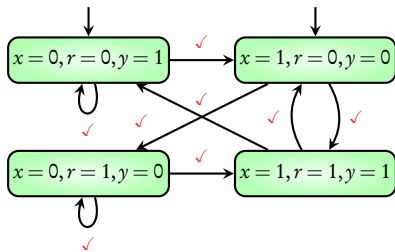
$$r_{\text{next}} = \text{XOR}(x, r)$$



| | | | | | | | | | | | |
|----------|---|---|---|---|---|---|---|---|---|---|-----|
| <i>x</i> | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | |
| <i>r</i> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | ... |
| <i>y</i> | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | |



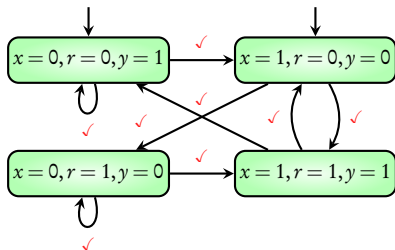




More than one initial state

States with **more than one transition** on an action

*Non-deterministic
transition system*



More than one initial state

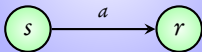
States with more than one transition on an action

Transition Systems

Deterministic

Single initial state

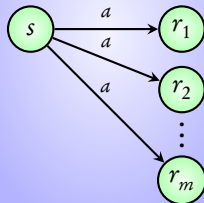
and



Non-deterministic

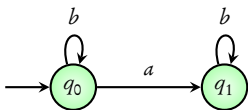
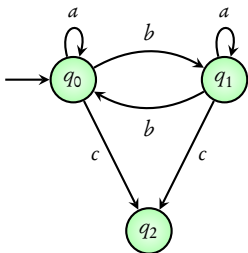
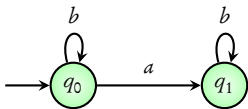
Multiple initial states

or

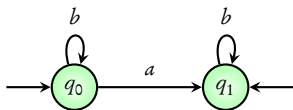
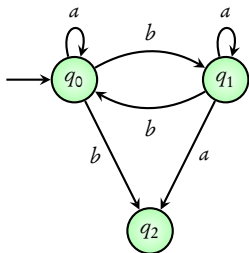
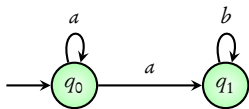


Coming next: examples of deterministic and non-deterministic transition systems

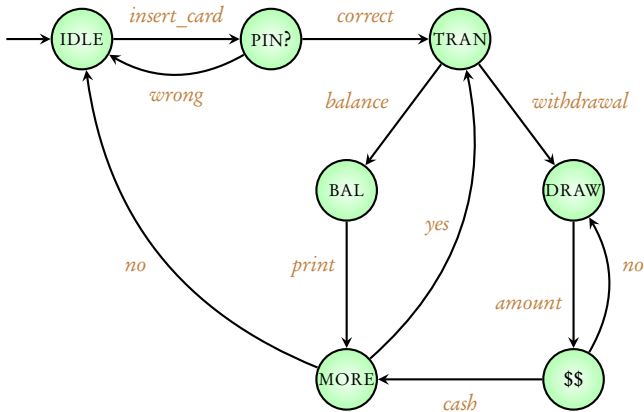
Deterministic



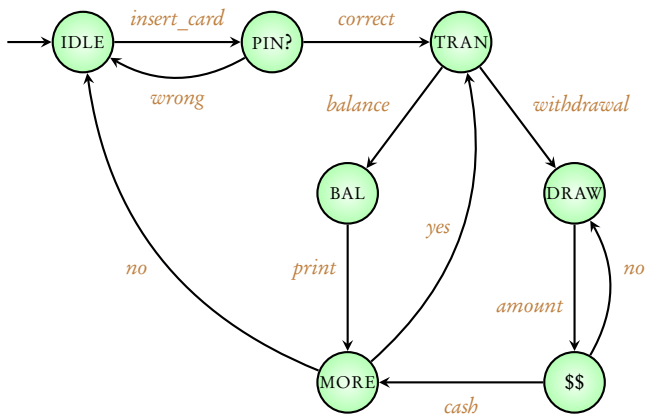
Non-deterministic



Model of ATM

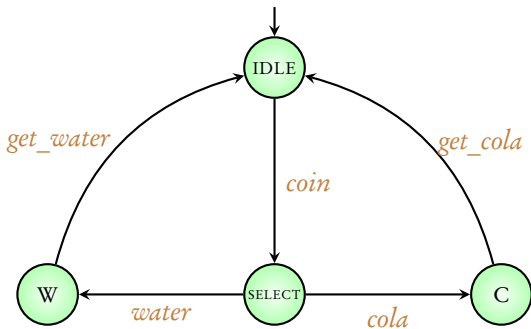


Model of ATM

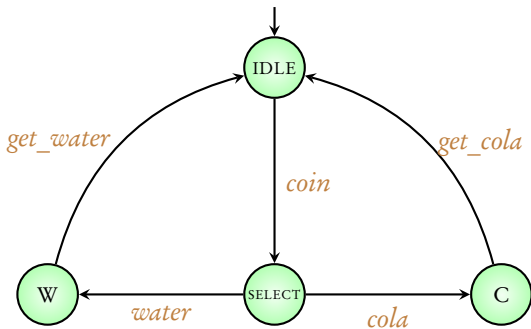


Deterministic transition system

Model of vending machine

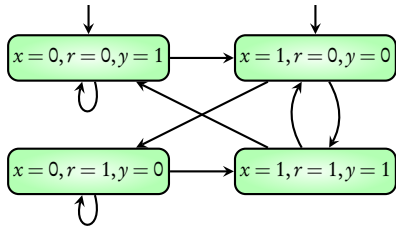


Model of vending machine

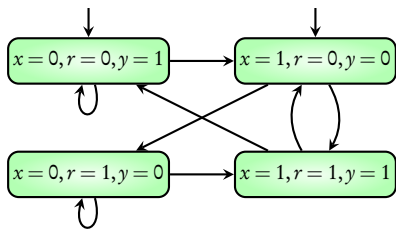


Deterministic transition system

Model of hardware circuit

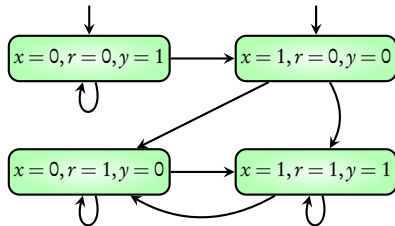
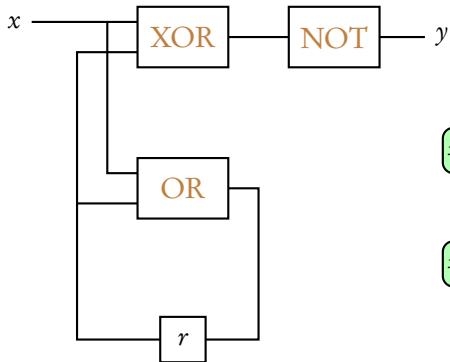


Model of hardware circuit



Non-deterministic transition system: to model incomplete information

Coming next: Another example of hardware circuit



Summary

Hardware Circuits

Modeling using transition systems

Non-determinism

Reference: Principles of Model Checking, *Baier and Katoen*, MIT Press (2008)

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