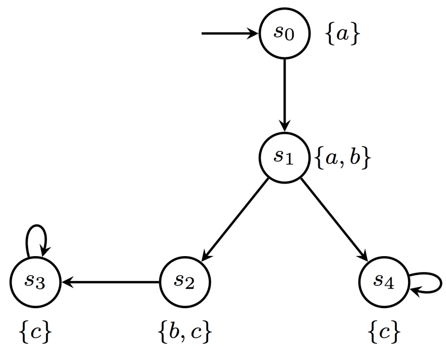
1. Let the set of atomic propositions be $\{a, b, c\}$.

   (a) Rewrite the CTL formula $A\ [\ a\ U\ (AF\ c)\ ]$ in existential normal form (that is, using only EX, EU and EG).

   (b) Which states of the transition system below satisfy the formula $EFAG\ c$?



Solution:

(a) Firstly, $AF\ c$ can be rewritten as $\neg EG\ \neg c$; let $\psi := \neg EG \neg c$. Then, $A\ [\ a\ U\ \psi]$ can be rewritten as:

$$\neg\ [\ EG\neg\psi\ \vee\ E\ (\neg a\ U\ (\neg a \wedge \neg\psi))\ ]$$

(b) All states satisfy $EF\ AG\ c$.

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2. (a) Let TS be a transition system, and let TS' be a transition system obtained by removing some state of TS and its associated transitions. Assume that TS' has at least one state, and there are no terminal states in both TS and TS'.

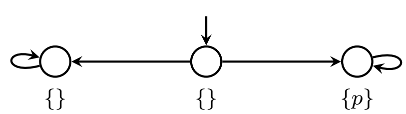
   Show that if TS satisfies an LTL property ϕ, then TS' satisfies ϕ.

   (b) Use the above observation to show that there is no equivalent LTL formula for the CTL property $\boldsymbol{EFAG}p$.

Solution:

(a) TS satisfies ϕ if $Traces(TS) \subseteq L(\phi)$. Note that by construction, $Traces(TS') \subseteq L(\phi)$. Hence TS' satisfies ϕ.

(b) Consider the following transition system: This satisfies $EFAGp$. Call this TS. By removing the



state with $\{p\}$ we get a transition system TS' which does not satisfy $EFAGp$. Therefore, if there is an LTL formula ϕ equivalent to $EFAGp$, we have that TS satisfies ϕ, but TS' does not. This contradicts the observation in the previous question.

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3. The $\boldsymbol{F}$ operator in LTL is used to say that a property is true sometime in the *future*. Let us now introduce the $\boldsymbol{O}$ operator (short form for *Once*) to say that a property was true sometime in the *past*.

   The formal semantics of $\boldsymbol{O}$ can be defined as follows. For an ω-word α, let α^i denote the suffix of α starting from the i^{th} position. Then:

   $$\alpha^i \models \boldsymbol{O}\phi \quad \text{if} \quad \exists j \leq i \text{ s.t. } \alpha^j \models \phi \qquad \text{and} \qquad \alpha \models \boldsymbol{O}\phi \quad \text{if} \quad \alpha^0 \models \boldsymbol{O}\phi$$

   Let p_1 and p_2 be atomic propositions. Take the alphabet $\mathbb{B}^2 = \{\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{1}{1}\}$ where the top element indicates the value for p_1 and the bottom one indicates the value of p_2.

   Let $\Psi := \boldsymbol{G}\ (p_1 \rightarrow \boldsymbol{O}p_2)$.

   i) Give two examples of ω-words over $\mathbb{B}^2$: one which satisfies Ψ and one which does not satisfy Ψ.

   ii) Show that Ψ can be rewritten into an equivalent LTL formula which uses only the standard Until operator $\boldsymbol{U}$ and the boolean connectives $(\neg, \wedge, \vee, \rightarrow)$.

   Solution:

   (a) $\{p_2\}^\omega$ satisfies Ψ, $\{p_1\}^\omega$ does not satisfy Ψ.

   (b) Let us look at the negation of Ψ. A word satisfies $\neg\Psi$ if there exists a p_1 at some position i, and there is no p_2 in the interval $[0, i]$. This corresponds to the LTL formula $\neg p_2 U(\neg p_2 \wedge p_1)$. Therefore, Ψ is the negation of this formula:

   $$\neg\ (\neg p_2 U(\neg p_2 \wedge p_1))$$

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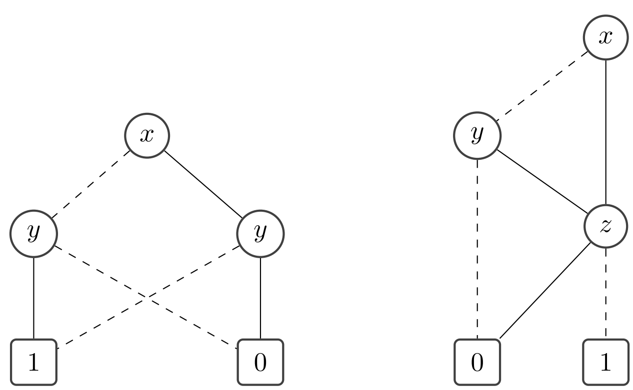
4. Draw the ROBDD for the following boolean functions, with the specified order for variables:

   (a) $x.\overline{y} + \overline{x}.y$ with order $[x, y]$ (b) $(x + y).\overline{z}$ with order $[x, y, z]$

   Solution:



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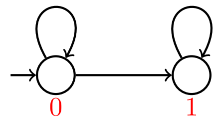
5. Represent the following transition system as an ROBDD.



Solution: