1. Consider the following functions `f()` and `g()`.

```
f(){                    g(){
  w = 5;                   z = w+1;
  w = 2*z + 2;             z = 3*z - w;
}                          print(z);
                         }
```

We start with `w` and `z` set to `0` and execute `f()` and `g()` in parallel. What are the possible values printed by `g()`?

Solution: The possible values are : $\{-1, -2, 3, 4, 7, 13\}$, corresponding to the interleavings below.

```
w = 5; w = 2*z + 2; z = w+1; z = 3*z - w:  --- z = 7
w = 5; z = w+1; w = 2*z + 2; z = 3*z - w;  --- z = 4
w = 5; z = w+1; z = 3*z - w;               --- z = 13
z = w+1; z = 3*z - w;                      --- z = 3
z = w+1; w = 5; z = 3*z - w;               --- z = -2
z = w+1; w = 5; w = 2*z + 2; z = 3*z - w;  --- z = -1
```

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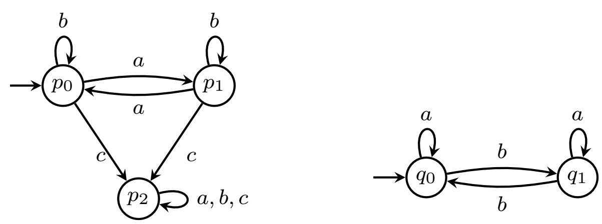
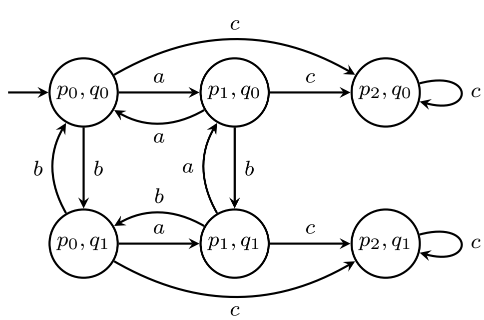
2. Given below are two transition systems which are executed in parallel. The two systems synchronize over shared actions $\{a, b\}$. Draw the transition system representing the joint behaviour of the concurrent system.



Solution:



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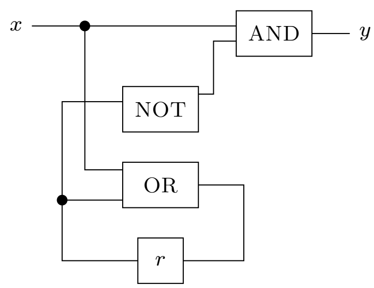
3. Consider the following sequential circuit: Use NuSMV tool to check if the above circuit satisfies the



property: *in every execution, the register r becomes* 0 *at some point of time (excluding the initial value).*

Solution:

```
MODULE main

VAR
x: boolean;
r: boolean;

DEFINE
y := x & (!r) ;

ASSIGN
next(r) := x | r ;
```

Property to be checked:

```
 X F !r
```

---

4. Let ϕ be a property.

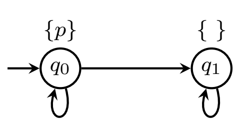
   (a) Can a (non-empty) transition system without terminal states satisfy both ϕ and $\neg\phi$?

   (b) Is it possible that a transition system satisfies neither ϕ nor $\neg\phi$?

Solution: Each property ϕ corresponds to a language L_ϕ over infinite words. A transition system TS satisfies ϕ if $Traces(TS) \subseteq L_\phi$.

(a) No, a transition system cannot satisfy both ϕ and its negation $\neg\phi$. The language $L_{\neg\phi}$ is the complement of L_ϕ. Since the transition system does not have terminal states, there is at least one infinite word in $Traces(TS)$. This word cannot belong to both L_ϕ and $L_{\neg\phi}$.

(b) Yes, it is possible that a transition system satisfies neither ϕ nor $\neg\phi$. Consider the following transition system: The transition system given below neither satisfies $\boldsymbol{G}p$ nor $\neg(Gp)$ (Why?).



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5. Which of the following are safety properties?

$$i)\ \boldsymbol{G}\,p \qquad ii)\ \boldsymbol{F}\,p \qquad iii)\ \boldsymbol{GF}\,p$$

where p is an atomic proposition.

Solution:

$\boldsymbol{G}\,p$ is a safety property. The set of bad prefixes is the set of all words containing $\neg p$.

$\boldsymbol{F}\,p$ is not a safety property. Suppose $\boldsymbol{F}\,p$ is a safety property. Then, there exists a set of bad prefixes B such that each word in $\boldsymbol{F}\,p$ contains no prefix from B. Let $w \in B$. Consider the word $\rho := w\{p\}\{\}\{\}\{\}\ldots$. The word $\rho \in \boldsymbol{F}\,p$, but contains a bad prefix w. This leads to a contradiction.

$\boldsymbol{GF}\,p$ is not a safety property. Similar reasoning as above.