1. Let $U = a(a+b)^*a$. Which of the following words belong to U^ω?
   i. $(abaabba)^\omega$
   ii. $(ab)^\omega$
   iii. a^ω

2. Let $U = a(a+b)^*a$. Which of the following words belong to $\lim U$?
   i. $(abaabba)^\omega$
   ii. $(ab)^\omega$
   iii. a^ω

3. Give an example of a regular language U such that $U^\omega \not\subseteq \lim U$.

4. Prove or disprove the following equations. Justify your answer: if you think the equation is true, provide a proof; otherwise give a specific counterexample to the equation. In the following, fix a finite alphabet Σ and assume that U and V are subsets of Σ^*. The notation V^+ stands for $V^* \setminus \epsilon$.
   i. $(U \cup V)^\omega = U^\omega \cup V^\omega$
   ii. $\lim(U \cup V) = \lim U \cup \lim V$
   iii. $U^\omega = \lim(U^+)$
   iv. $\lim(U \cdot V^+) = U \cdot V^\omega$

5. Let $\Sigma = \{a, b, c\}$. Give Büchi automata (deterministic or non-deterministic) for the following languages:
   i. set of all ω-words where abc occurs at least once
   ii. set of all ω-words where abc occurs infinitely often
   iii. set of all ω-words where abc occurs finitely often

6. Let $\Sigma = \{a, b\}$. Give a Büchi automaton for the following language:

   $$\{ \alpha \in \Sigma^\omega \mid \forall i \text{ if } \alpha(i) = a \text{ then } \exists j > i \text{ s.t. } \alpha(j) = b \}$$

   Recall that we write α as $\alpha(0)\alpha(1)\alpha(2)\ldots$ where $\alpha(i)$ denotes the letter at the i^{th} position.

7. Suppose U is the regular language $a(a+b)^*a$. What is the NBA for U^ω?

8. Let L be an ω-language. Suppose L is of the form U^ω for some U. Does it mean that there is no deterministic Büchi automaton which can recognize L?

9. Let $\mathcal{A}$ be an NFA. Let $\mathcal{B}$ be an NBA whose structure is identical to $\mathcal{A}$. Answer the following:
   i. Is $\lim(\mathcal{L}(\mathcal{A})) \supseteq \mathcal{L}(\mathcal{B})$?
   ii. Is $\lim(\mathcal{L}(\mathcal{A})) \subseteq \mathcal{L}(\mathcal{B})$?

10. Let UP be the set of all ω-words over $\{0, 1\}$ that are ultimately periodic. Show that UP is not ω-regular.