Towards a Regular Theory of Parameterized Concurrent Systems

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Reports on joint works with Paul Gastin, Akshay Kumar, and Jana Schubert.

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Chennai Mathematical Institute

The verification problem for parmeterized systems:

«Is a system correct independently of
the number of processes / the way they are arranged?»

Talks by Arnaud Sangnier and Pierre Ganty.

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- Complementation
- Equivalent characterization in terms of MSO logic
- Nonemptiness

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- Complementation
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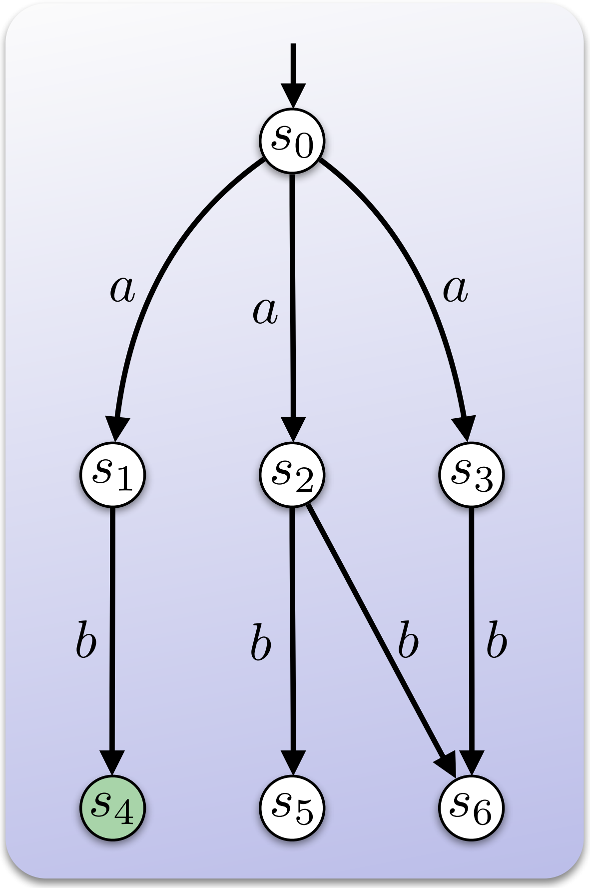
There have been robust models for fixed process architectures:

- Thomas: *On logical definability of trace languages.* ASMICS 1990.
- Henriksen-Mukund-Narayan Kumar-Sohoni-Thiagarajan: *A Theory of Regular MSC Languages.* I&C 2005.
- Genest-Kuske-Muscholl: *A Kleene theorem and model checking algorithms for existentially bounded communicating automata.* I&C 2006.

Finite Automata

finite automaton



Finite Automata

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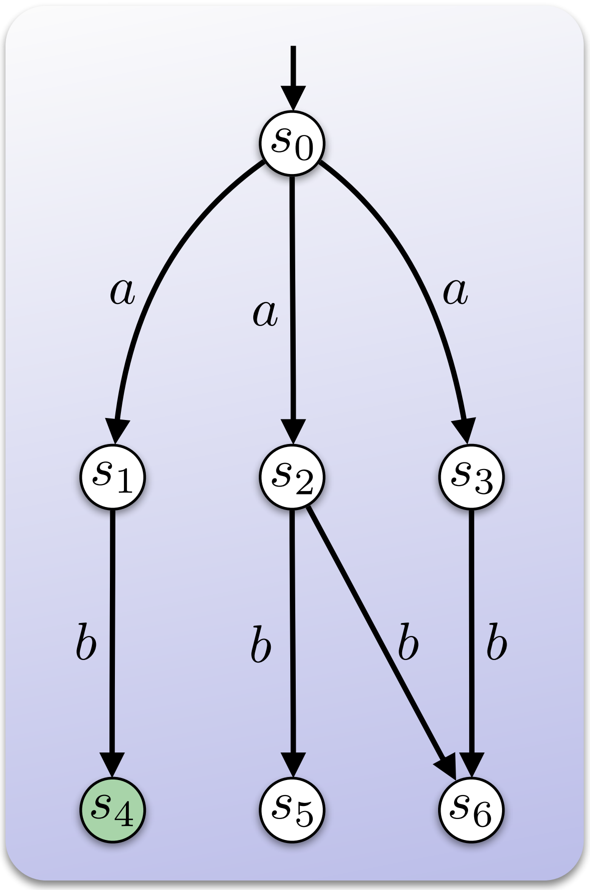
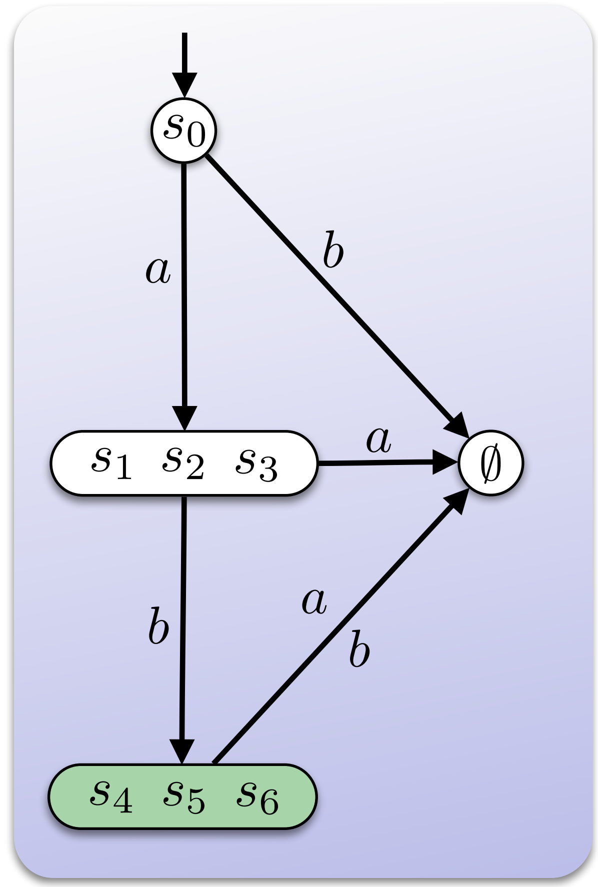
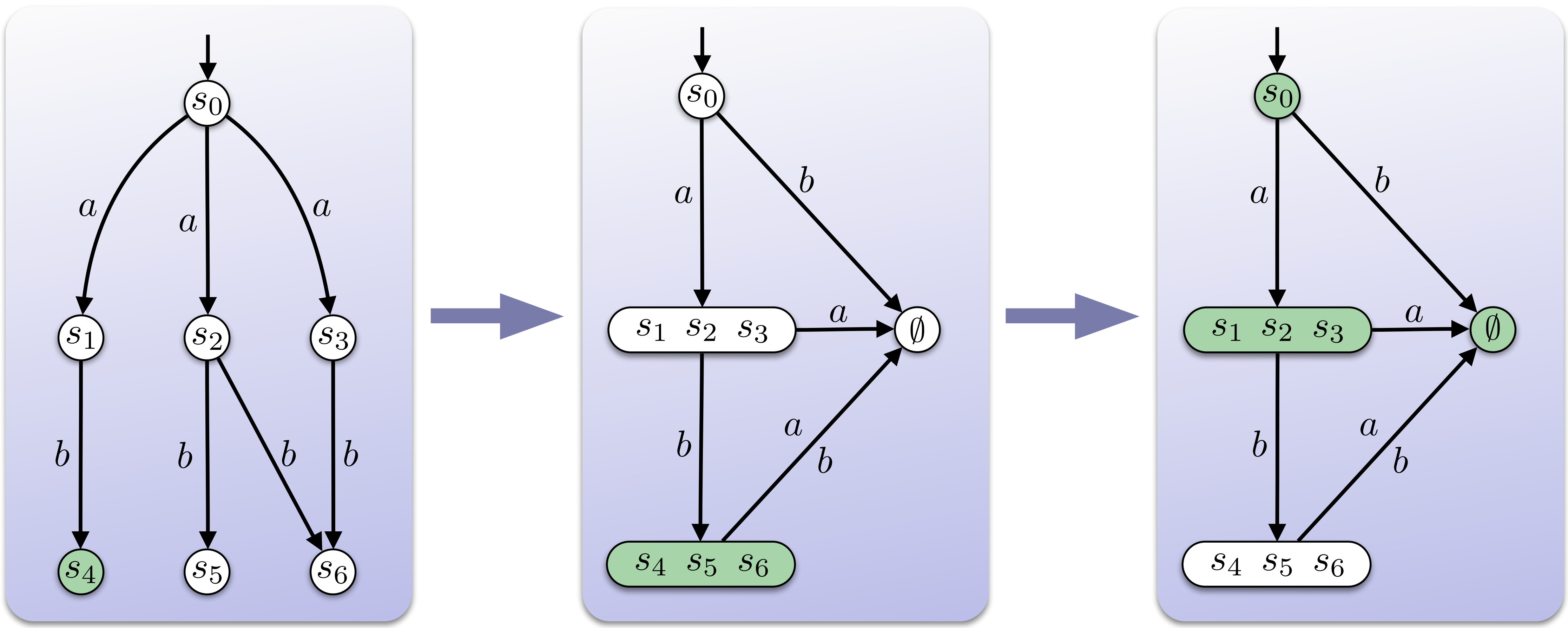


determinization



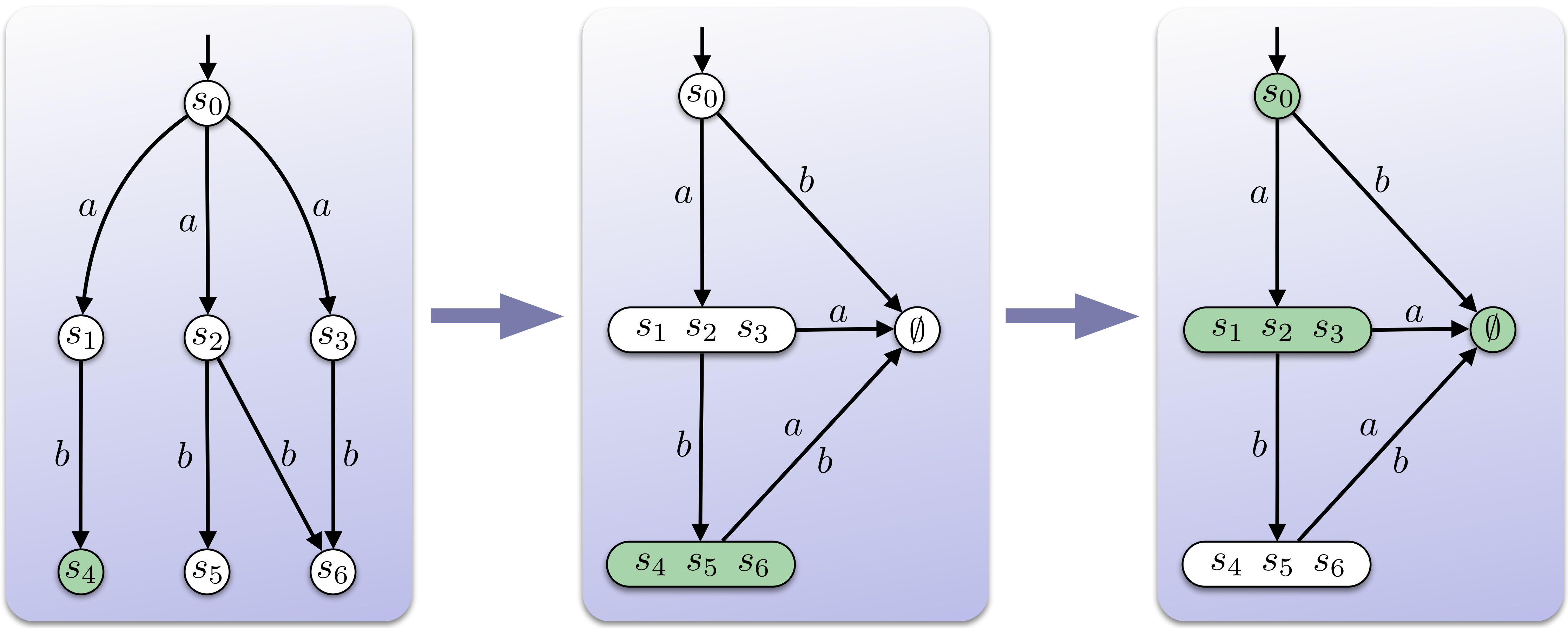
Finite Automata

finite automaton | determinization | complementation



Finite Automata

finite automaton — determinization — complementation



Theorem [Büchi-Elgot-Trakhtenbrot 1960s]:
Finite Automata = MSO

$$\forall x(a(x) \rightarrow \exists y(\mathsf{succ}(x,y) \wedge b(y)))$$

Finite Automata

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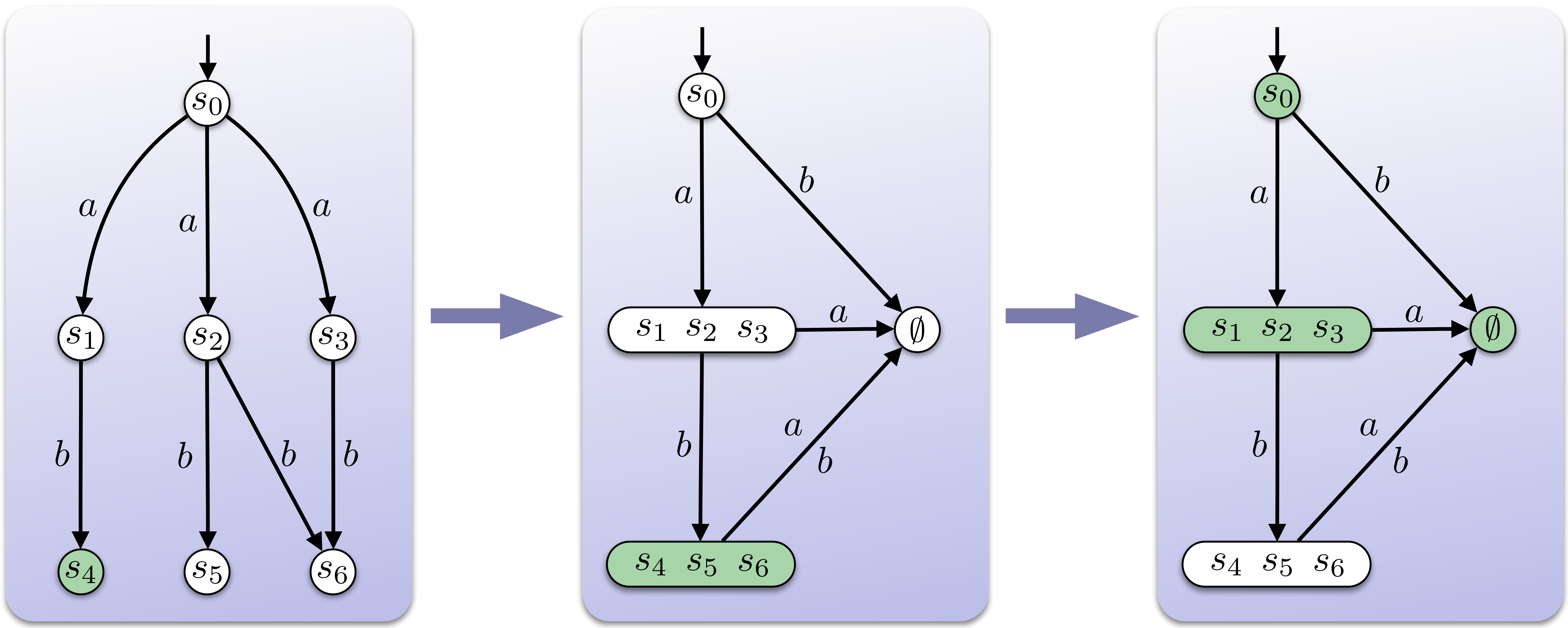

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Proof:

- free variables $\Rightarrow$ extended alphabet

Finite Automata

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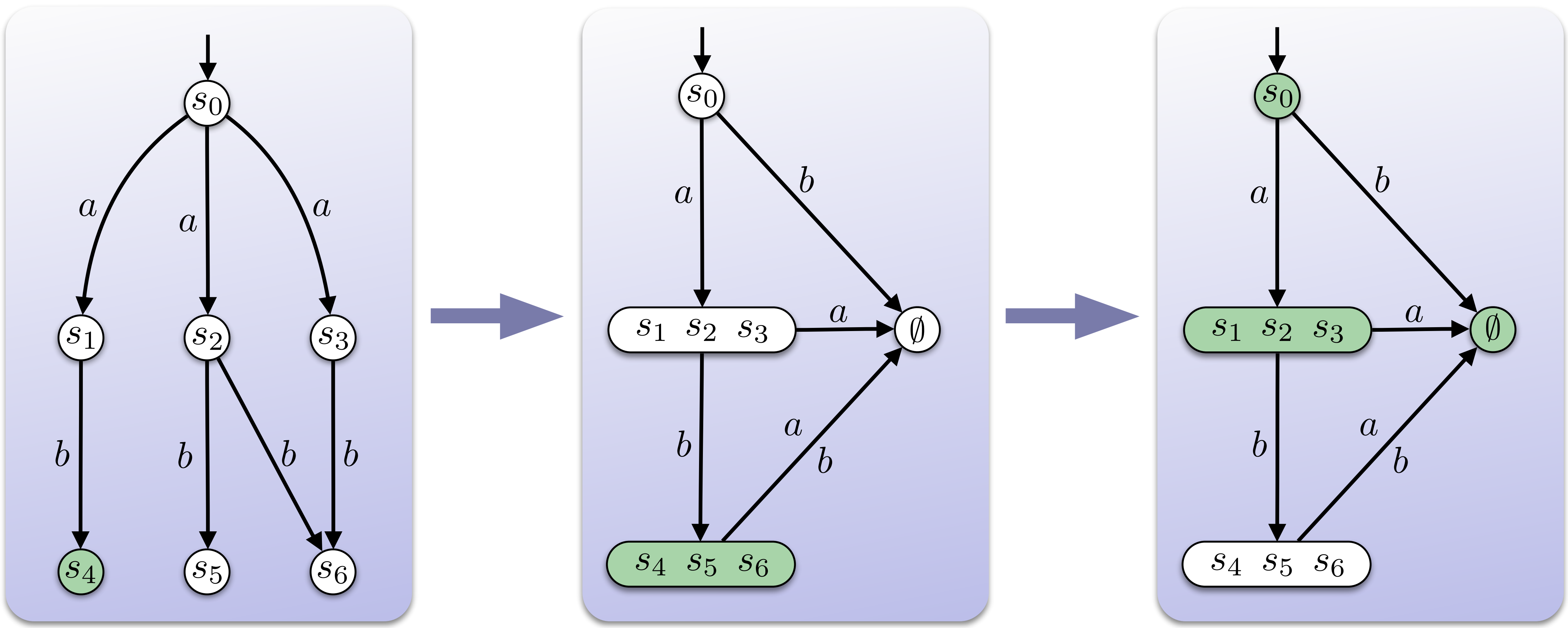

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- existential quantification → projection

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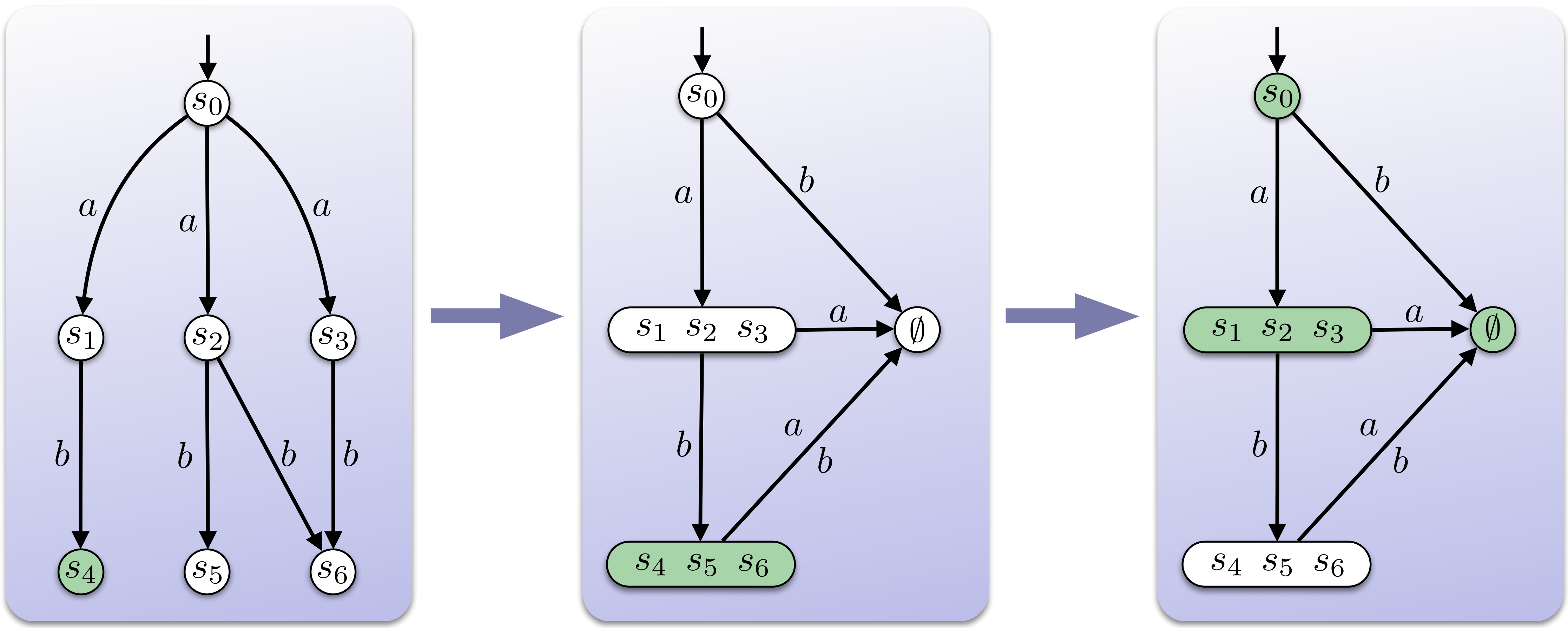
finite automaton | determinization | complementation



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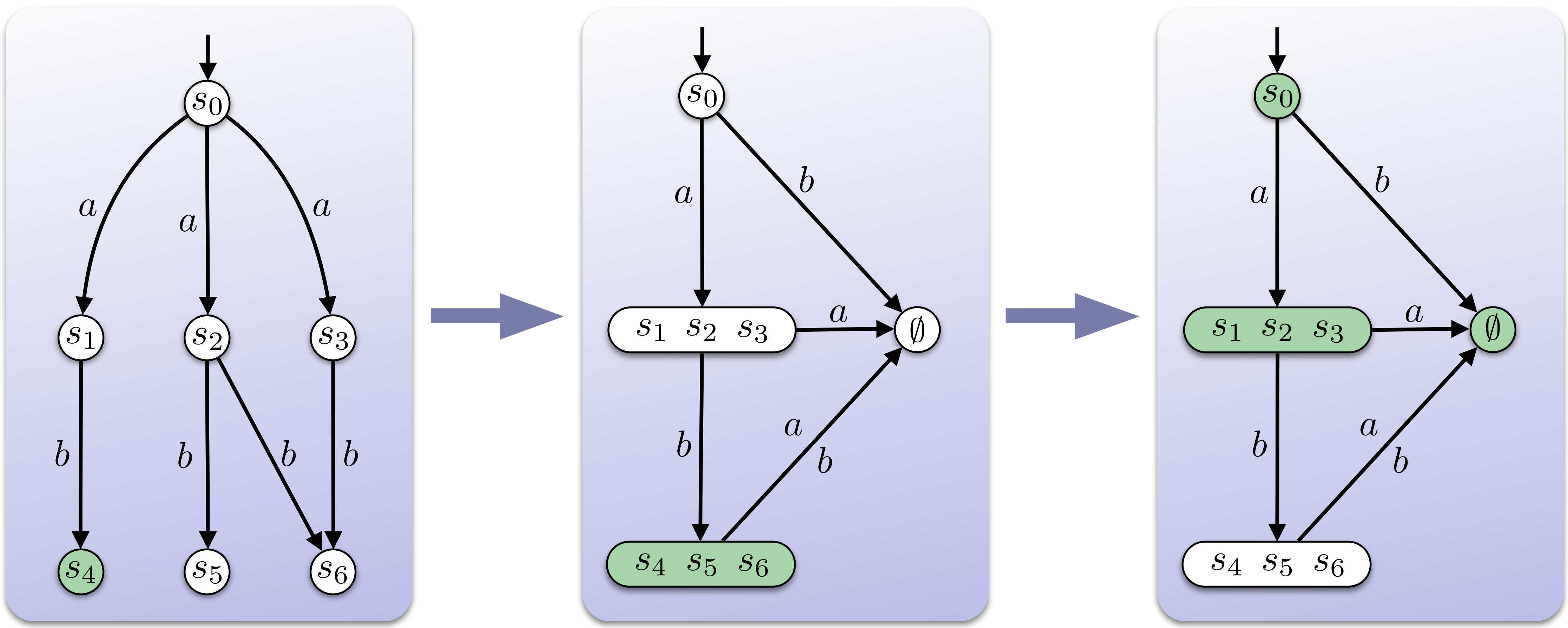
finite automaton determinization complementation



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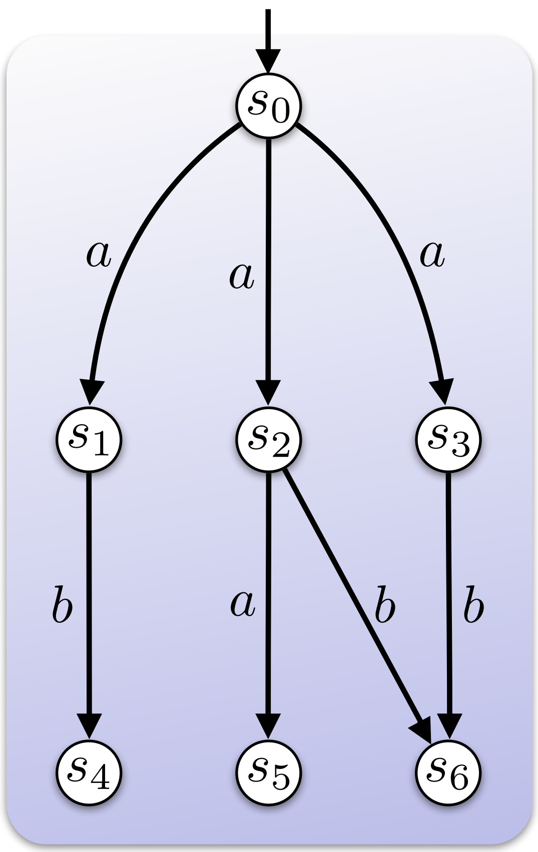
Parameterized Communicating Automata (PCA)



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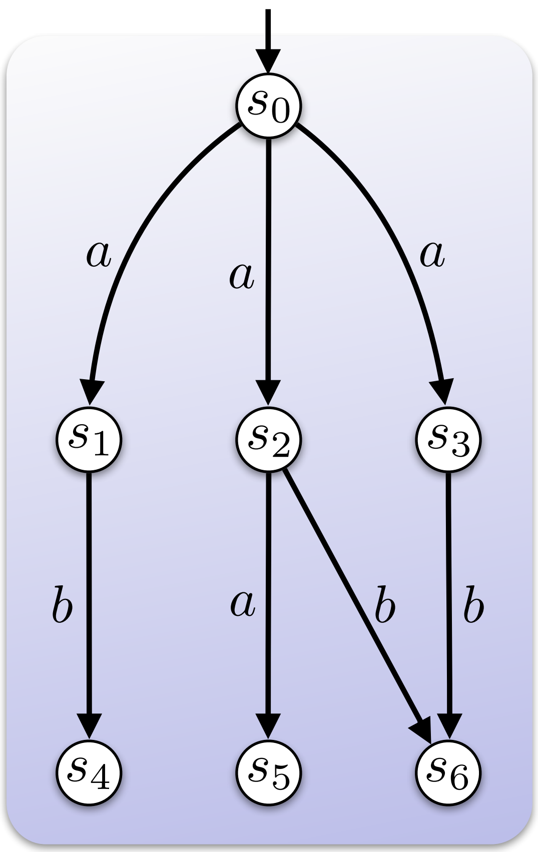

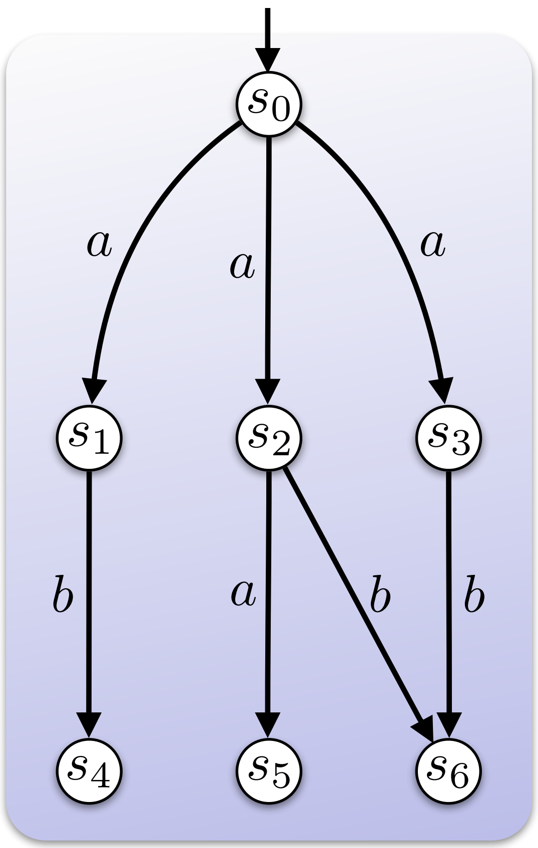

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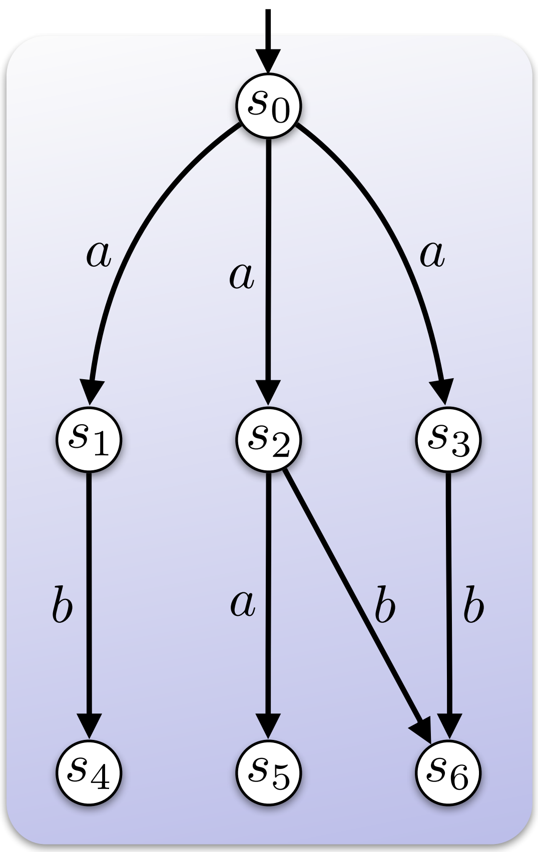

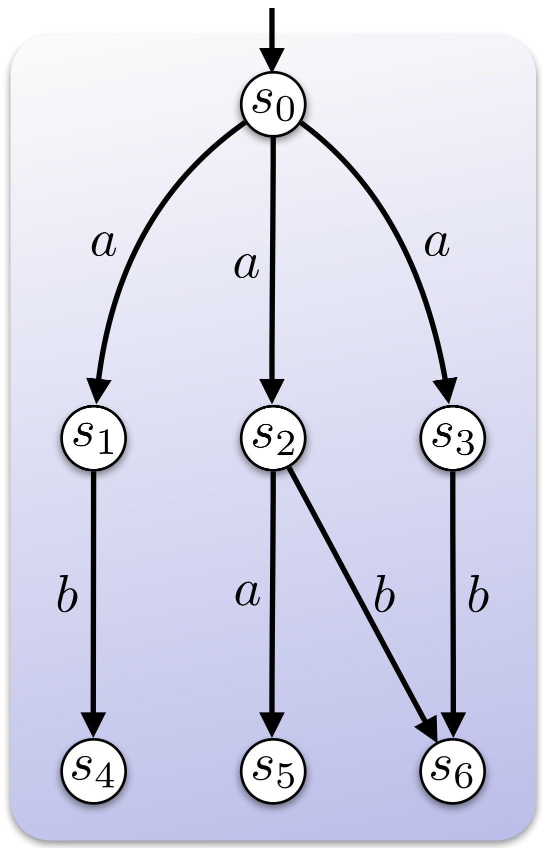

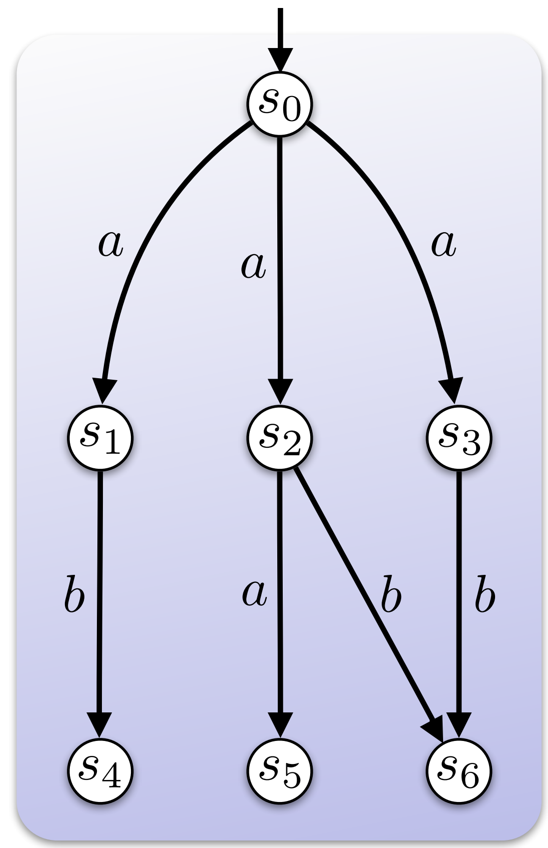

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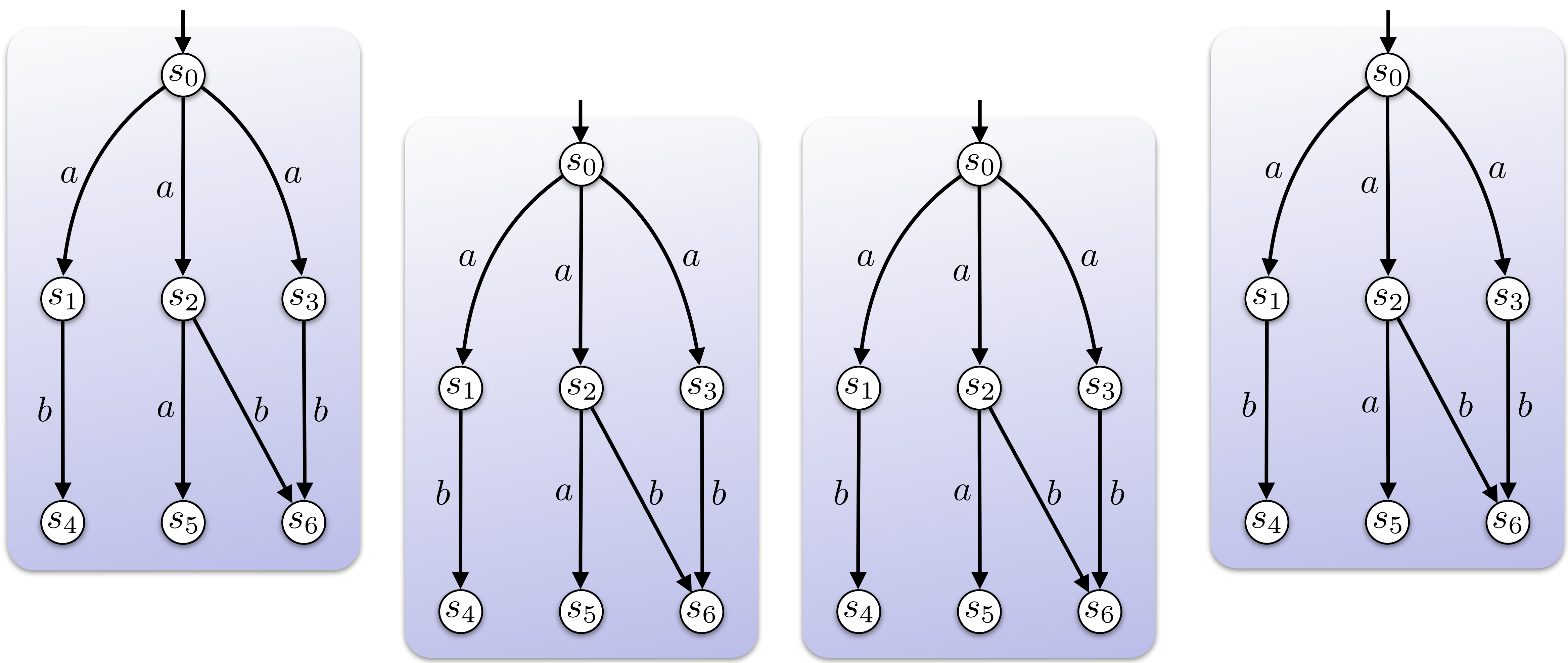

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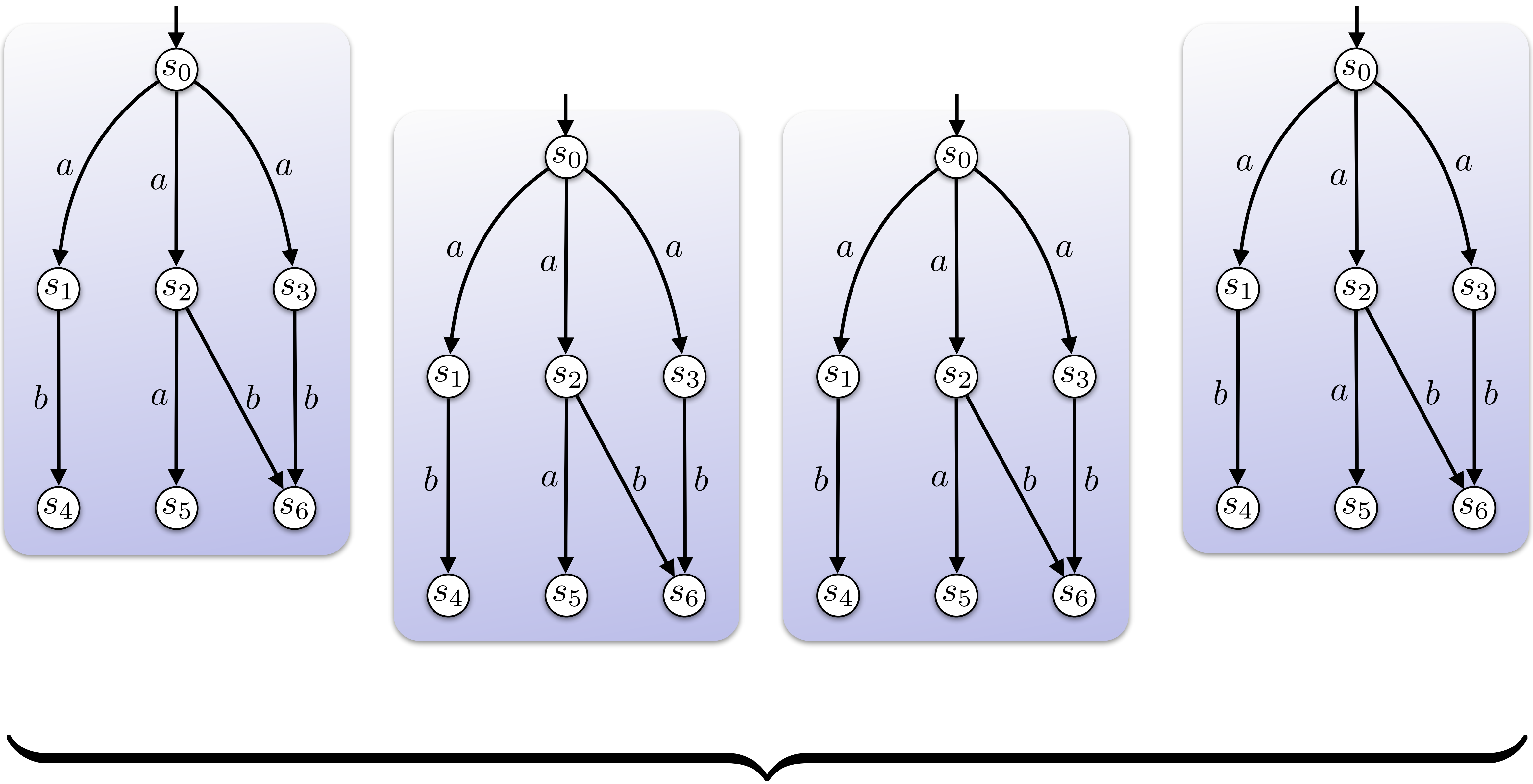
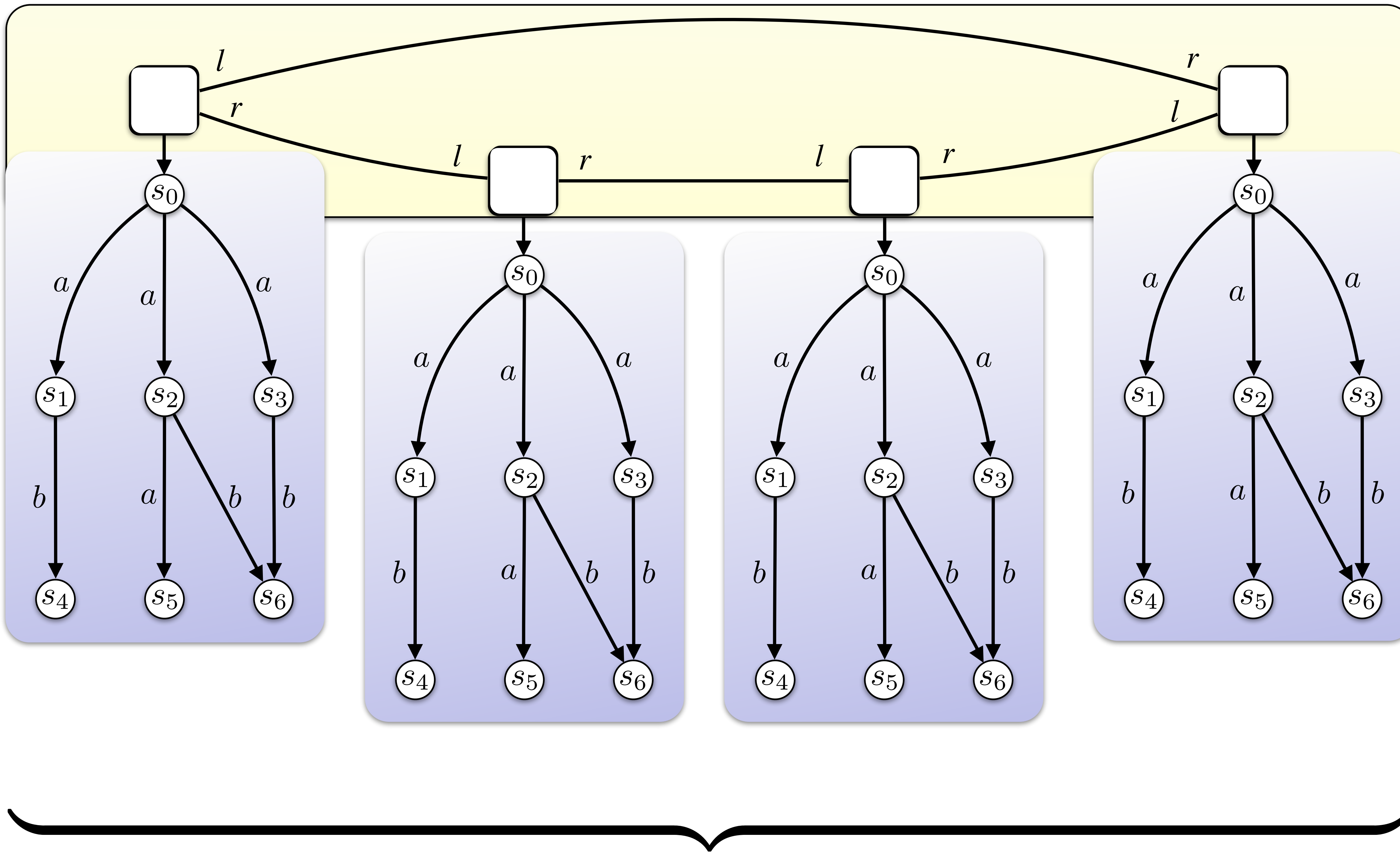


non-fixed & unbounded

Parameterized Communicating Automata (PCA) over Rings



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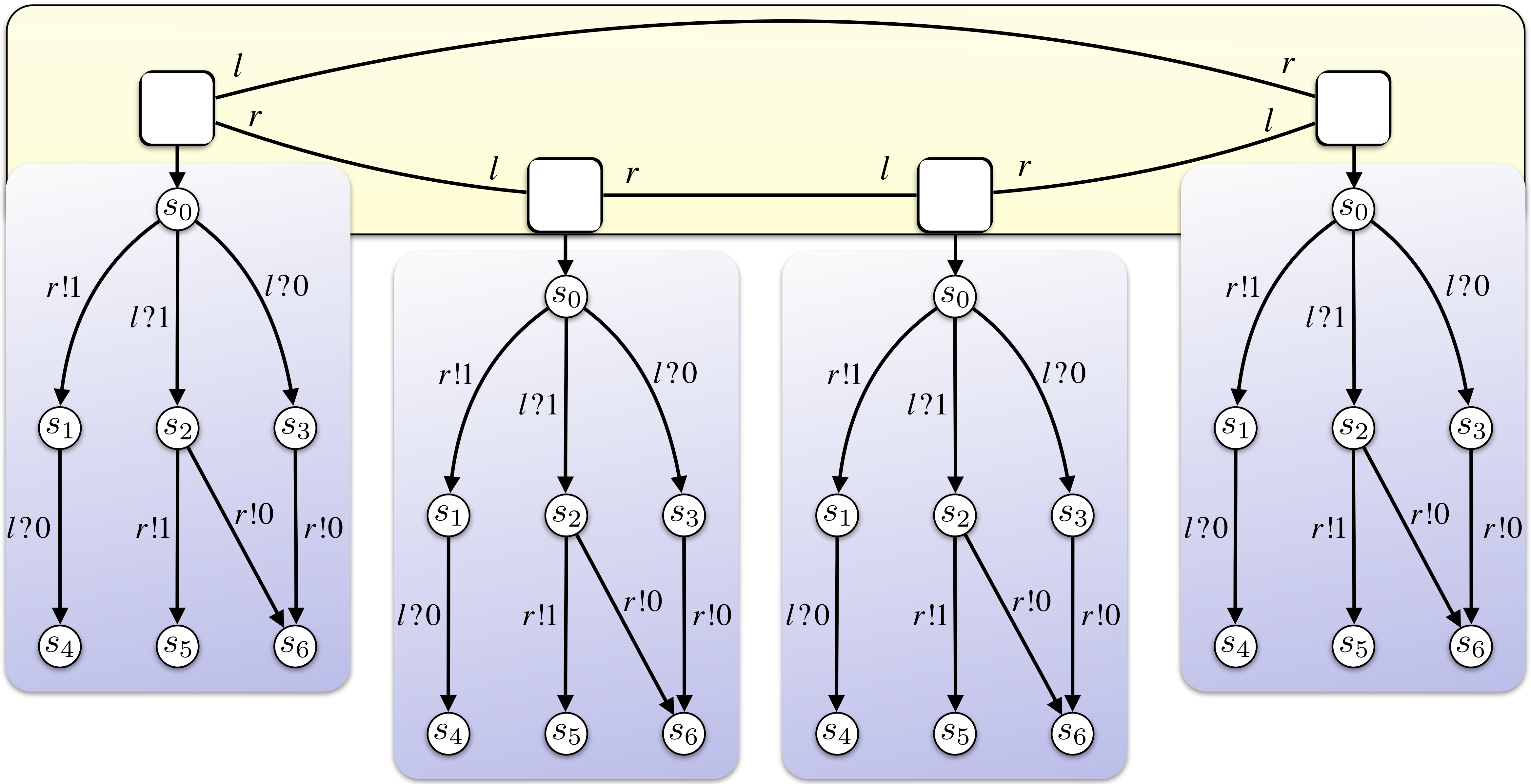

A PCA is given by:

- finite automaton over $\{l, r\} \times \{!, ?\} \times Msg$ (here: $Msg = \{0, 1\}$)
- acceptance condition

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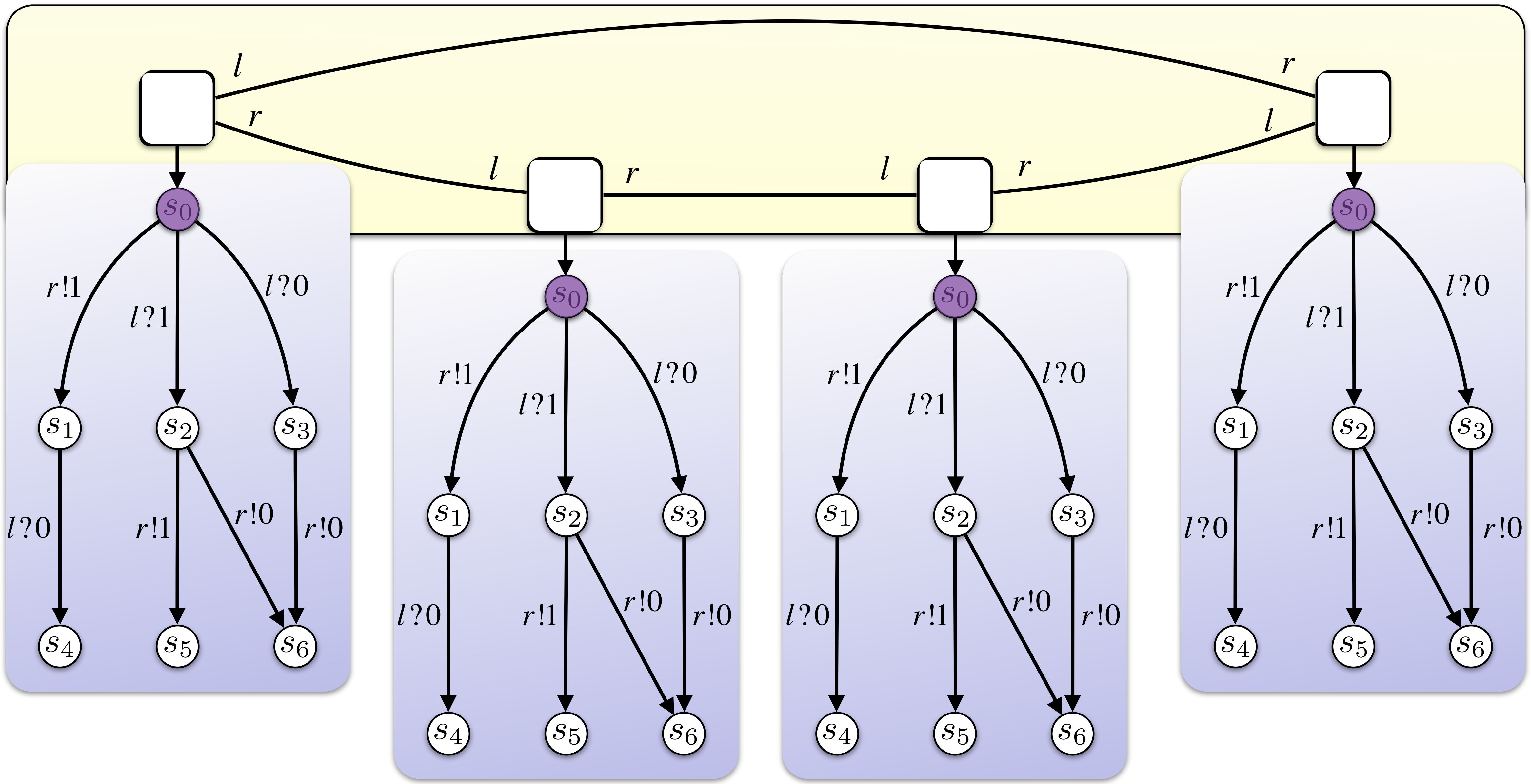

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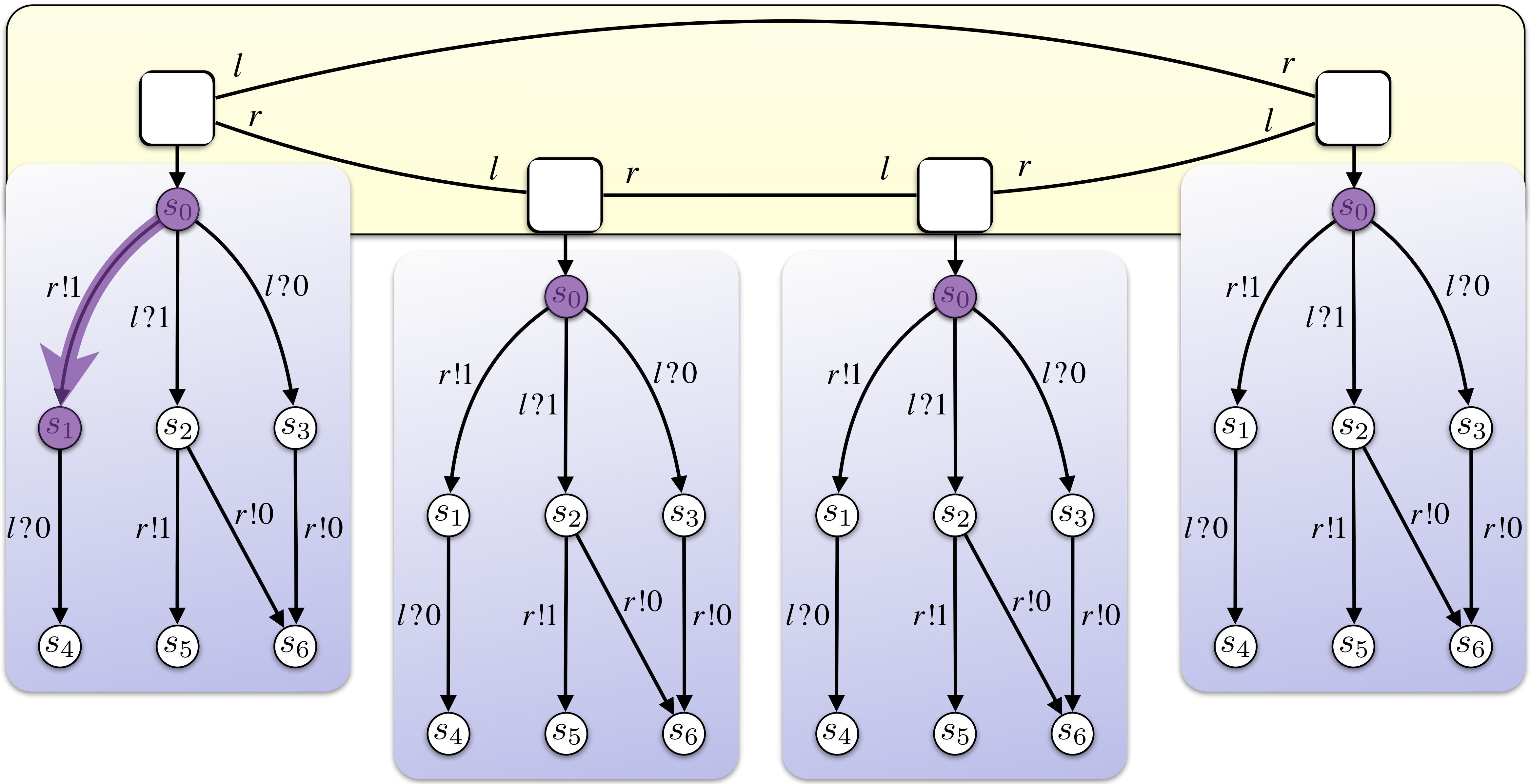

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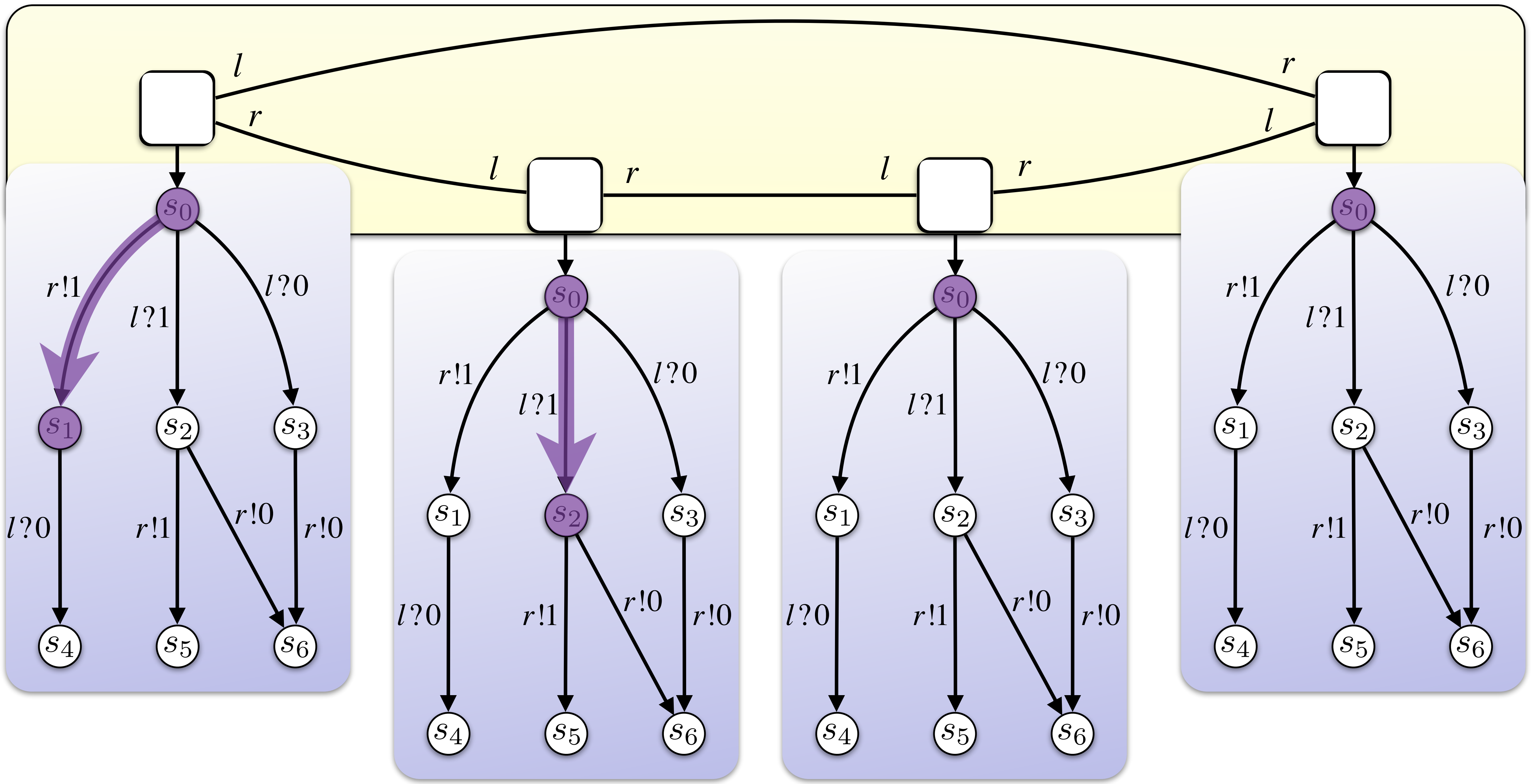

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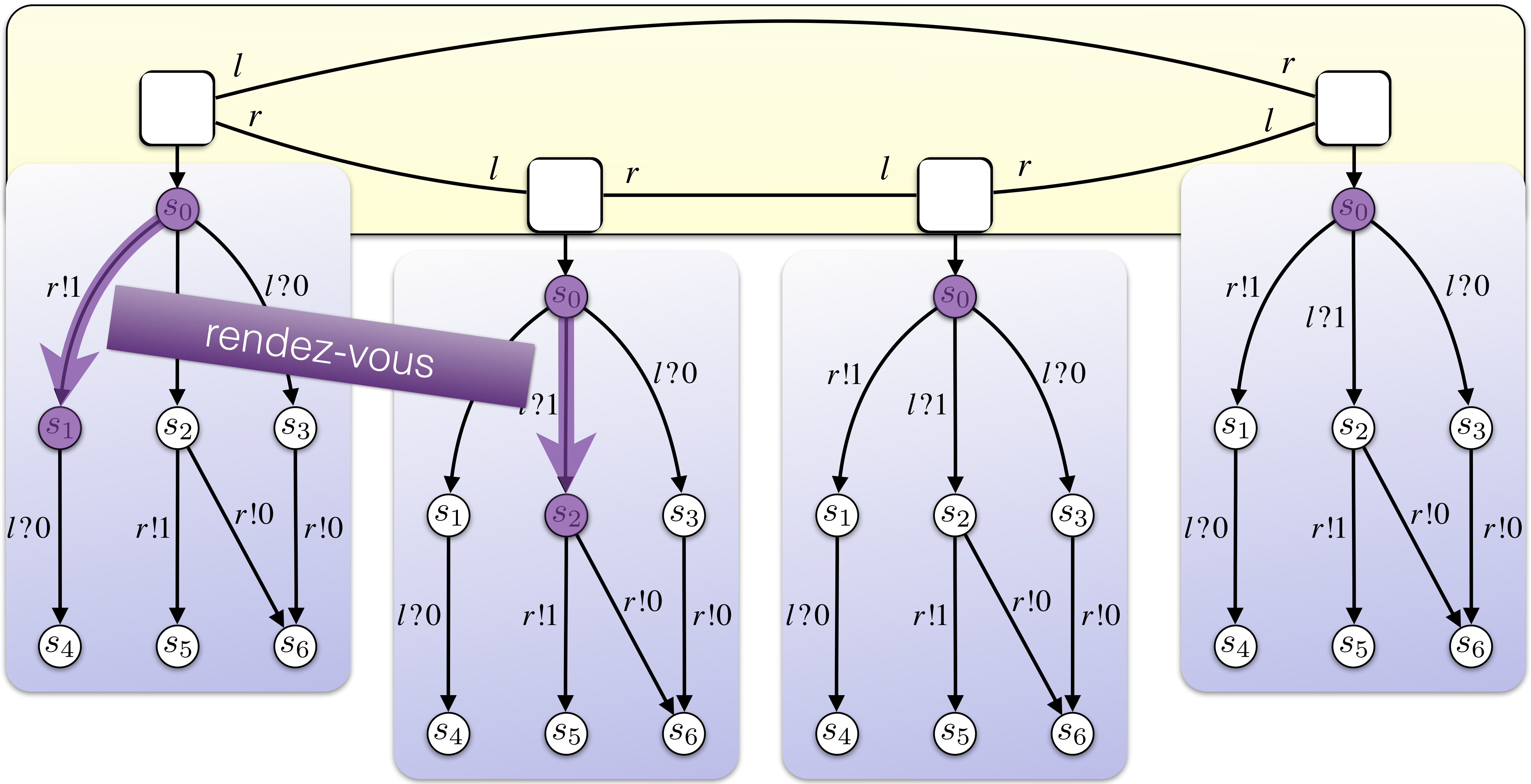

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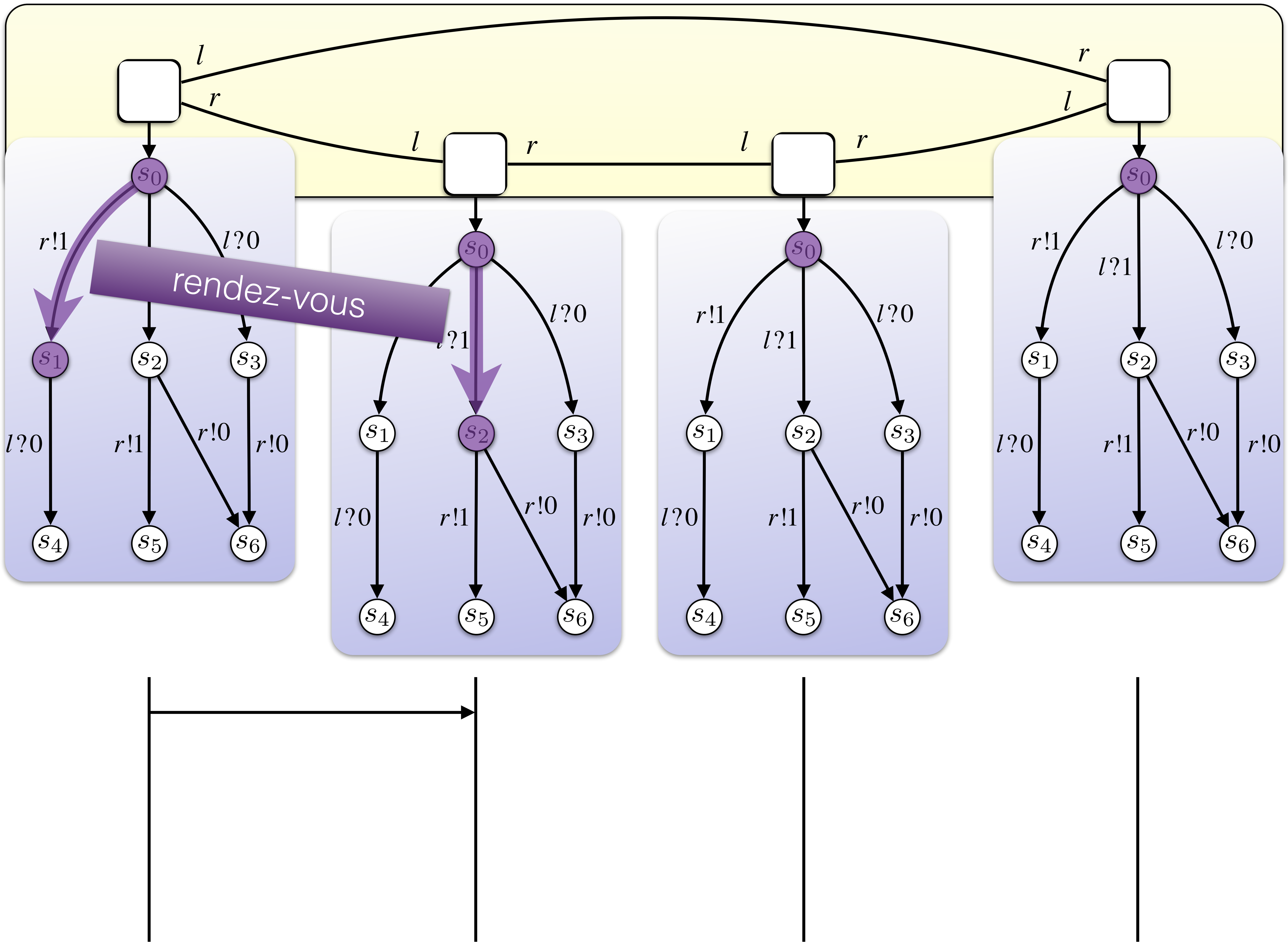

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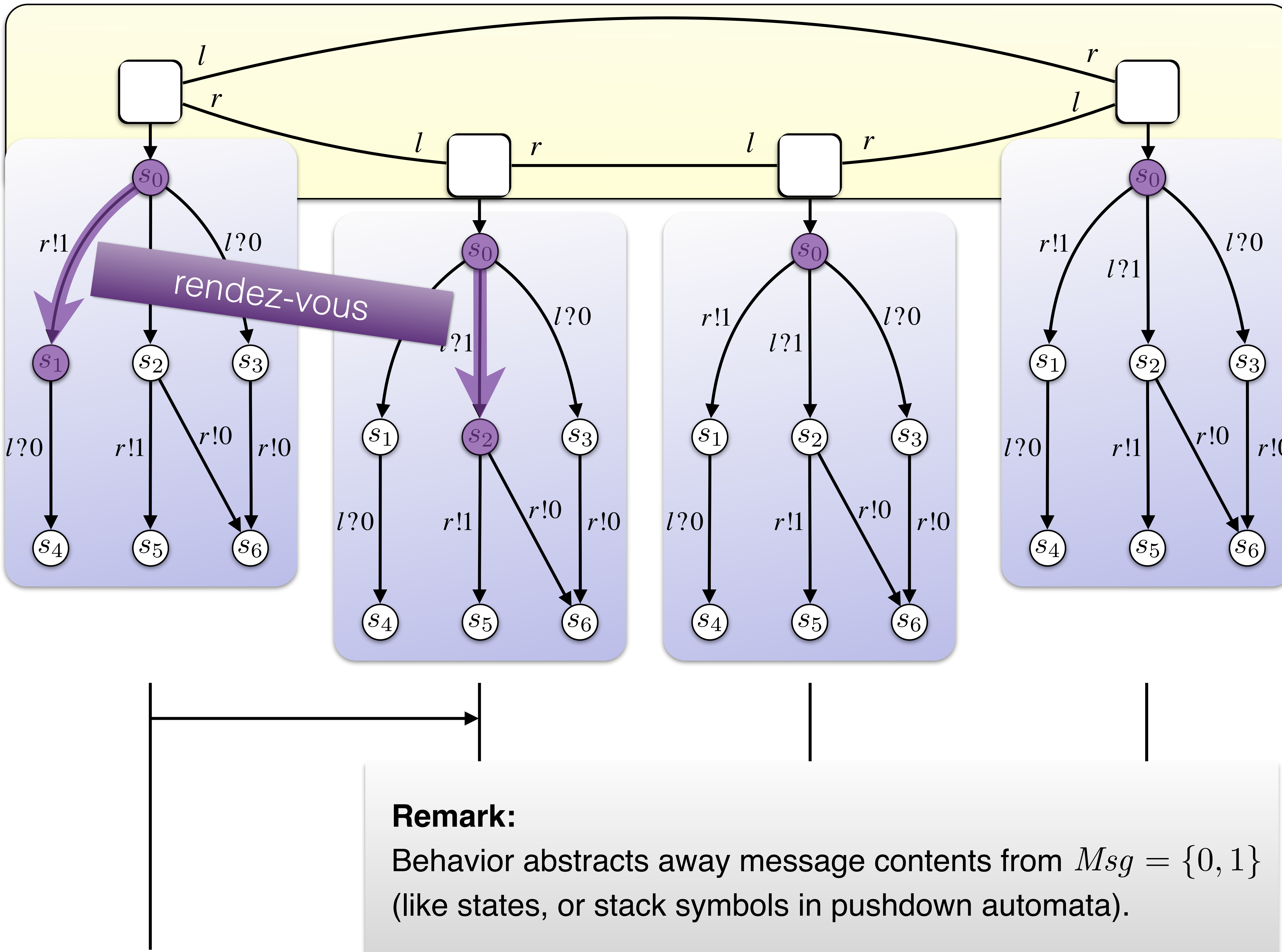

Remark:
Behavior abstracts away message contents from $Msg = \{0, 1\}$ (like states, or stack symbols in pushdown automata).

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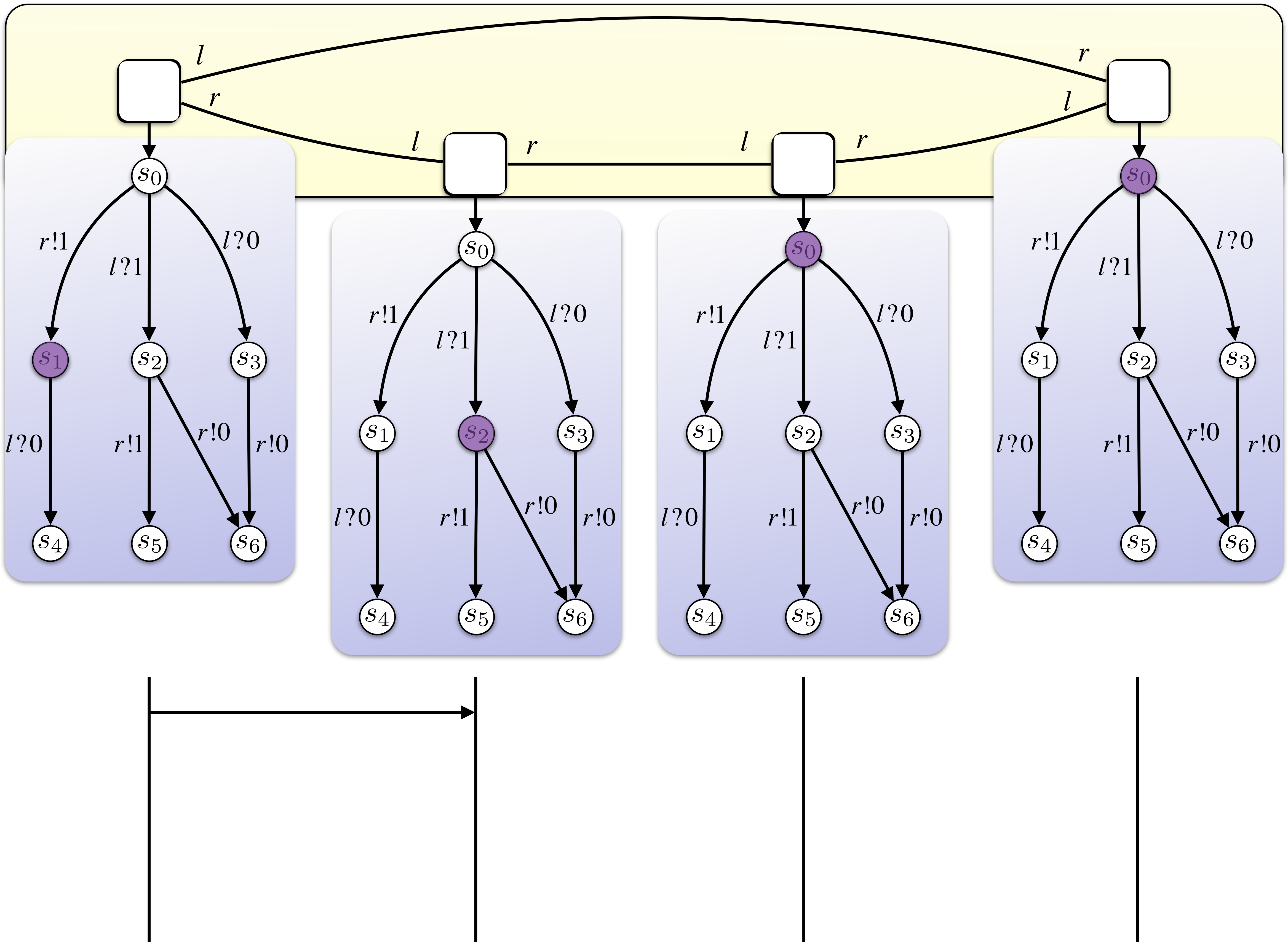

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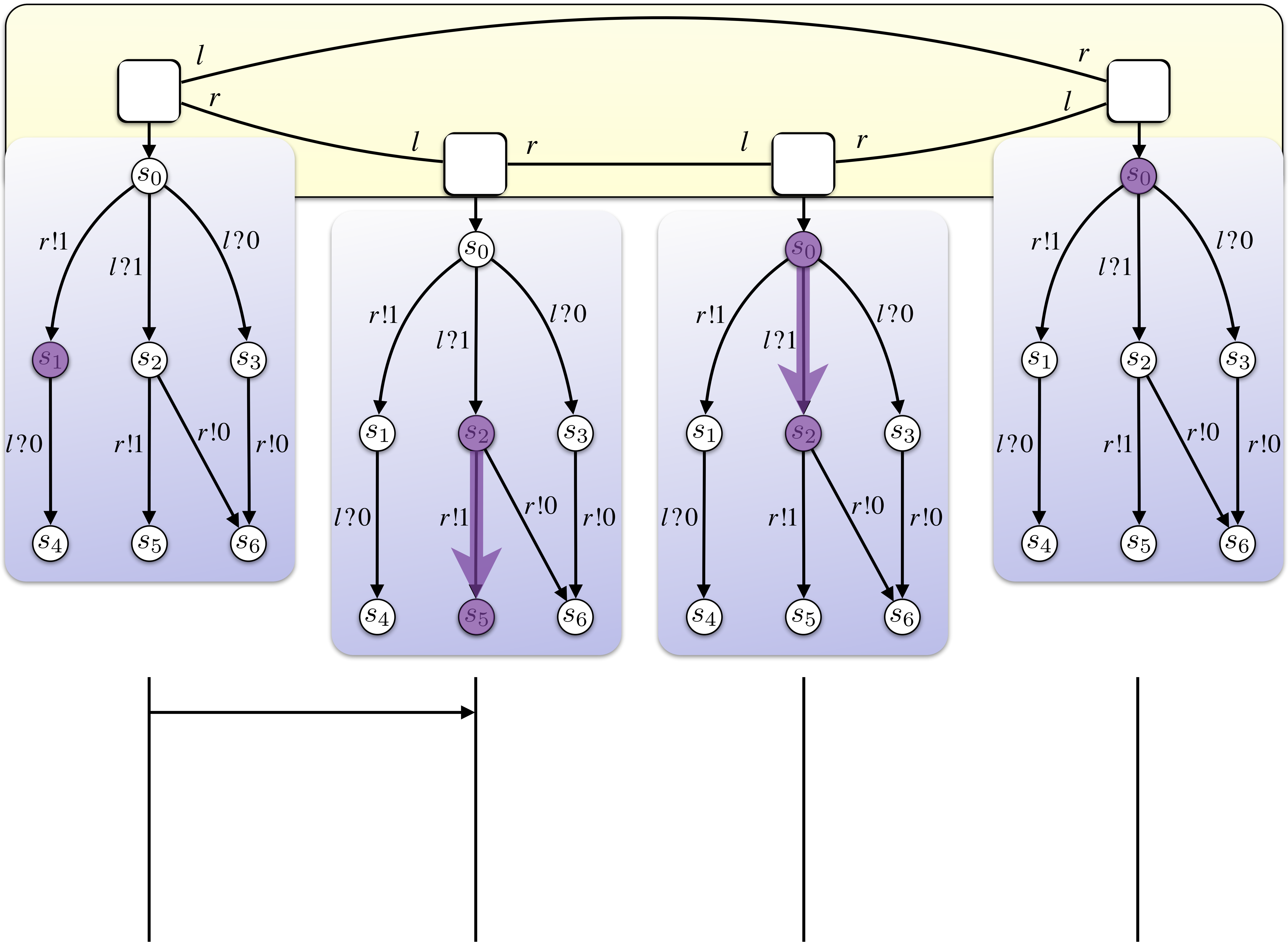

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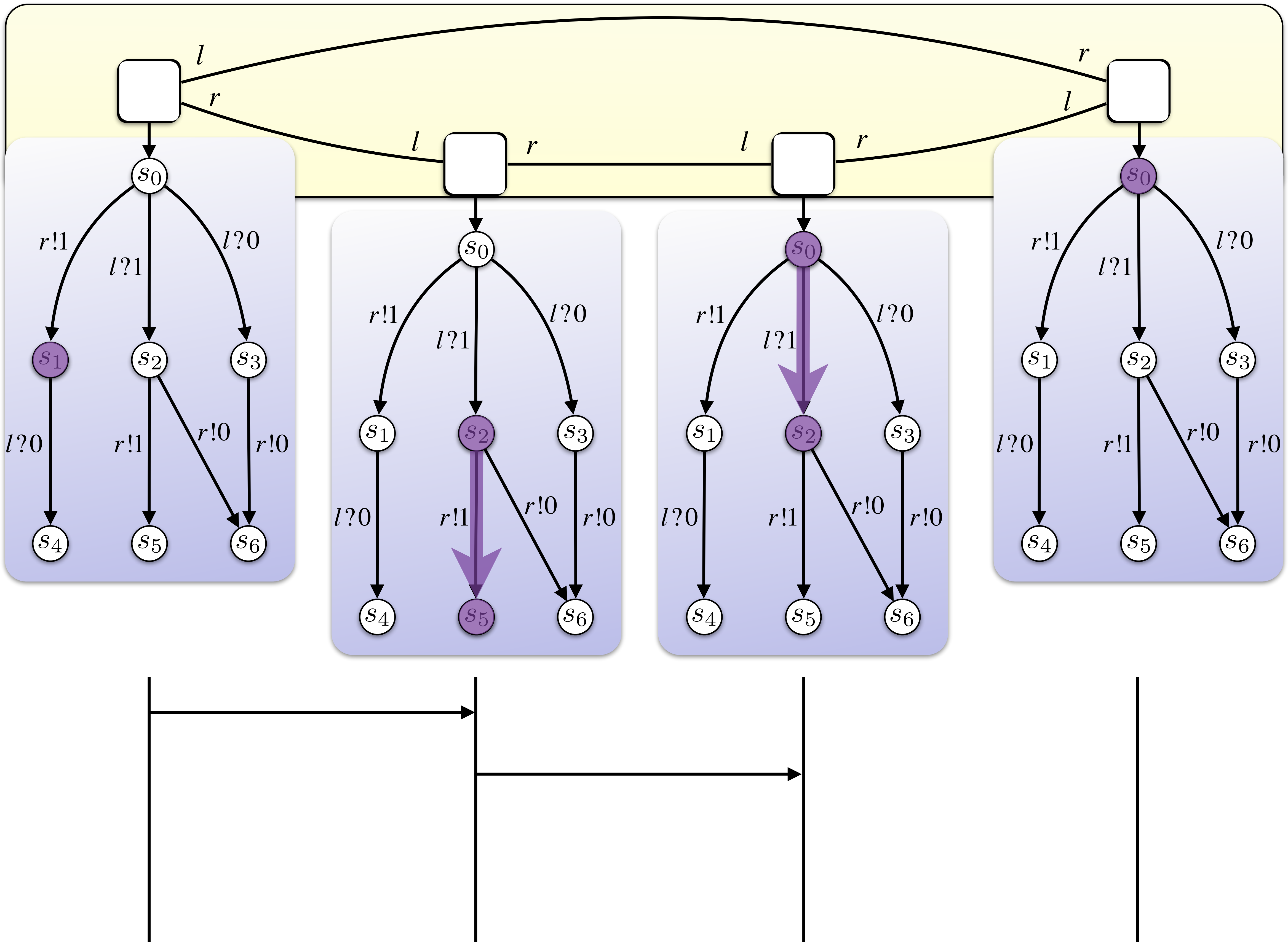

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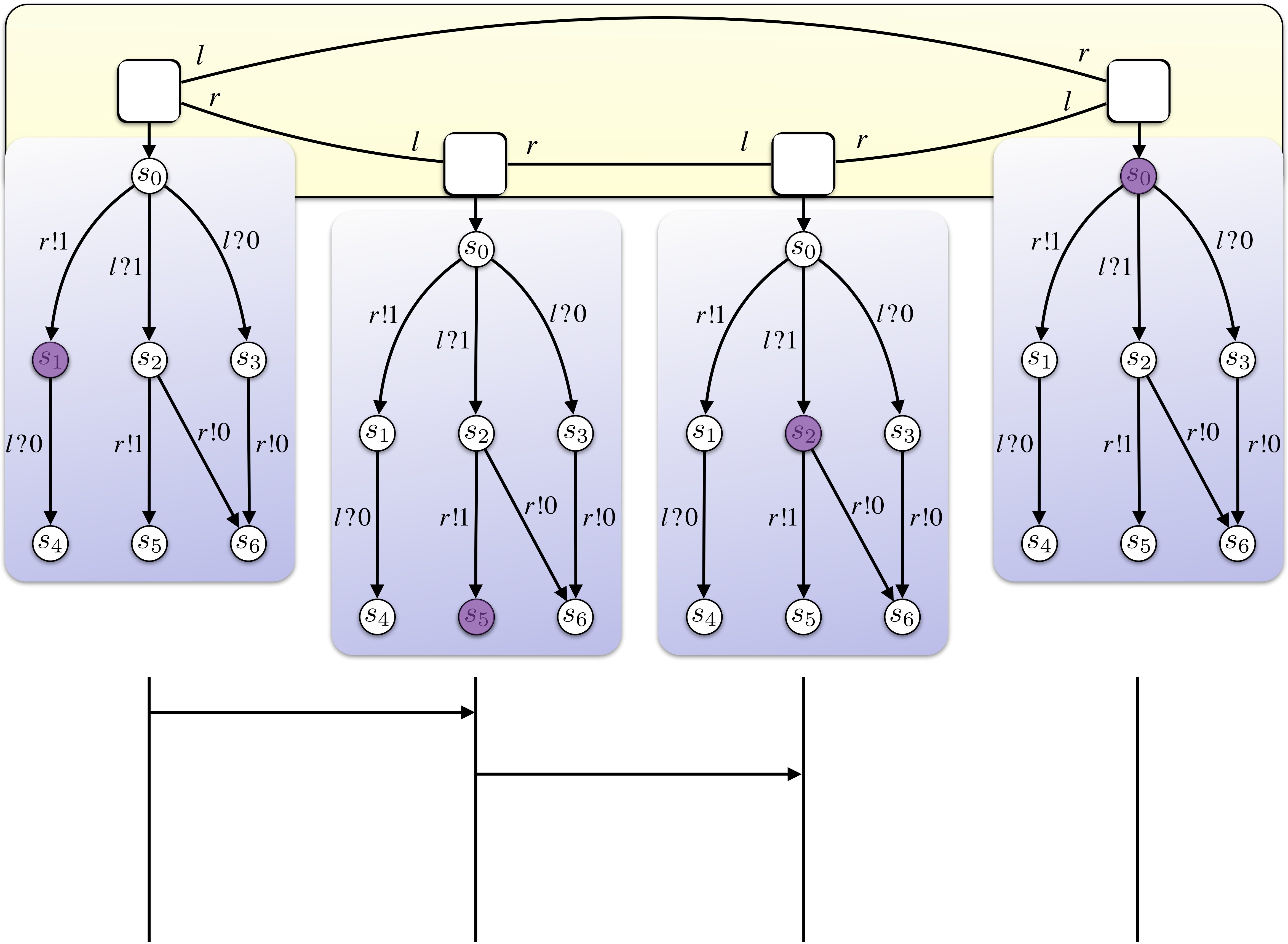

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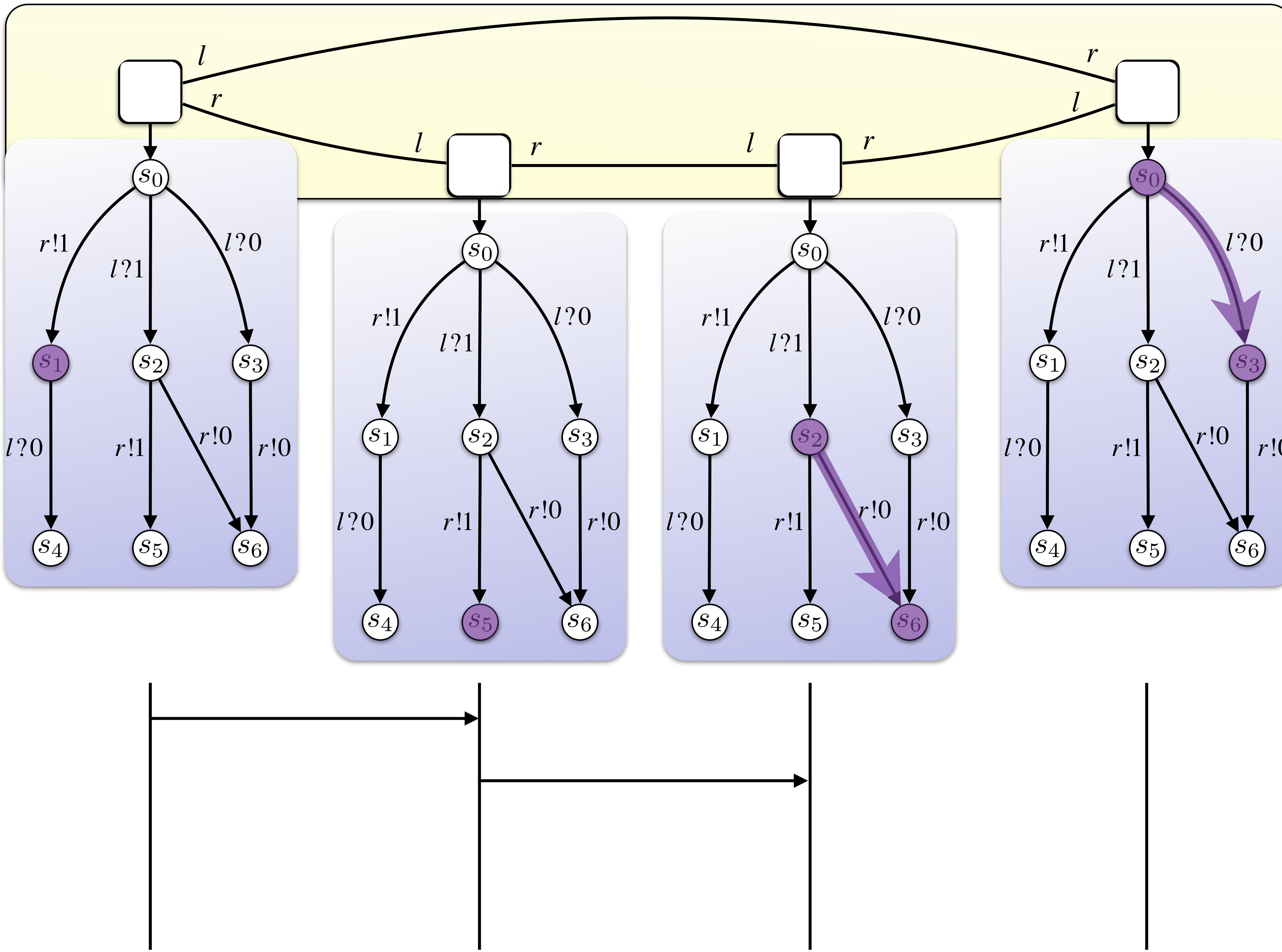

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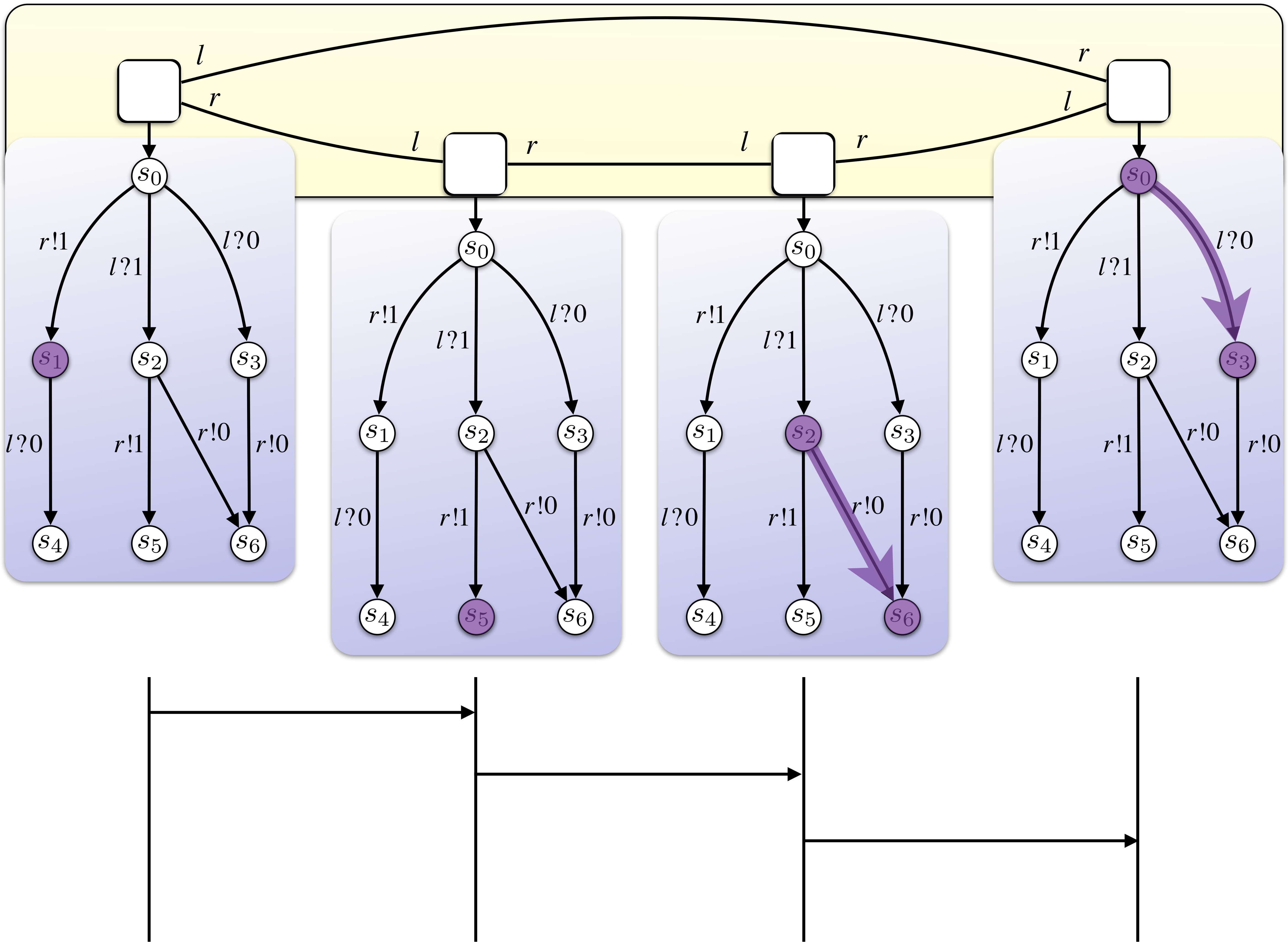

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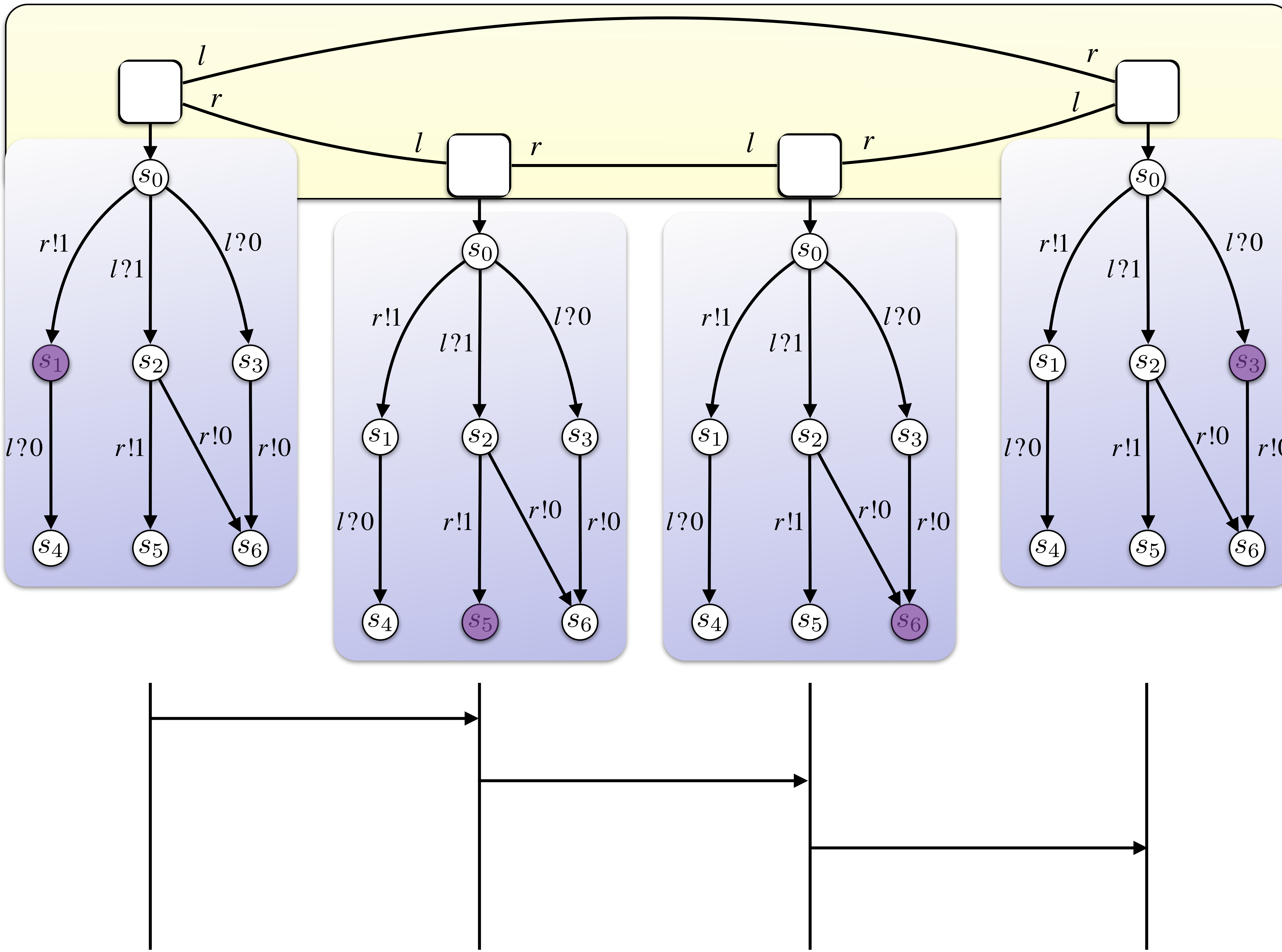

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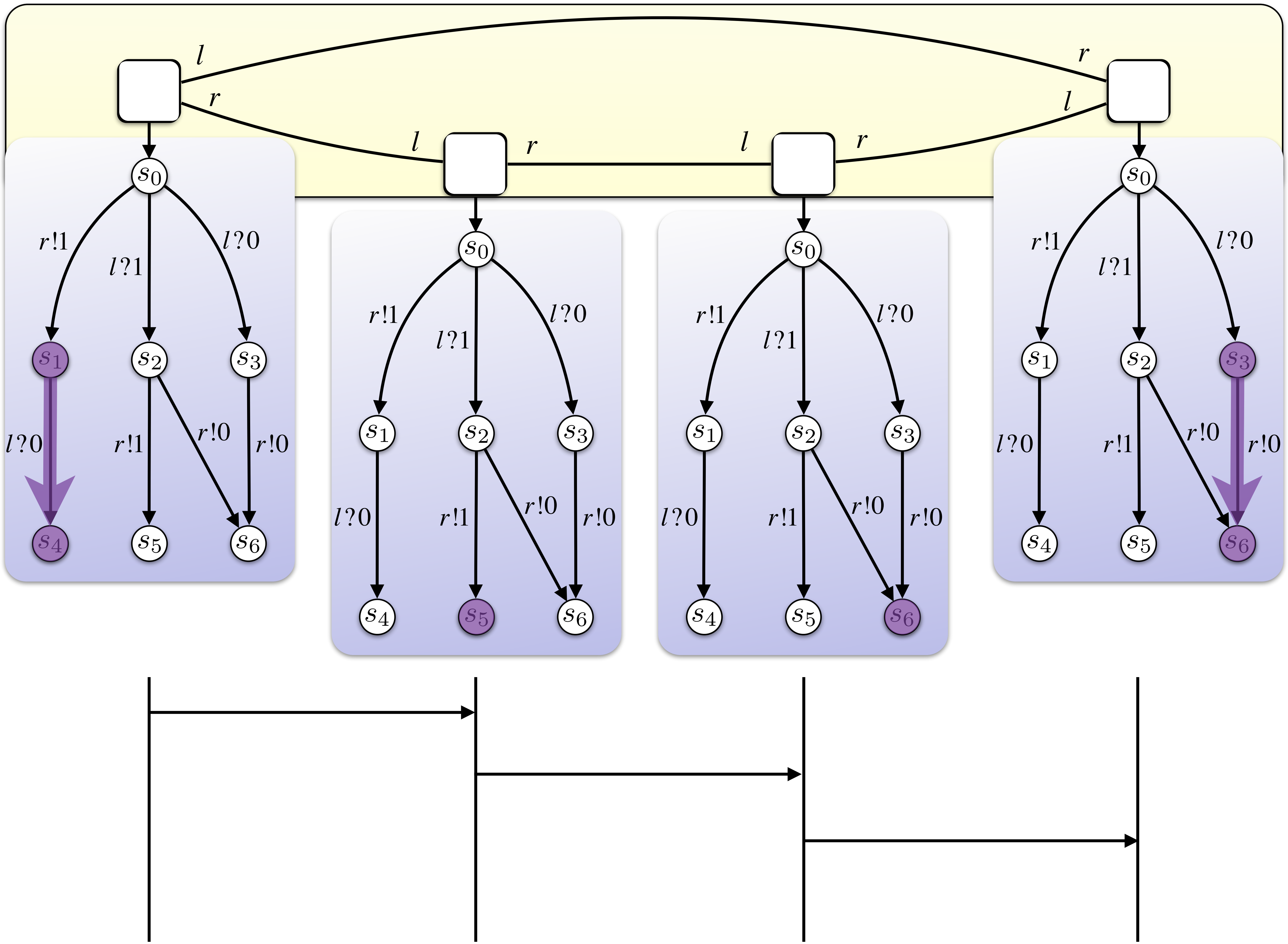

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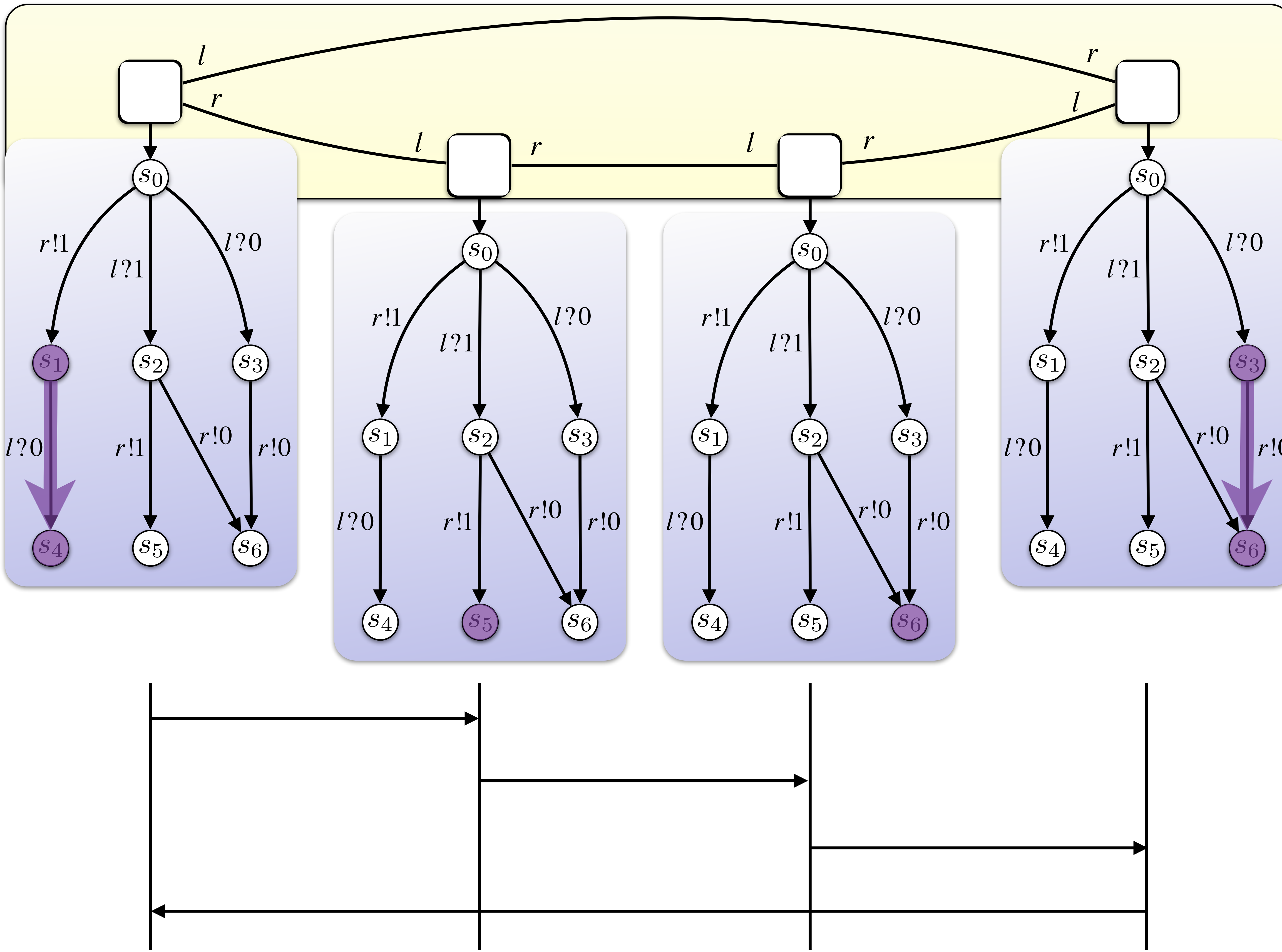

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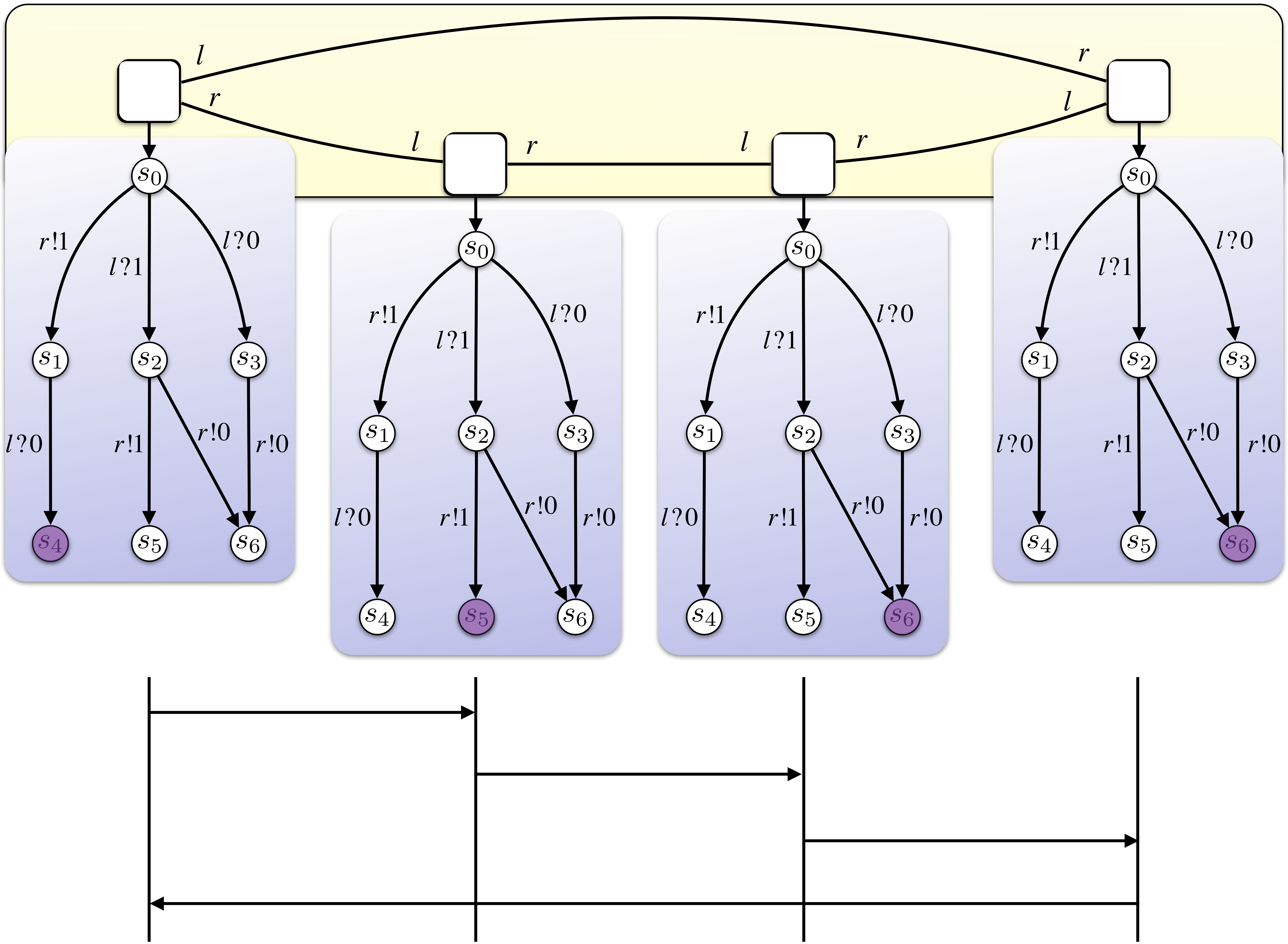

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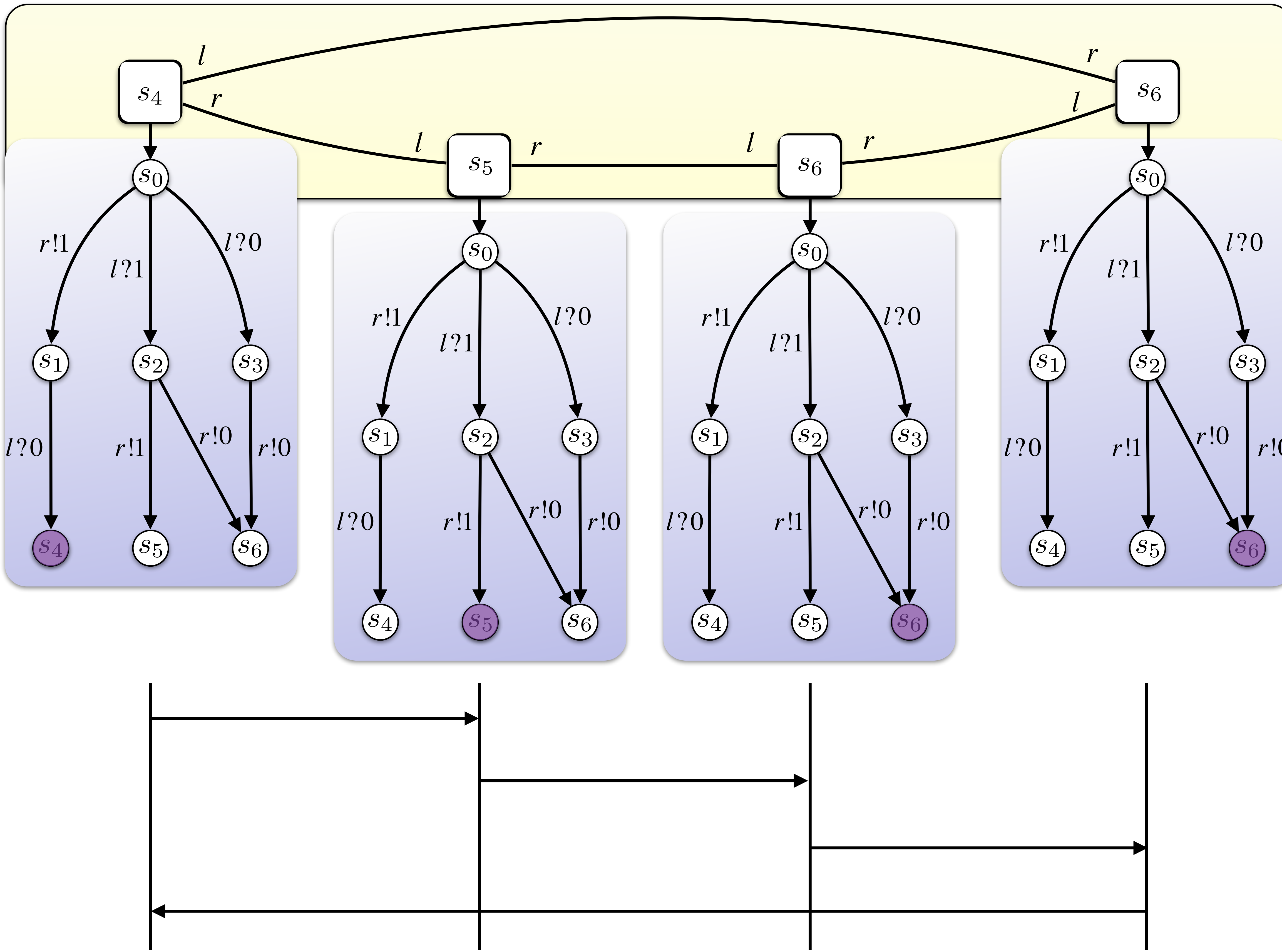

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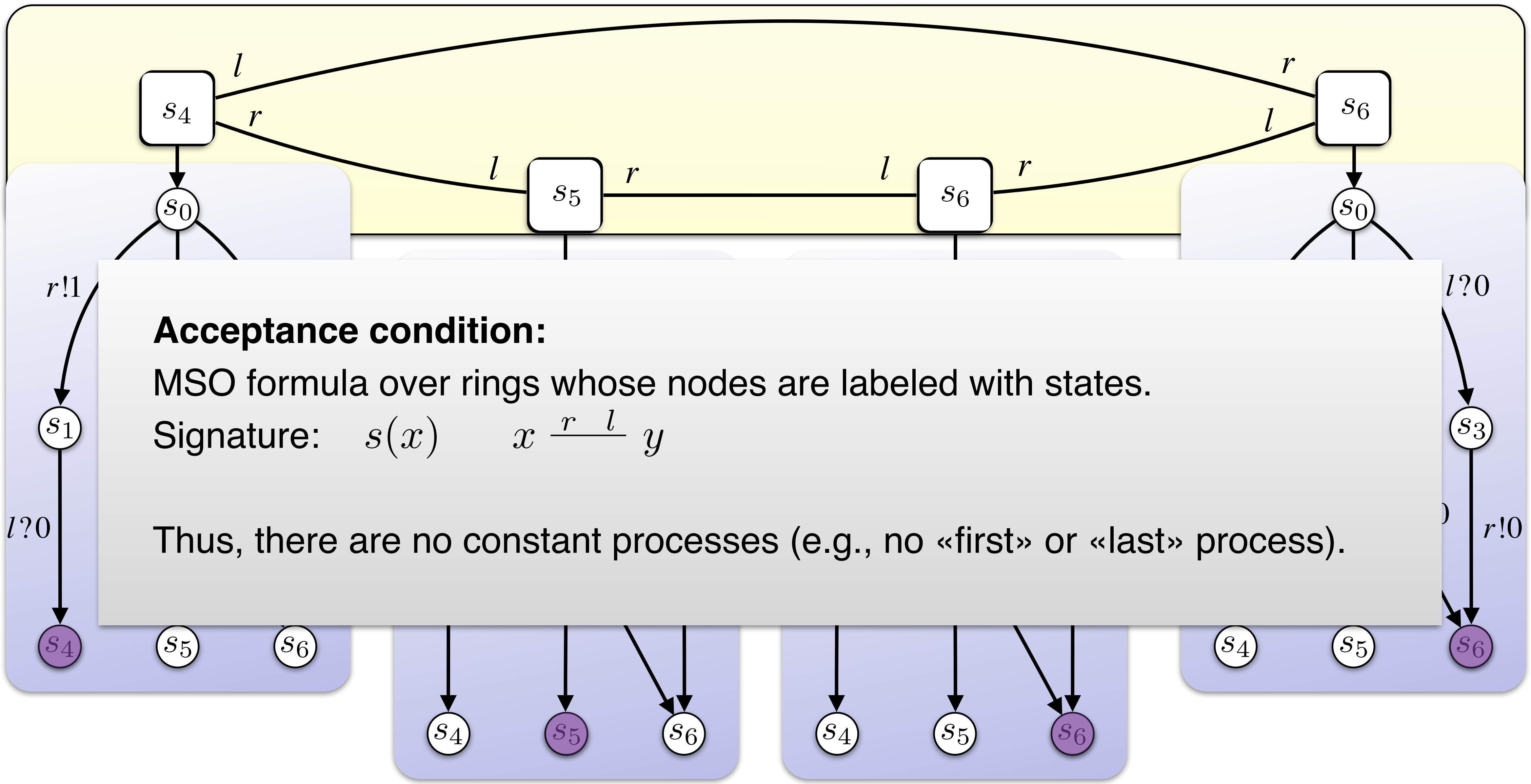
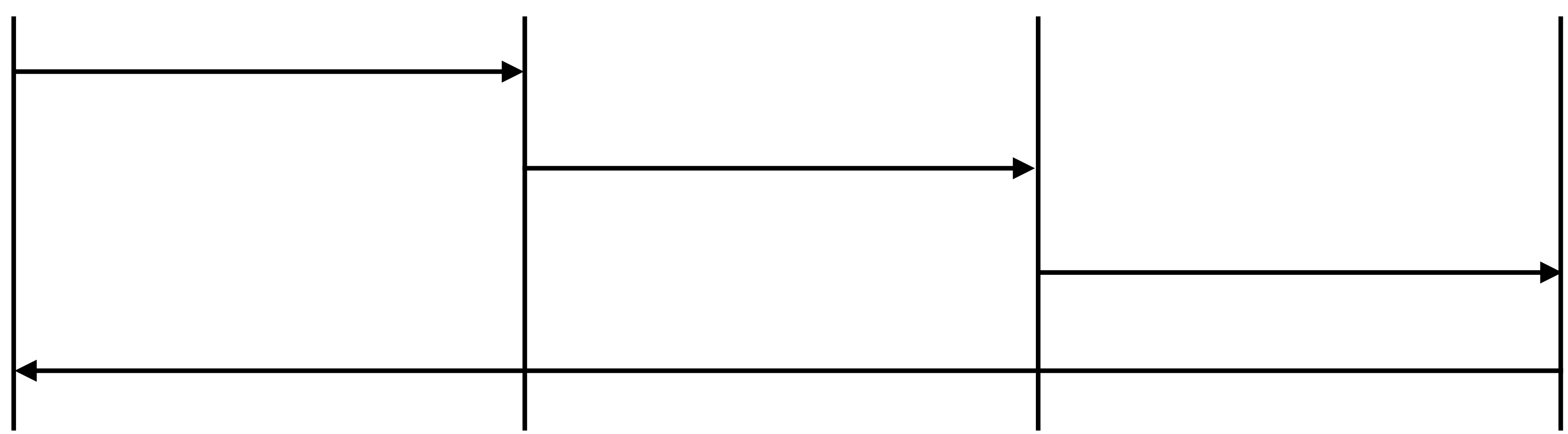


Acceptance condition:

MSO formula over rings whose nodes are labeled with states.

Signature: $s(x)$ $\quad x \xrightarrow{r \quad l} y$

Thus, there are no constant processes (e.g., no «first» or «last» process).



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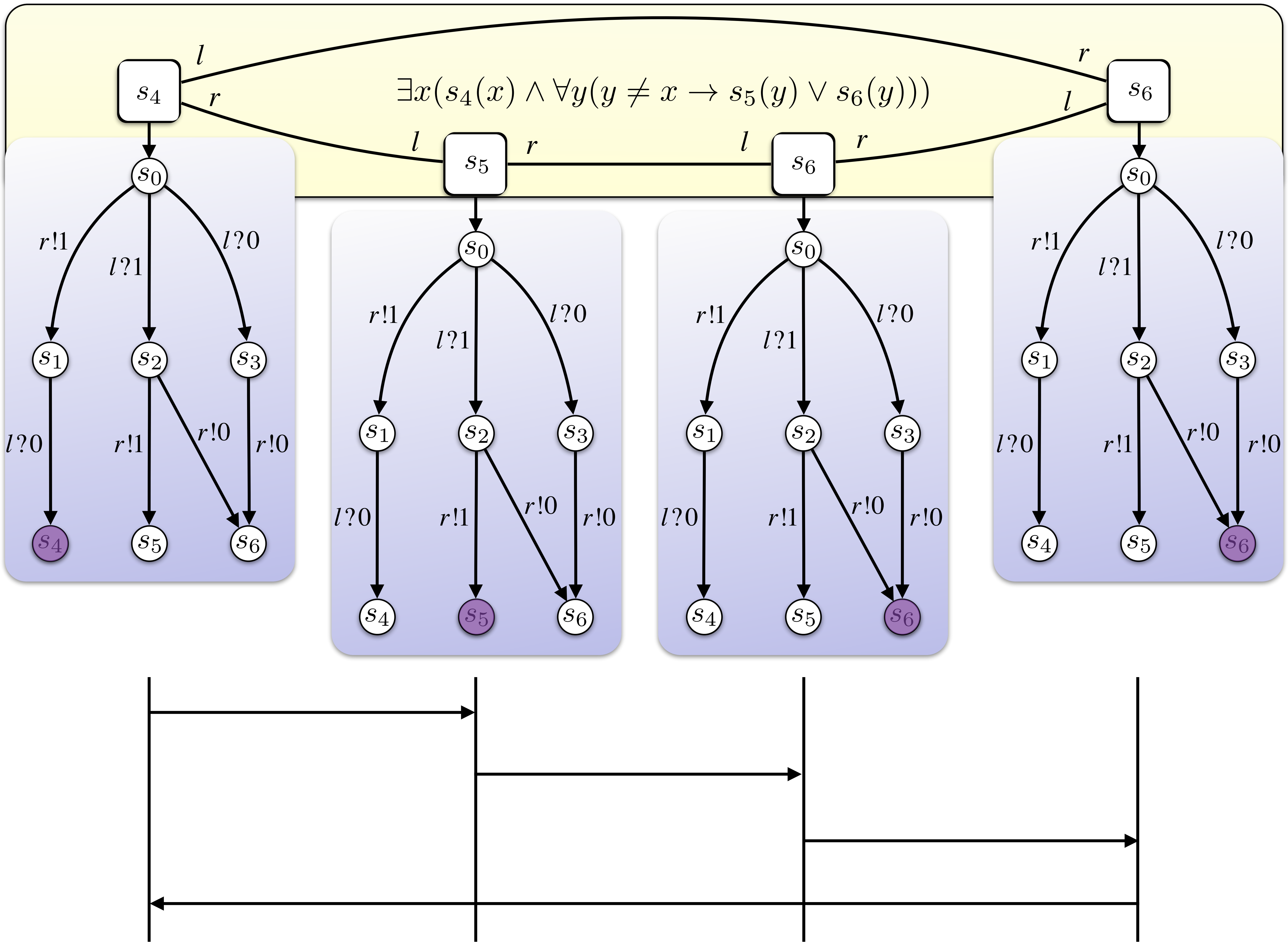

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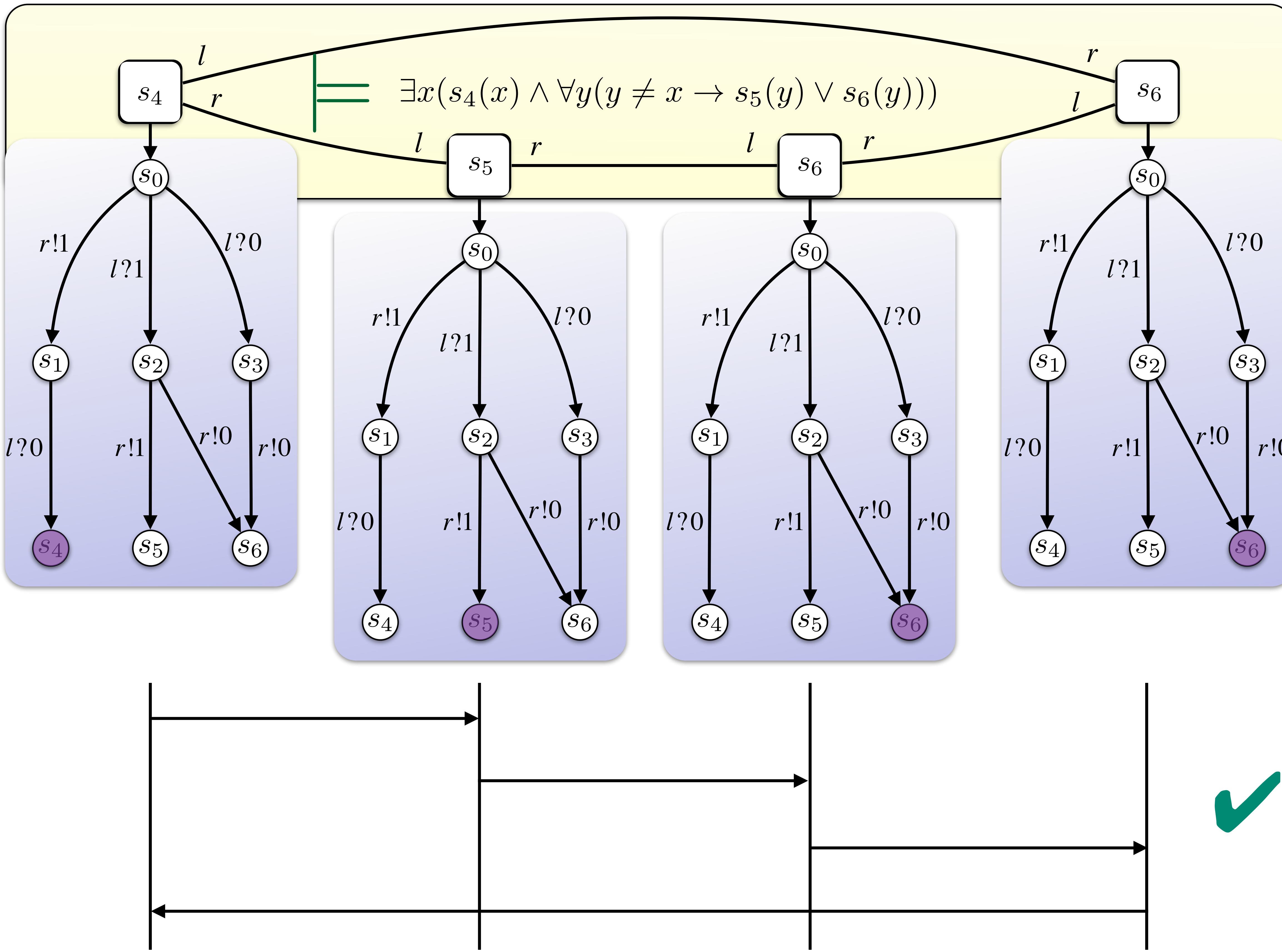

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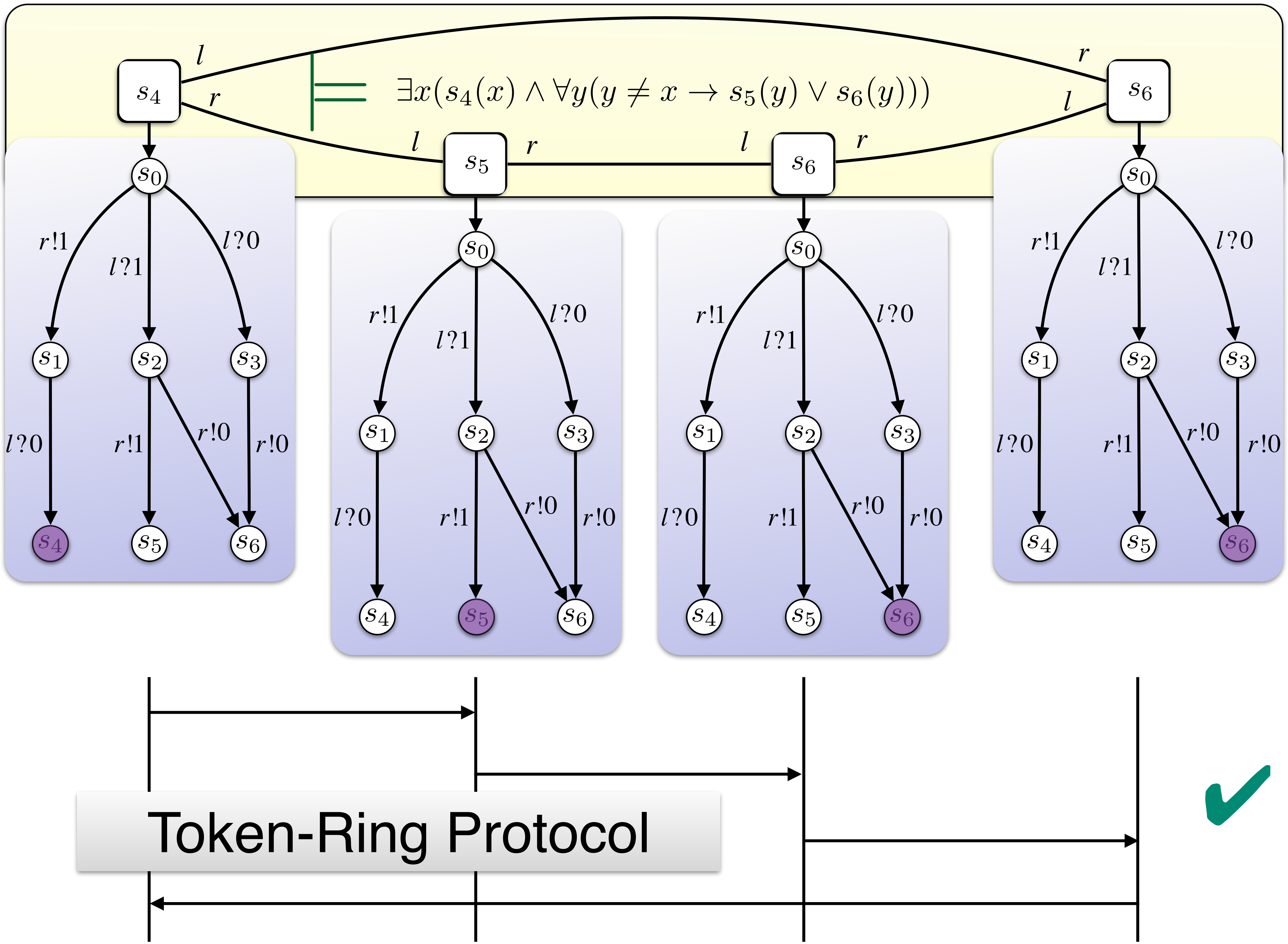

Token-Ring Protocol

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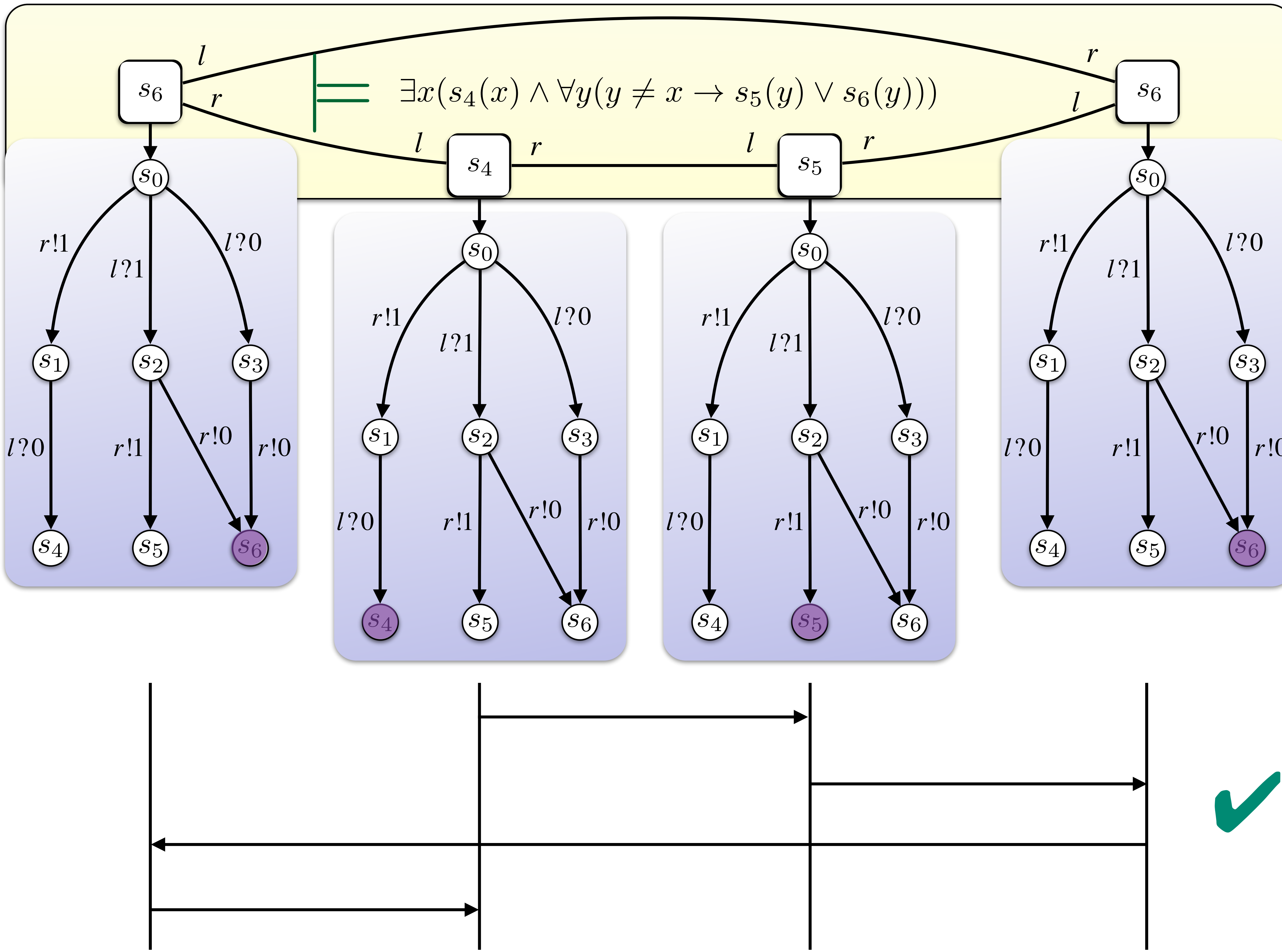
Parameterized Communicating Automata (PCA) over Rings



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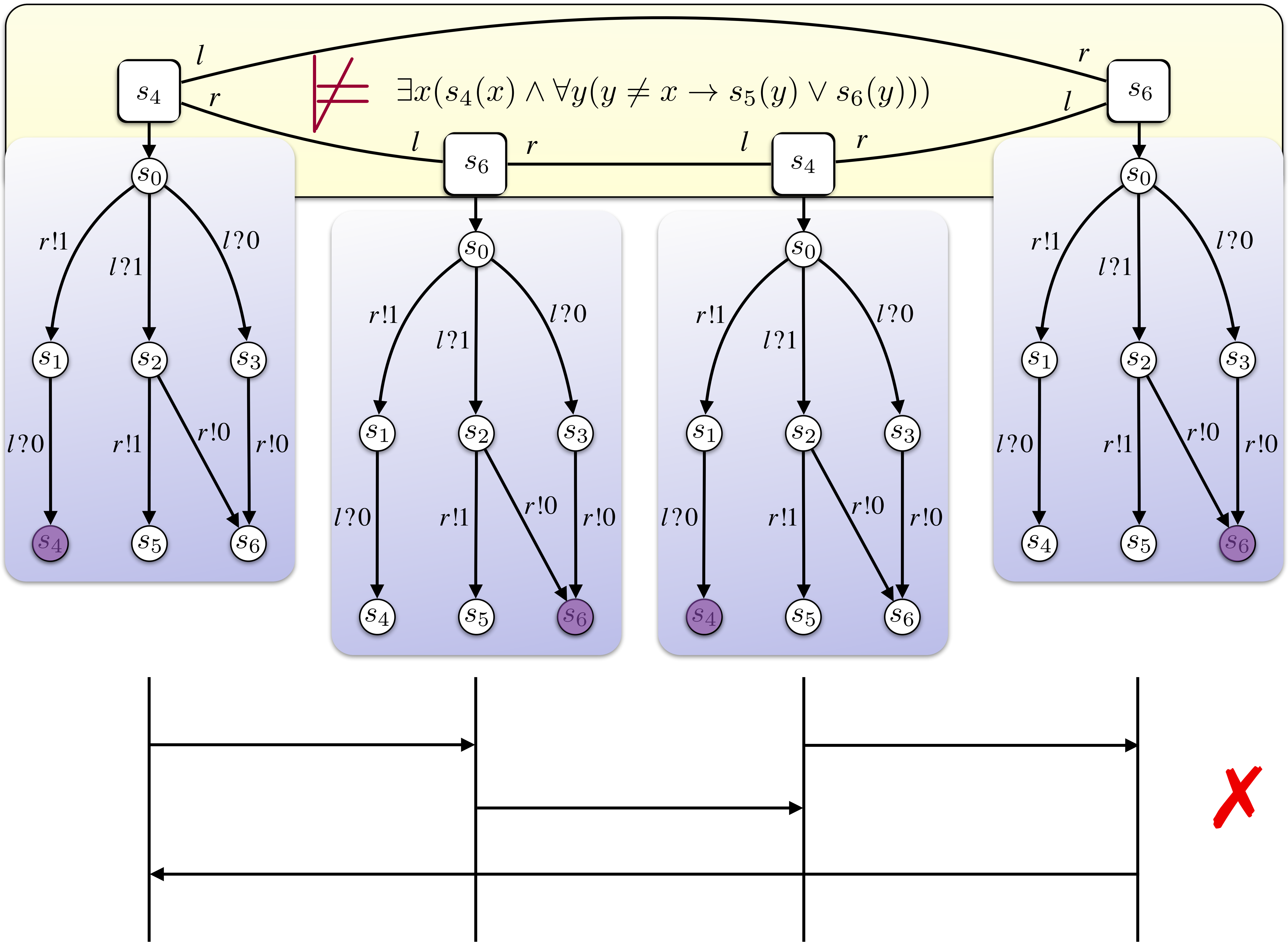

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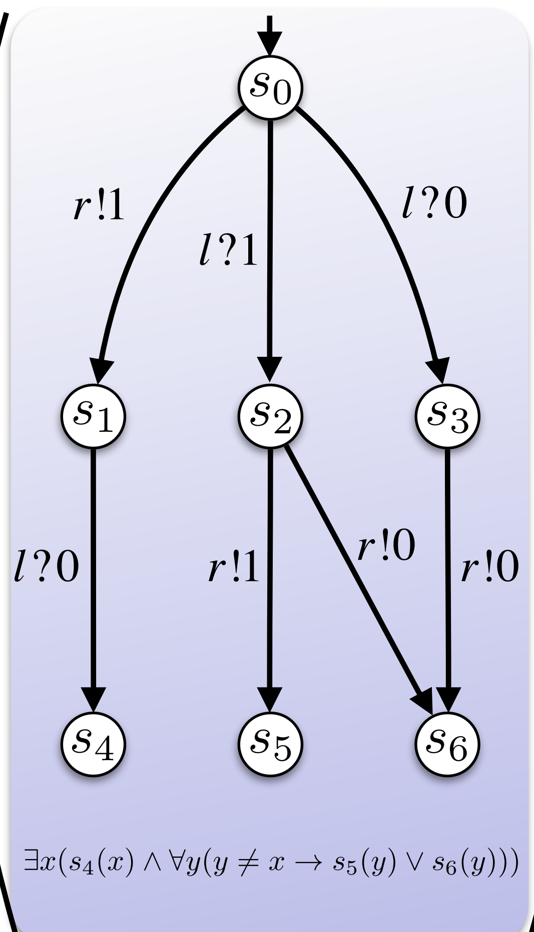

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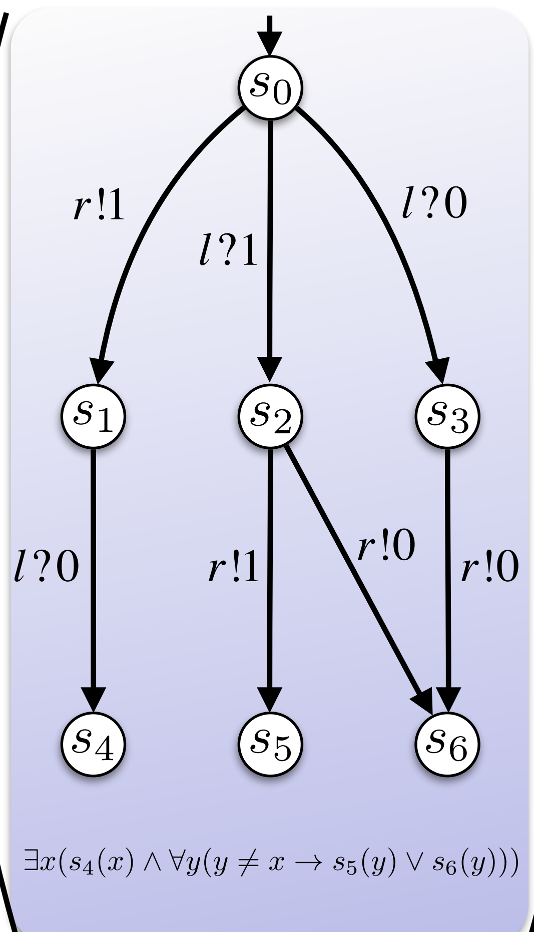

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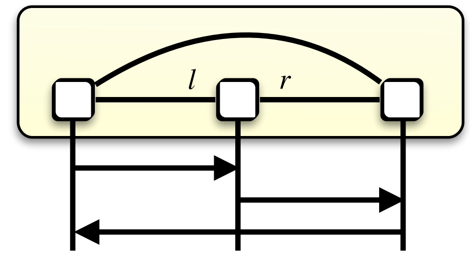

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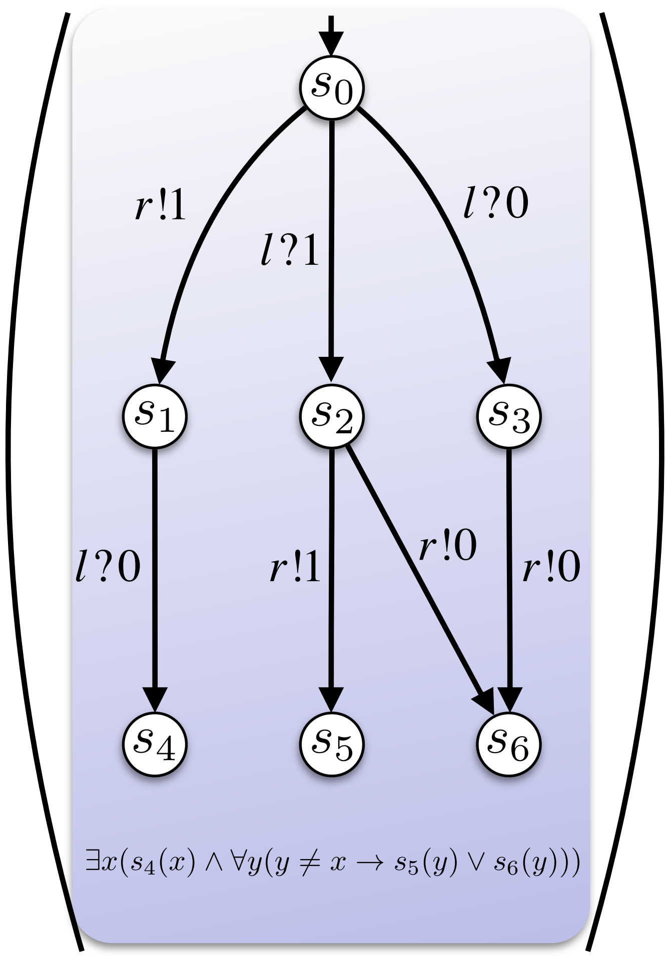

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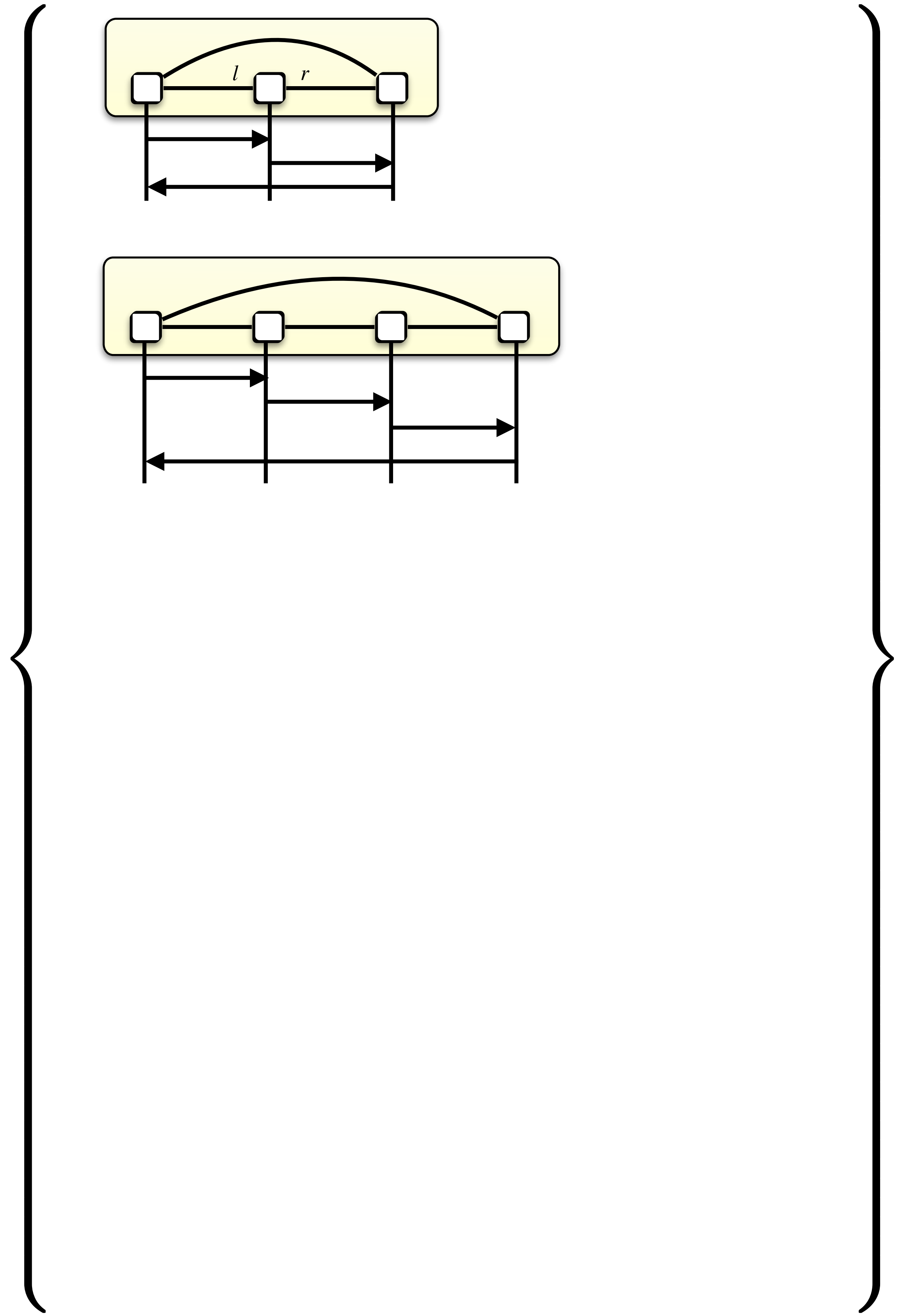

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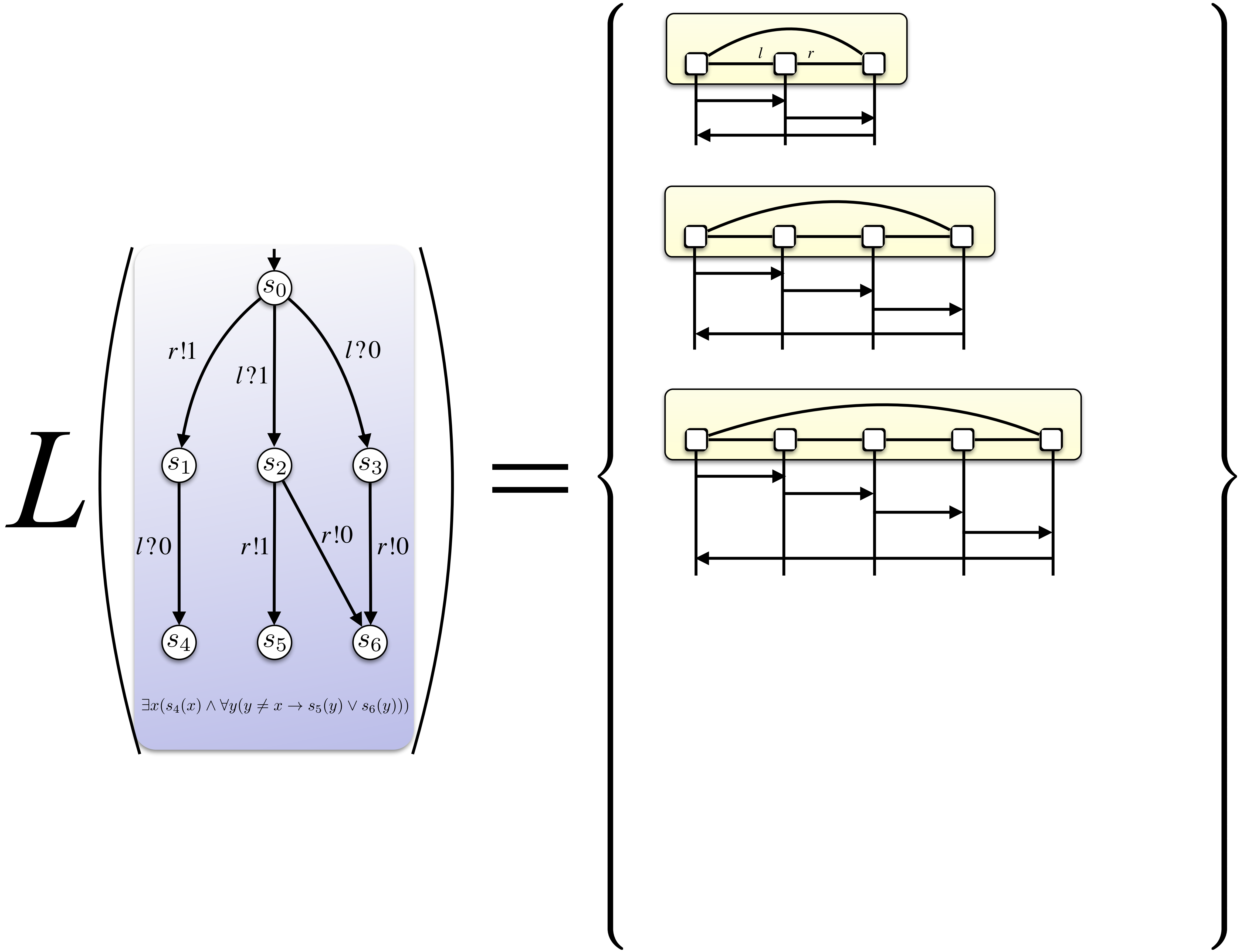

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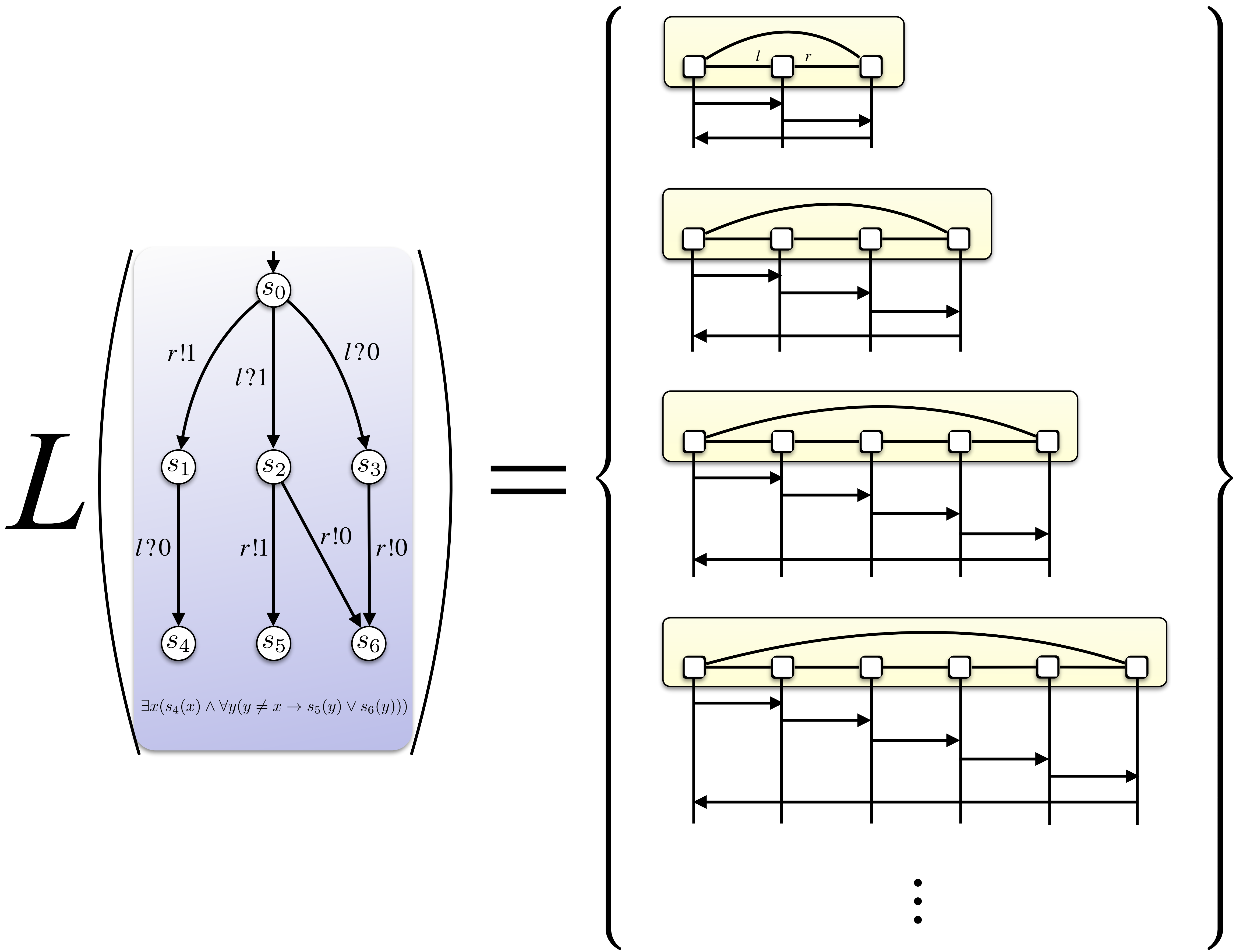
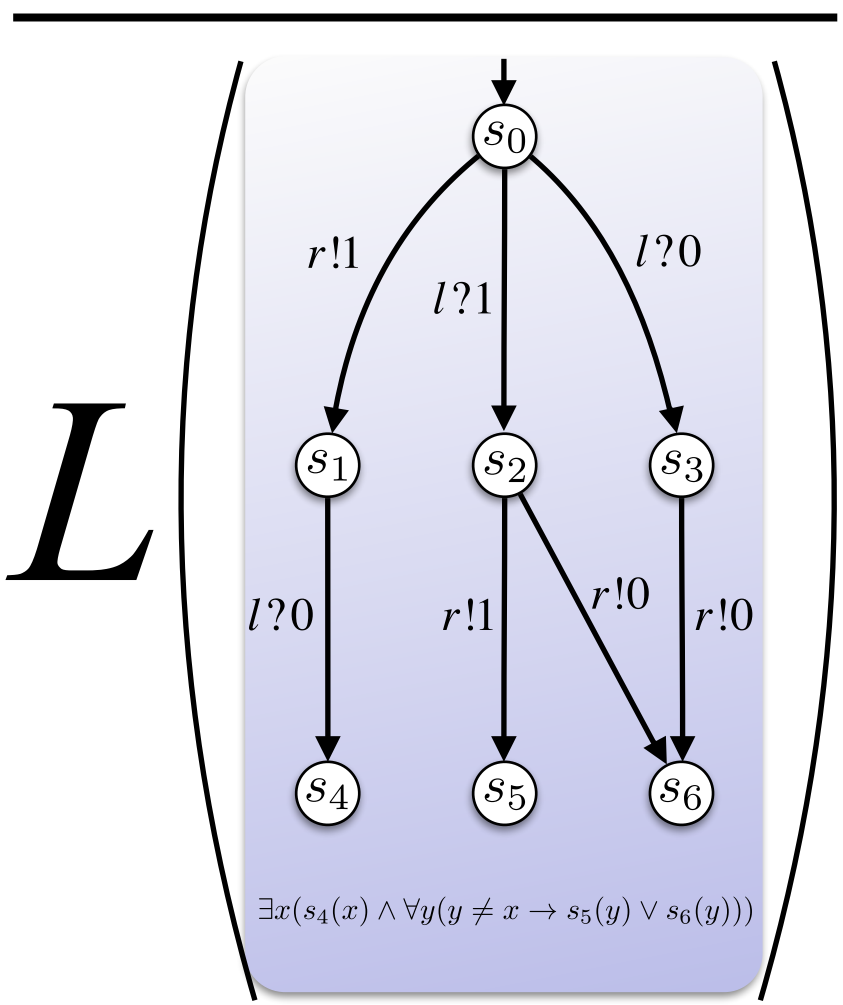


Complementation



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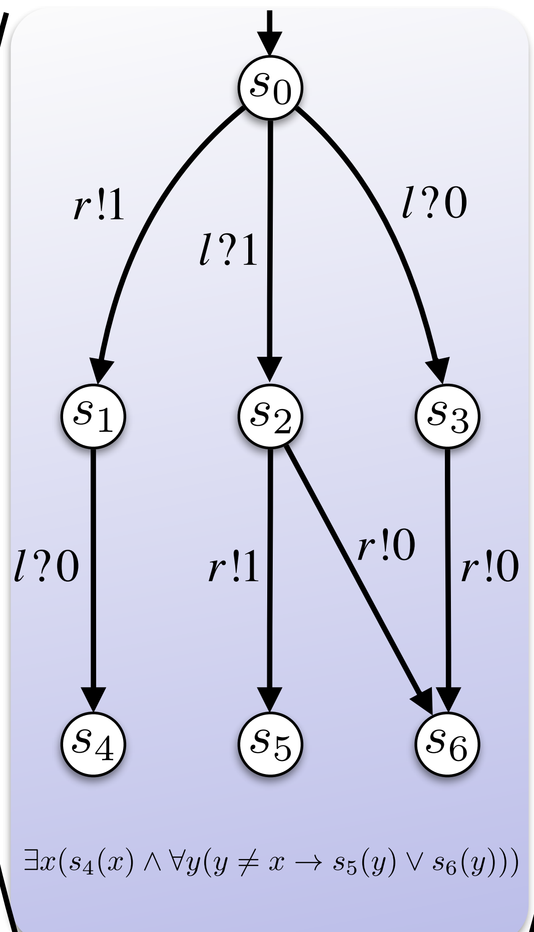

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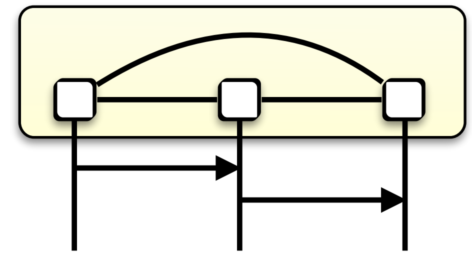

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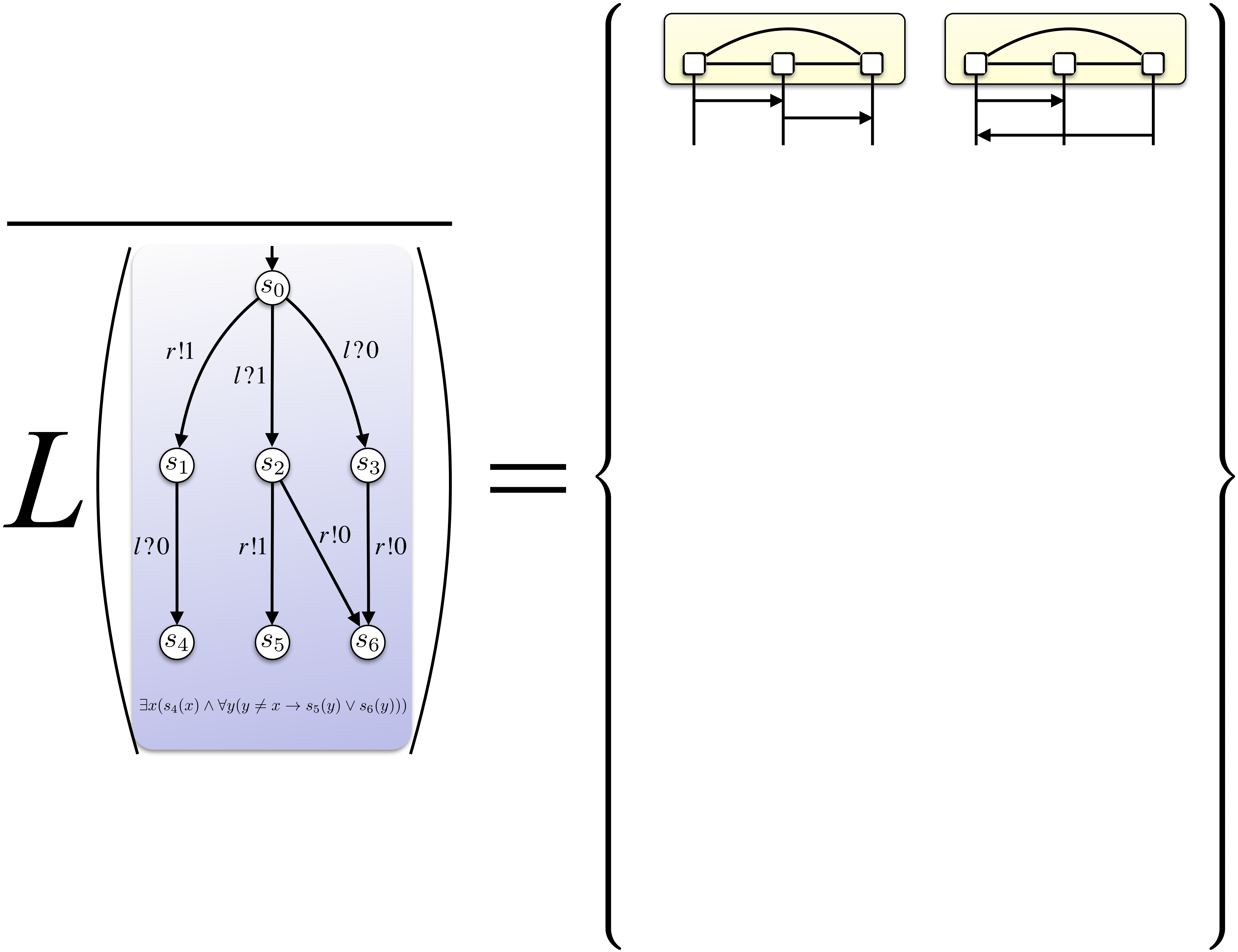

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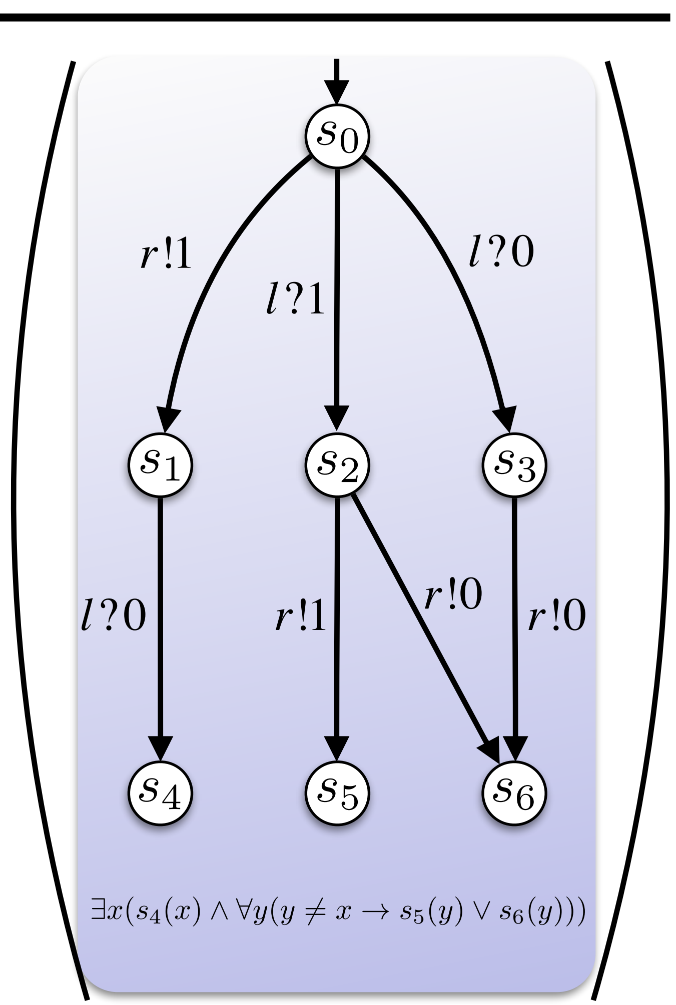

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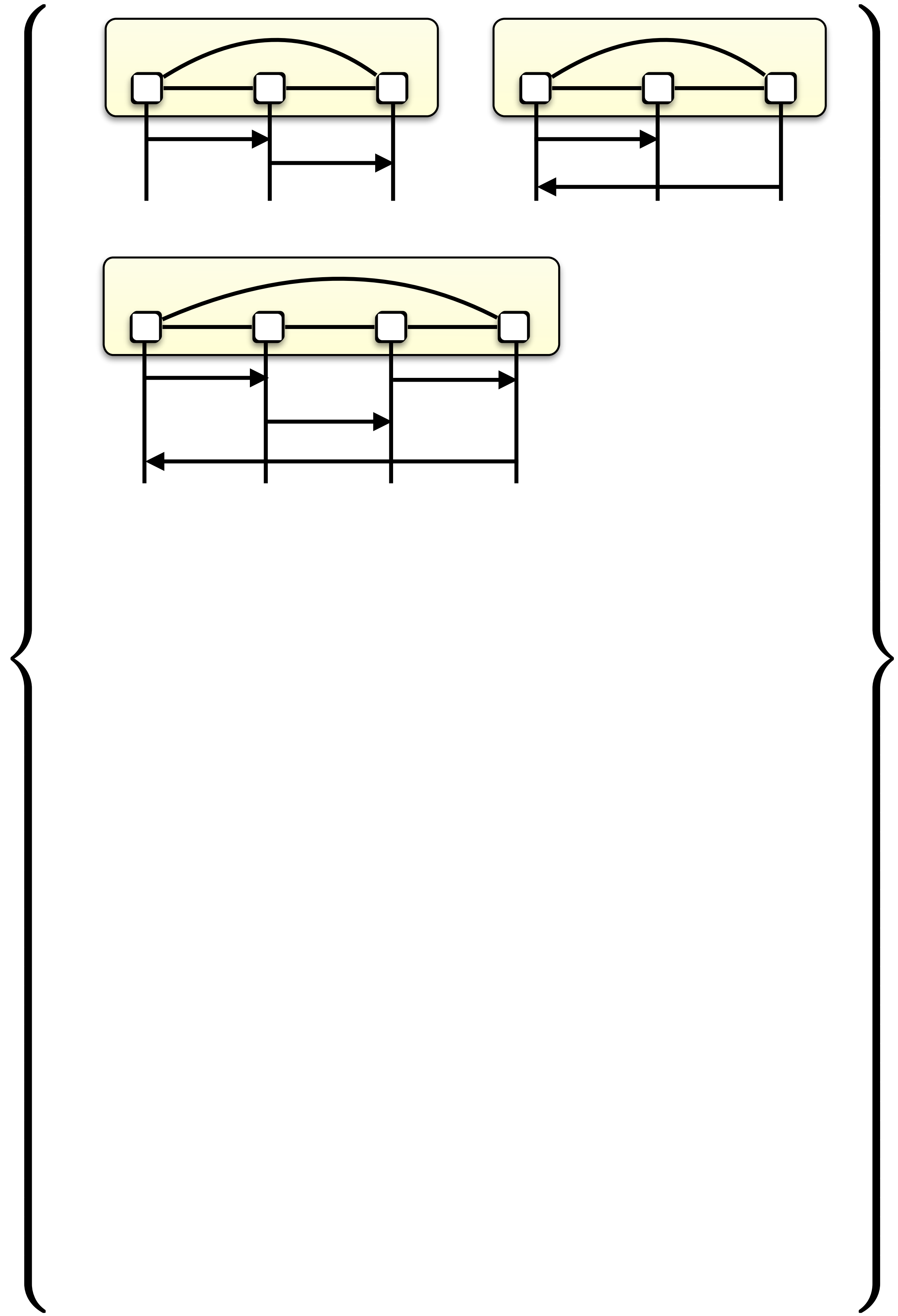

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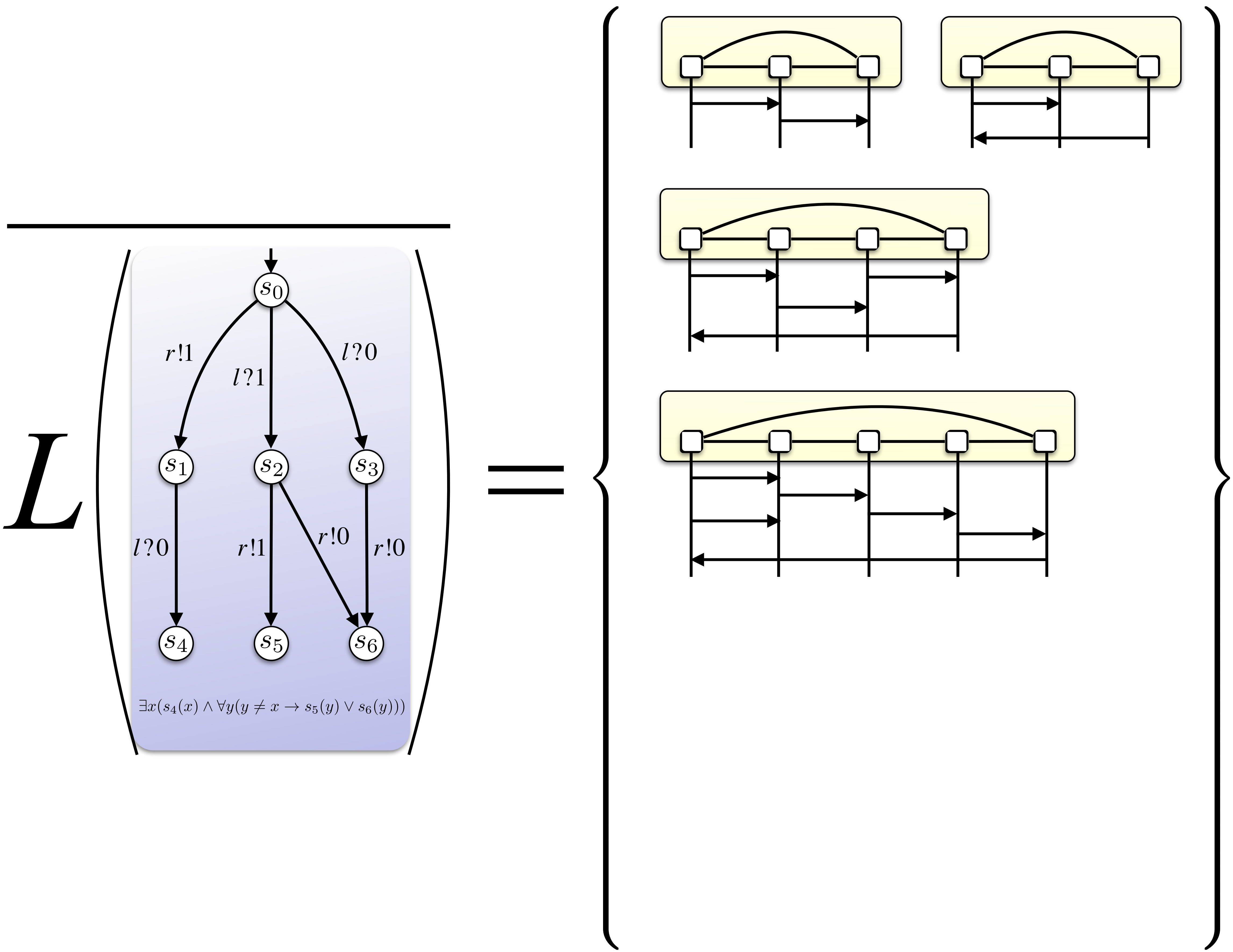

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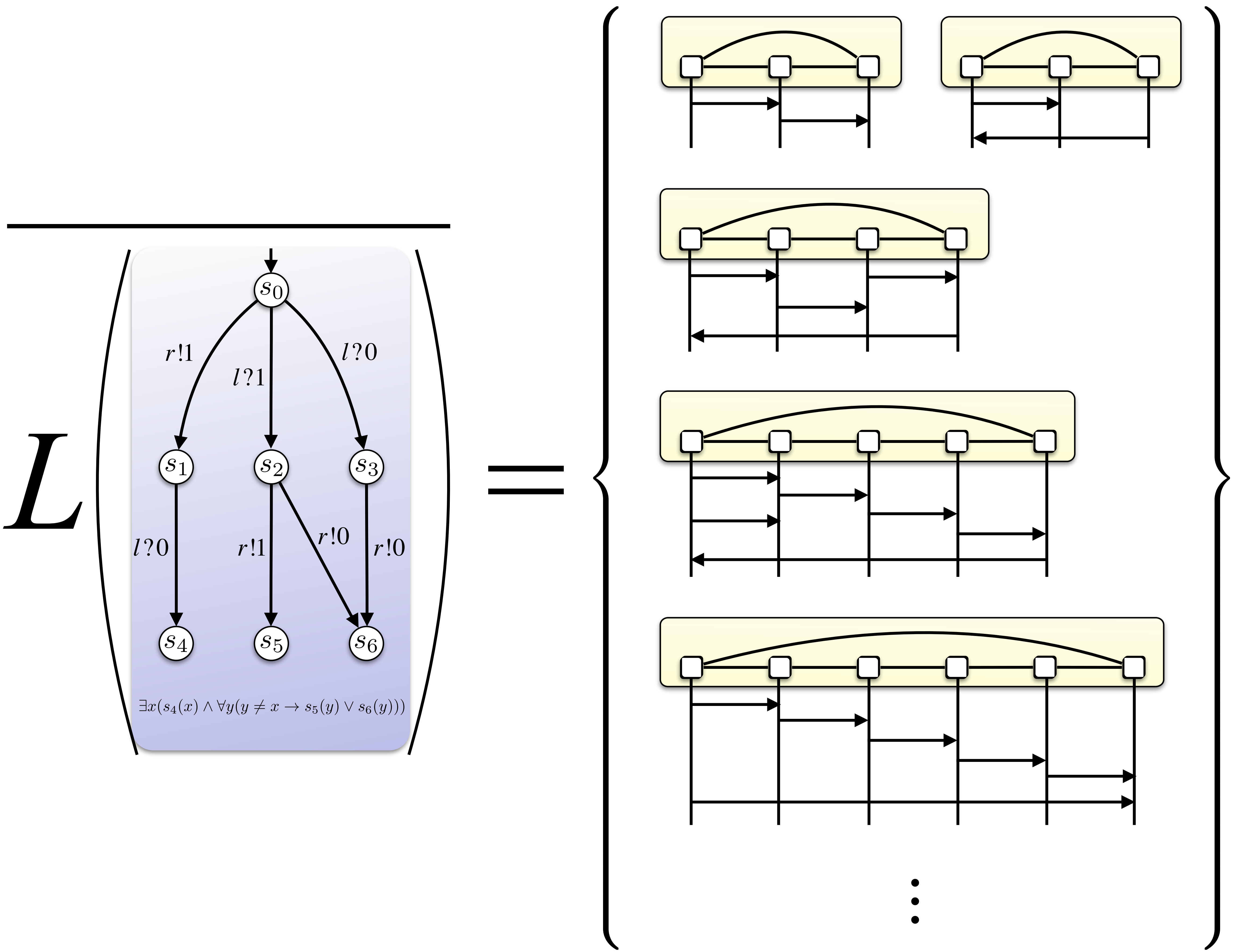

Negative Results

Theorem [B.-Gastin-Kumar; FSTTCS 2014]:

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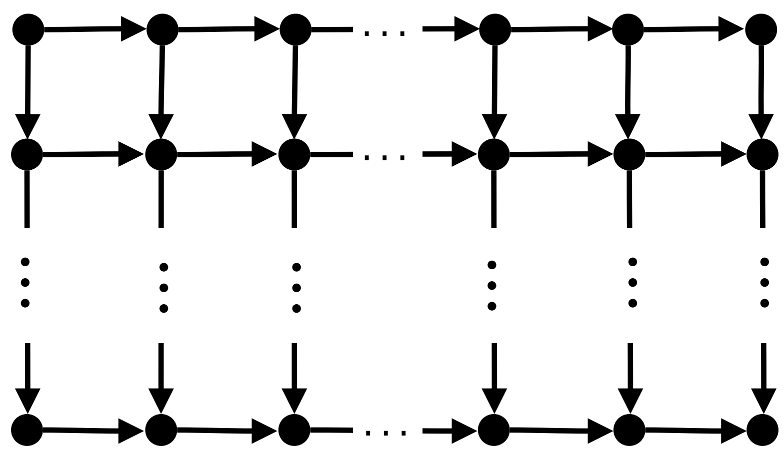

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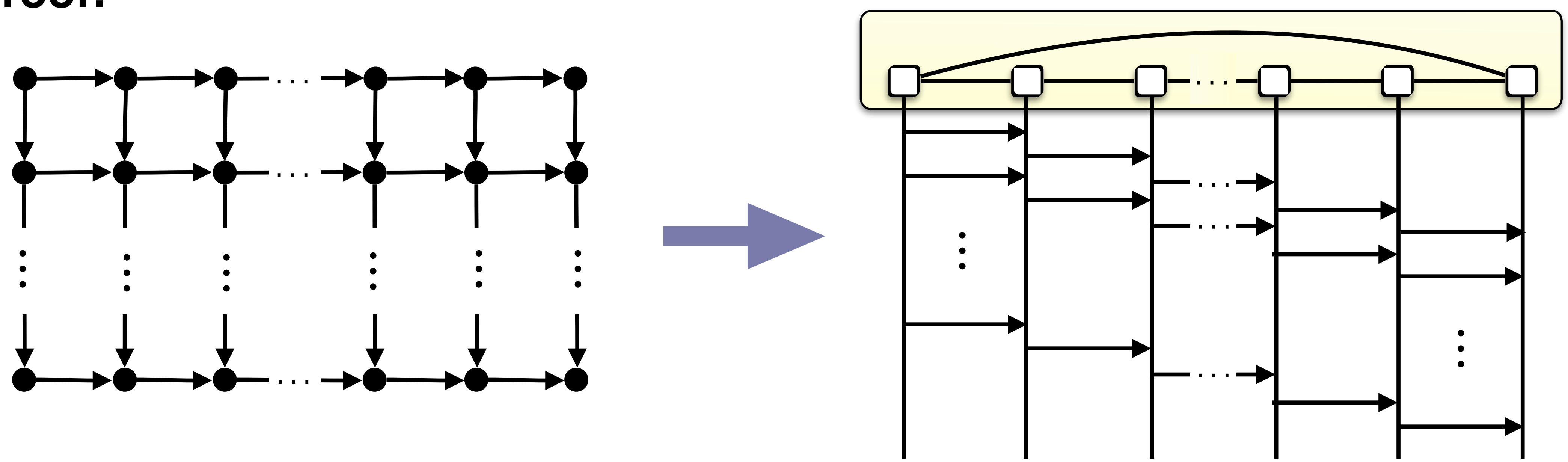

- Behaviors encode grids.

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Theorem [B.-Gastin-Kumar; FSTTCS 2014]:
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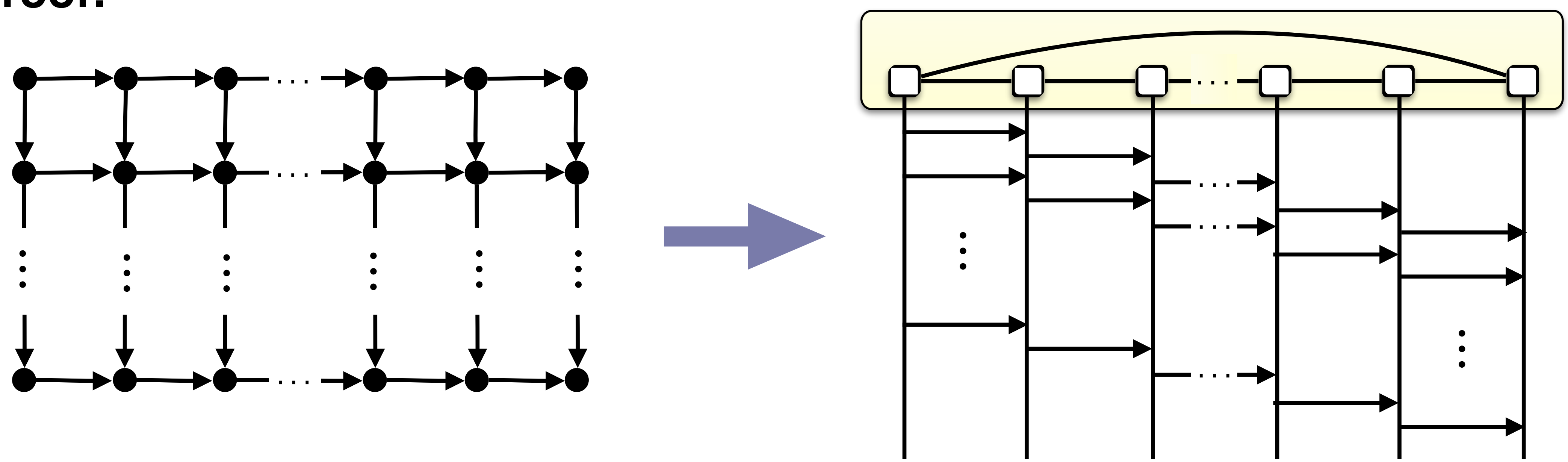

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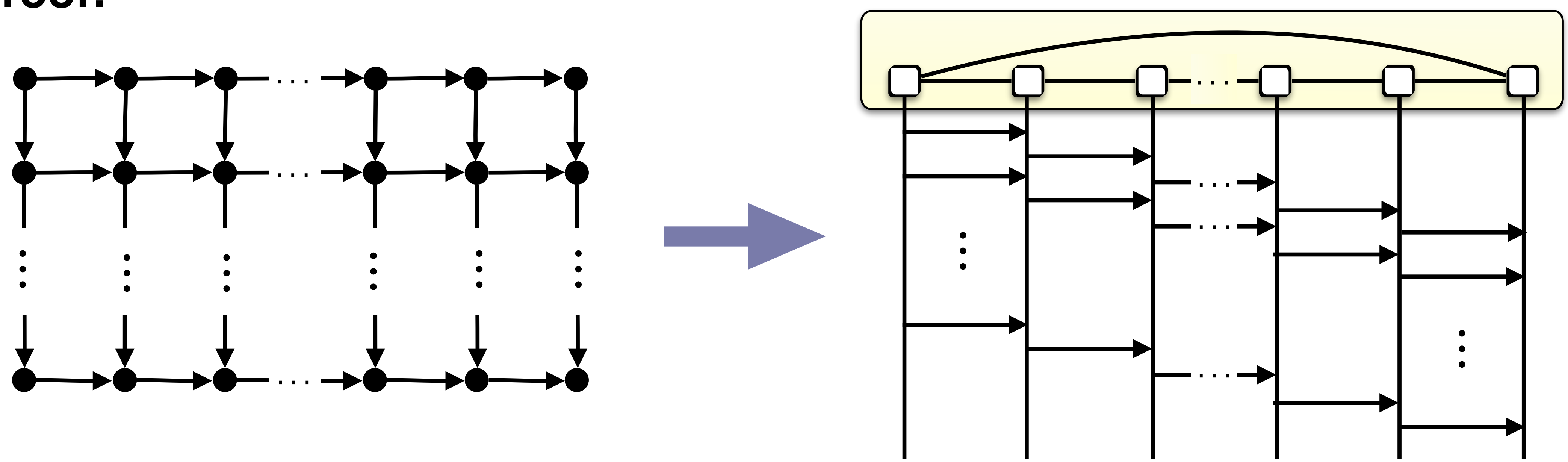

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Theorem [Emerson-Namjoshi 2003]:
Emptiness is undecidable for PCAs over rings
(even token-passing systems, unless $|Msg| = 1$).

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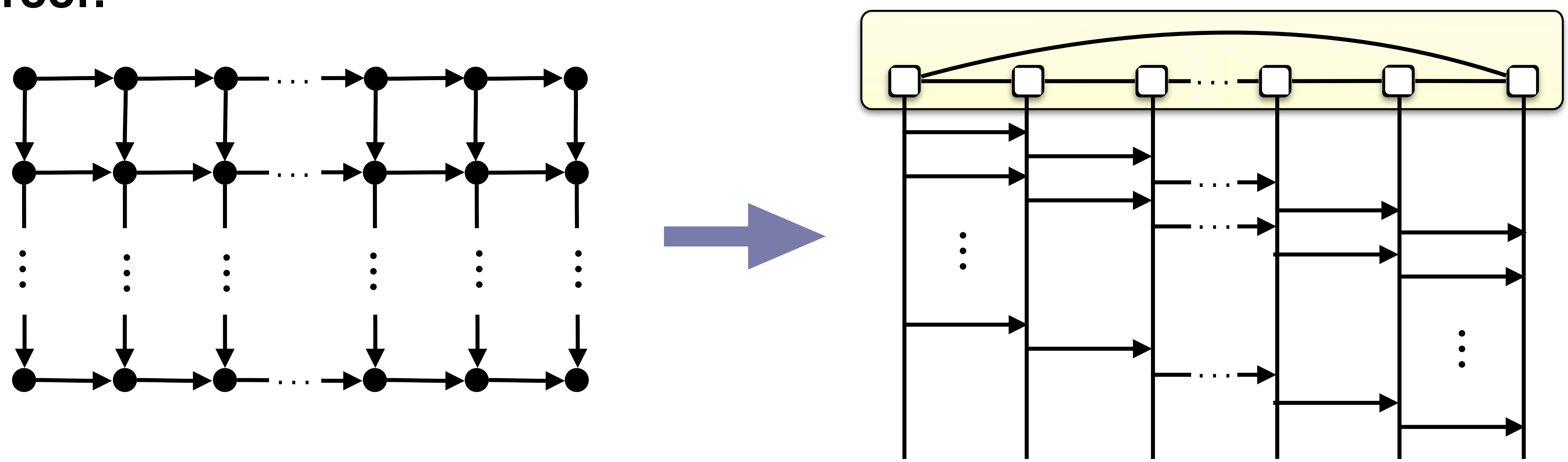

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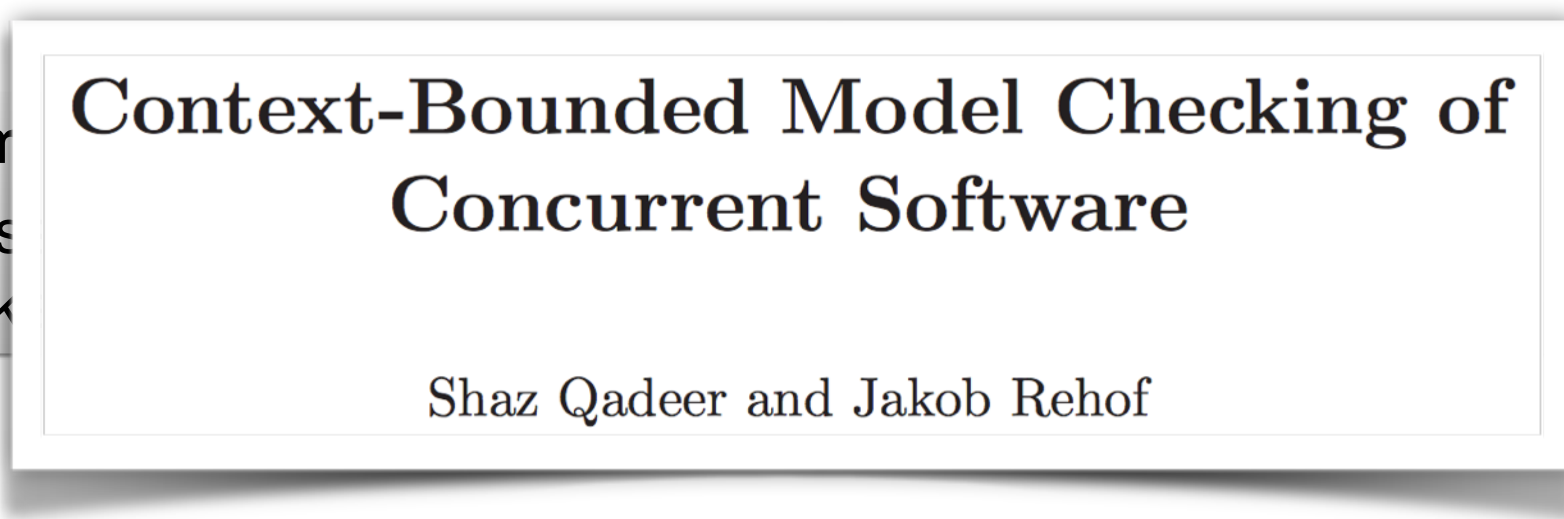

Context-Bounded Model Checking of Concurrent Software

Shaz Qadeer and Jakob Rehof

Context-Bounded PCAs

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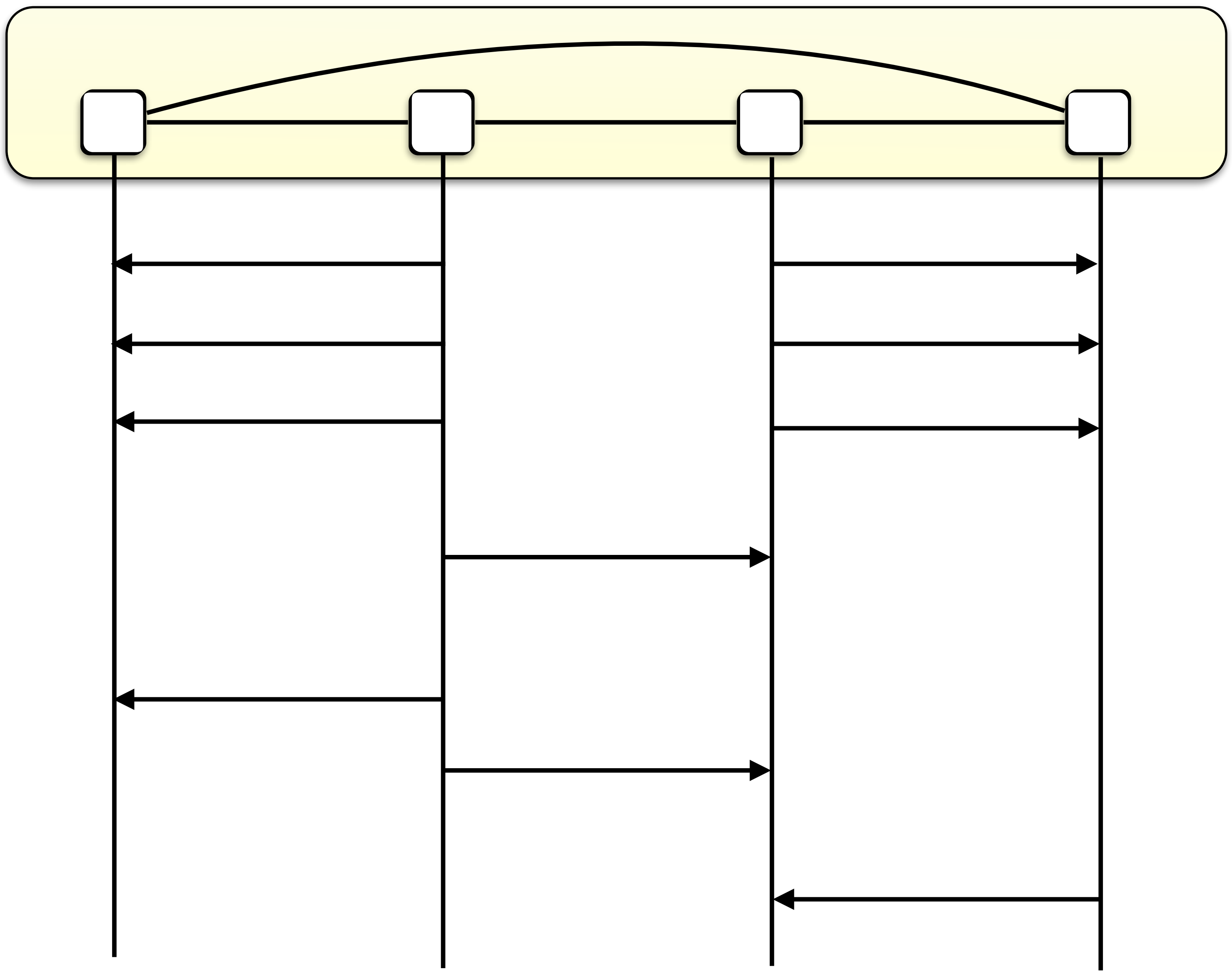

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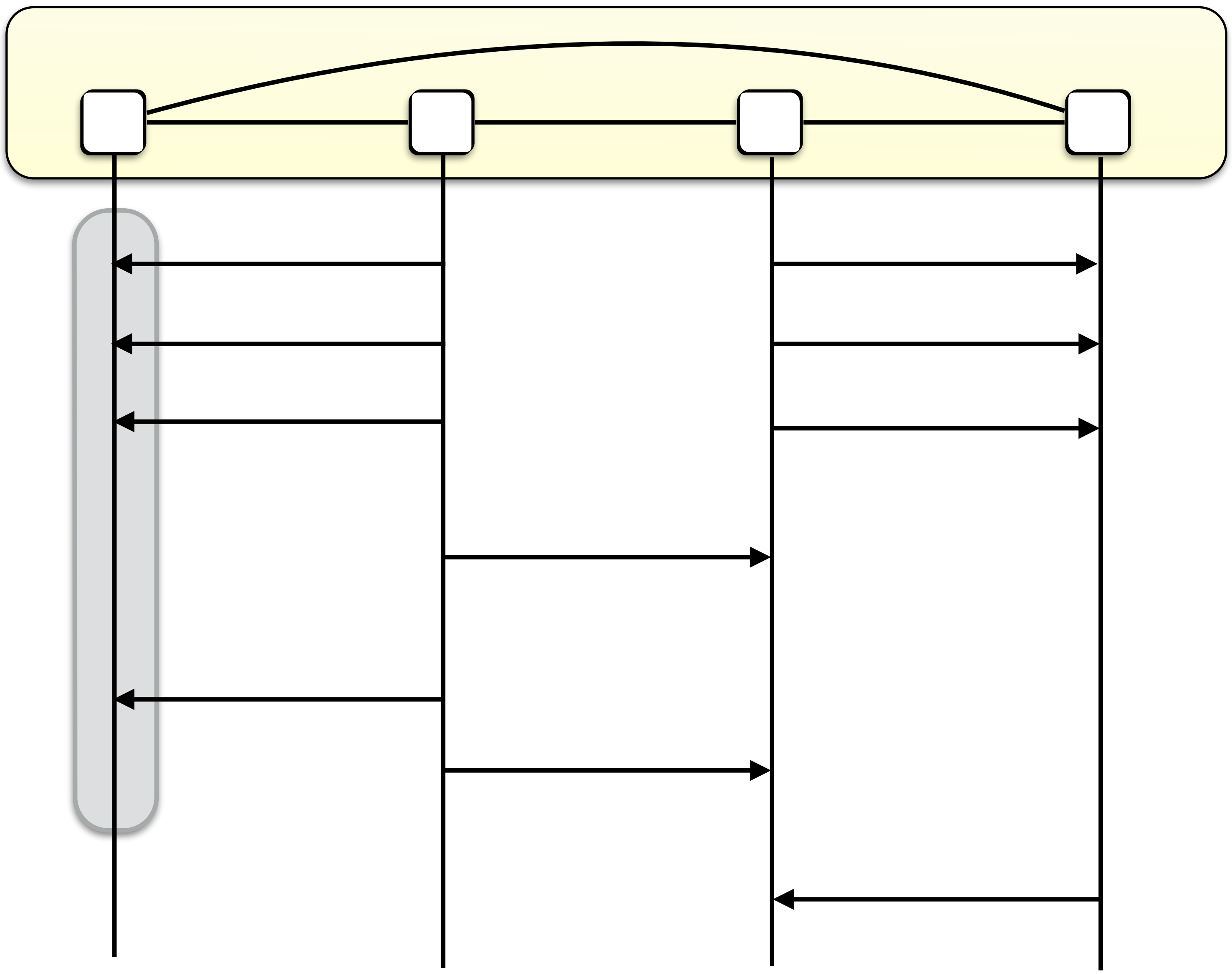

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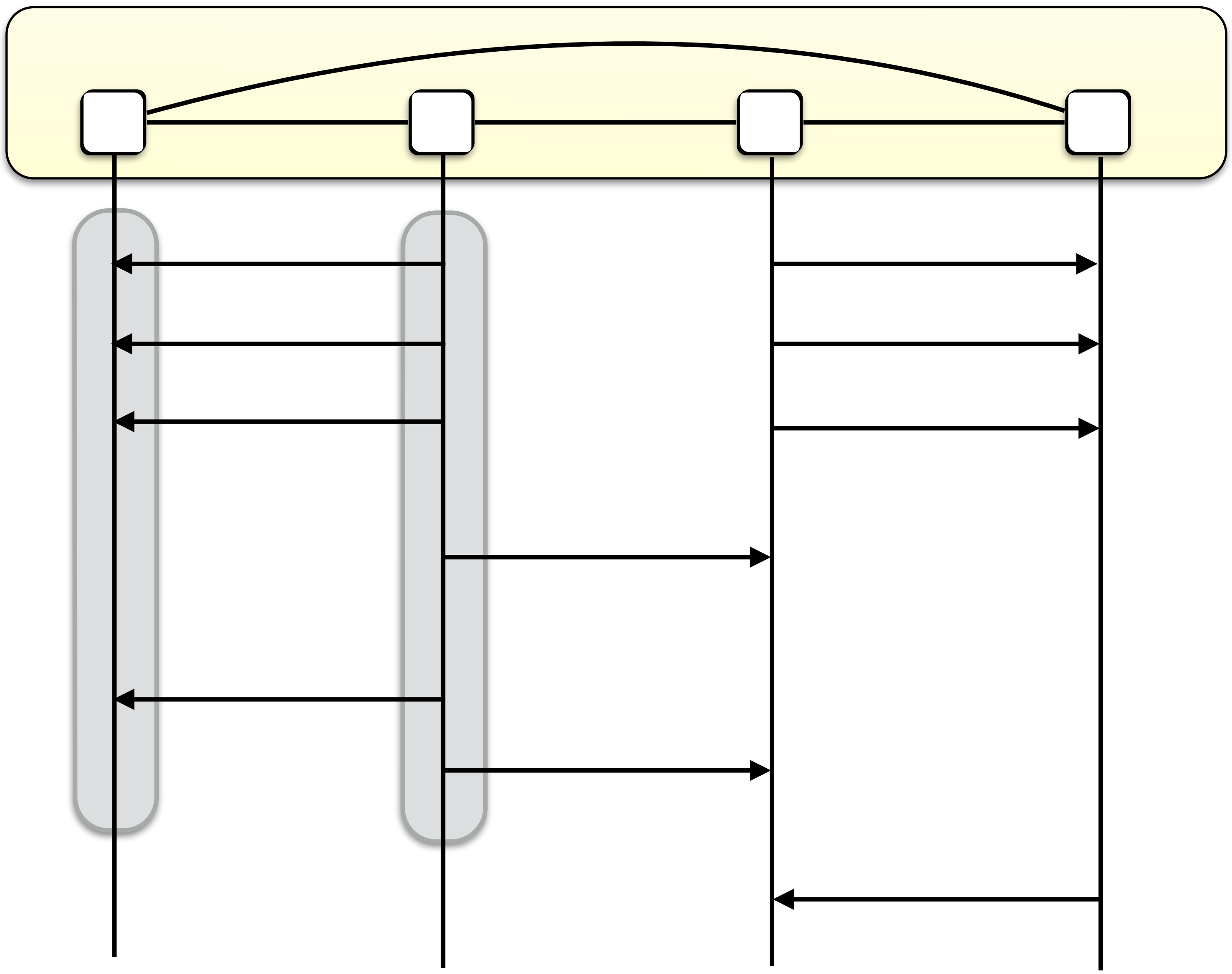

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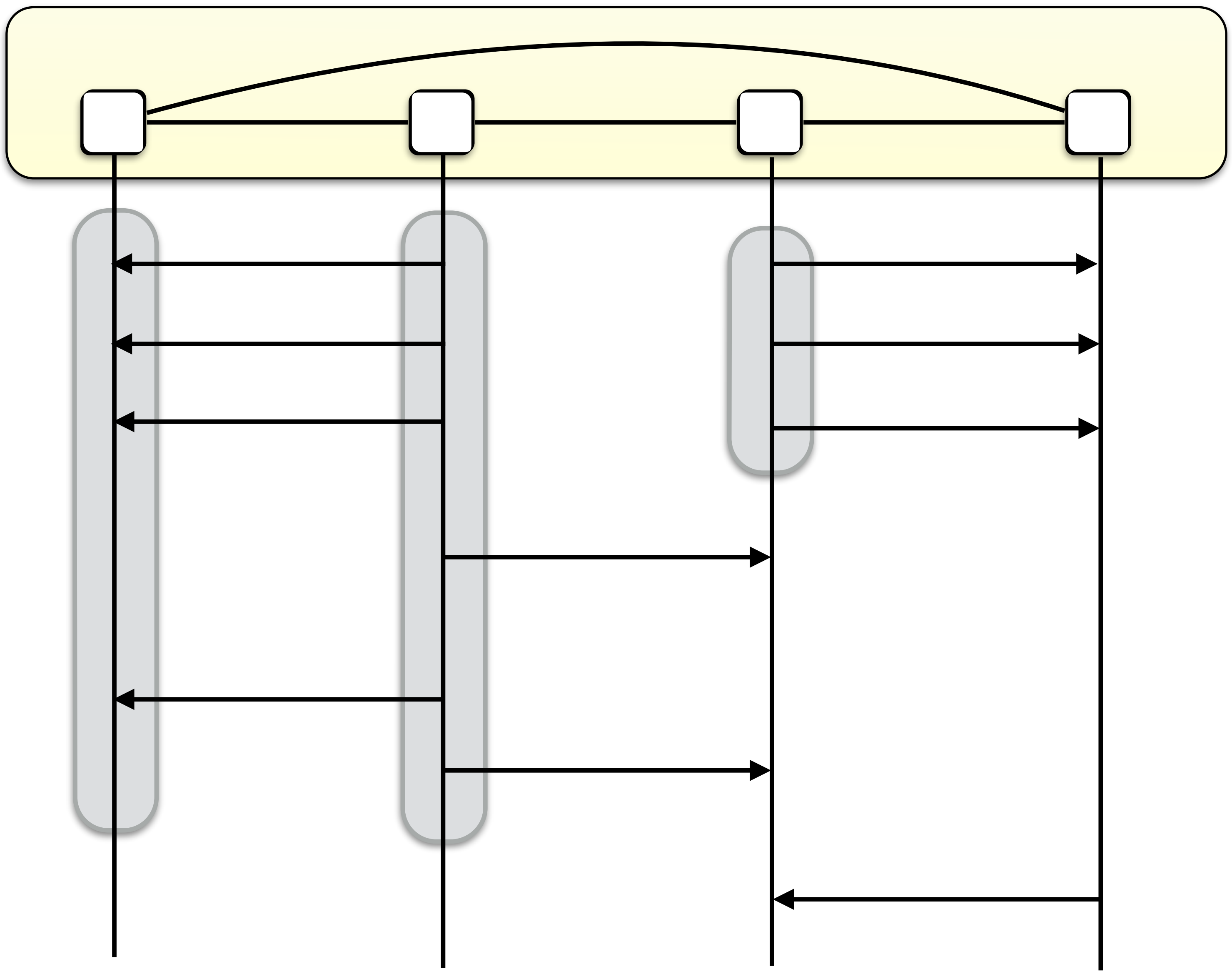

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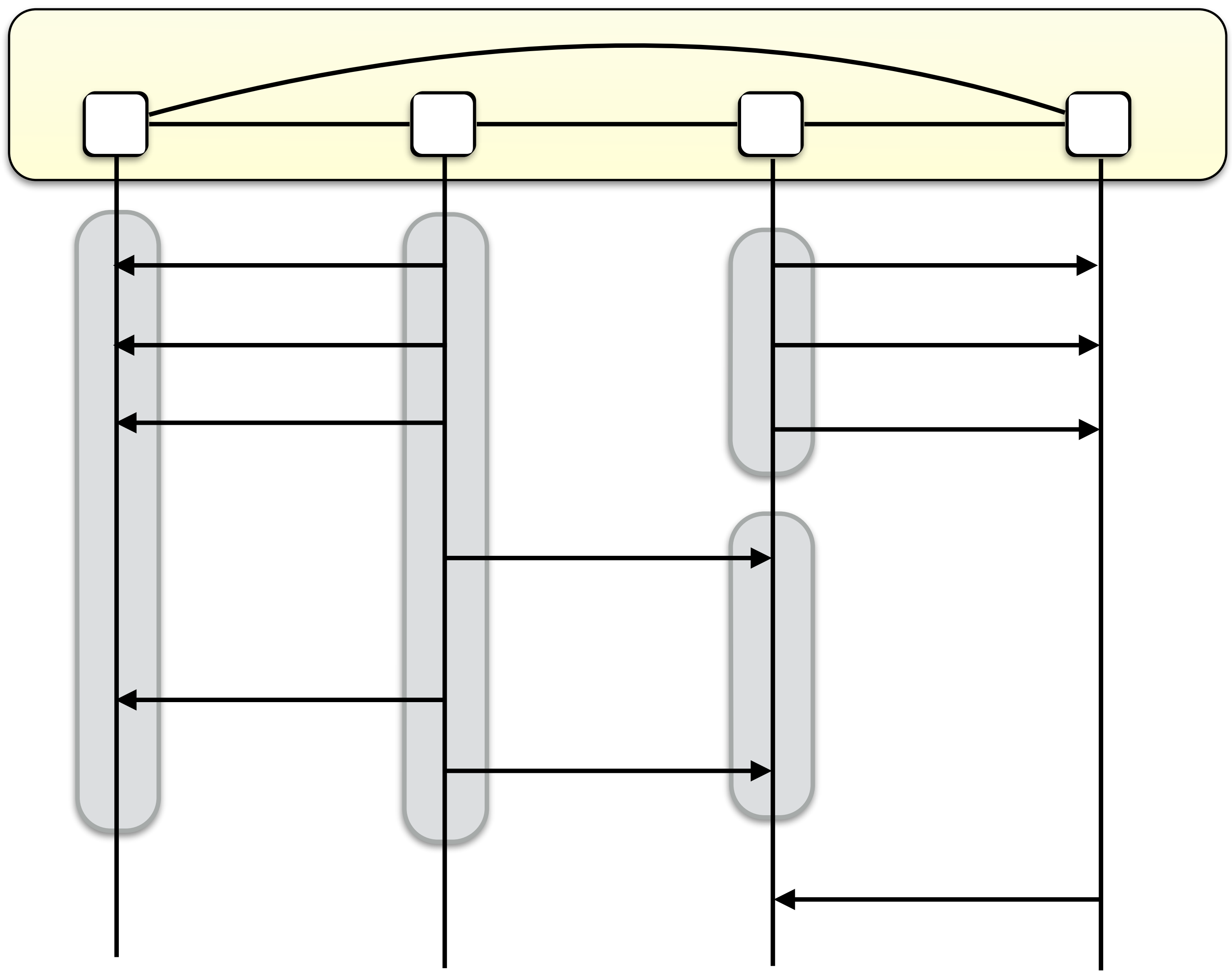

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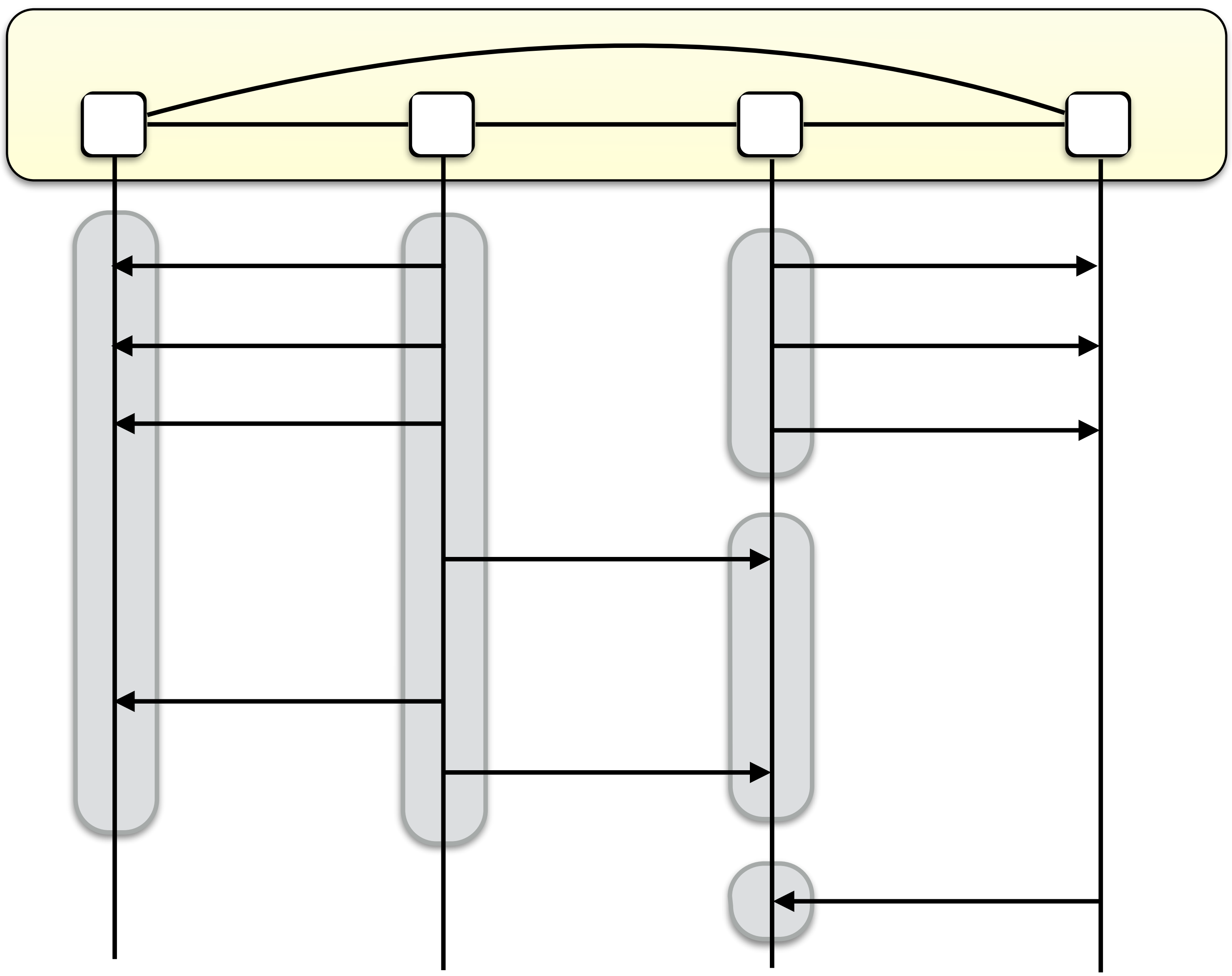

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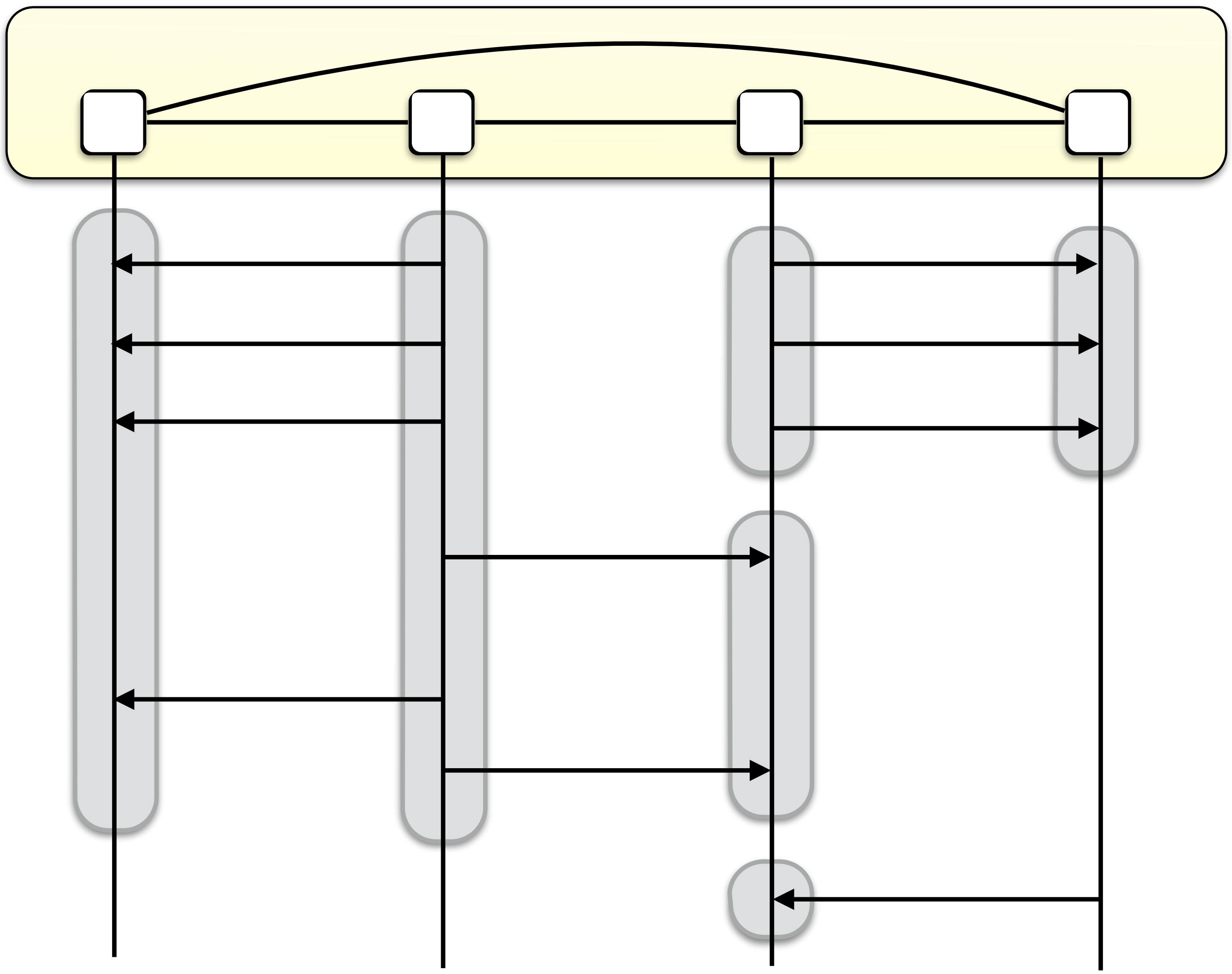

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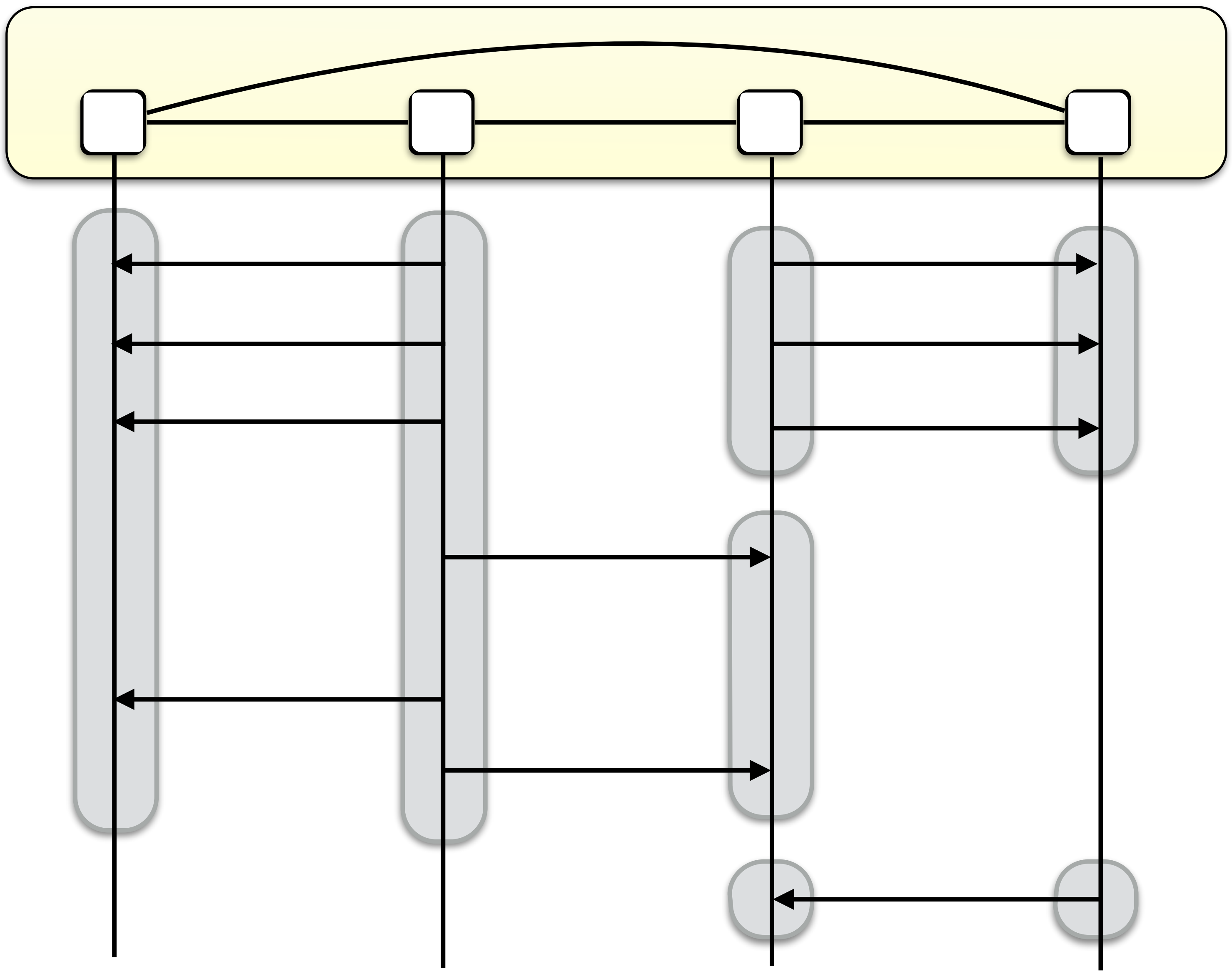

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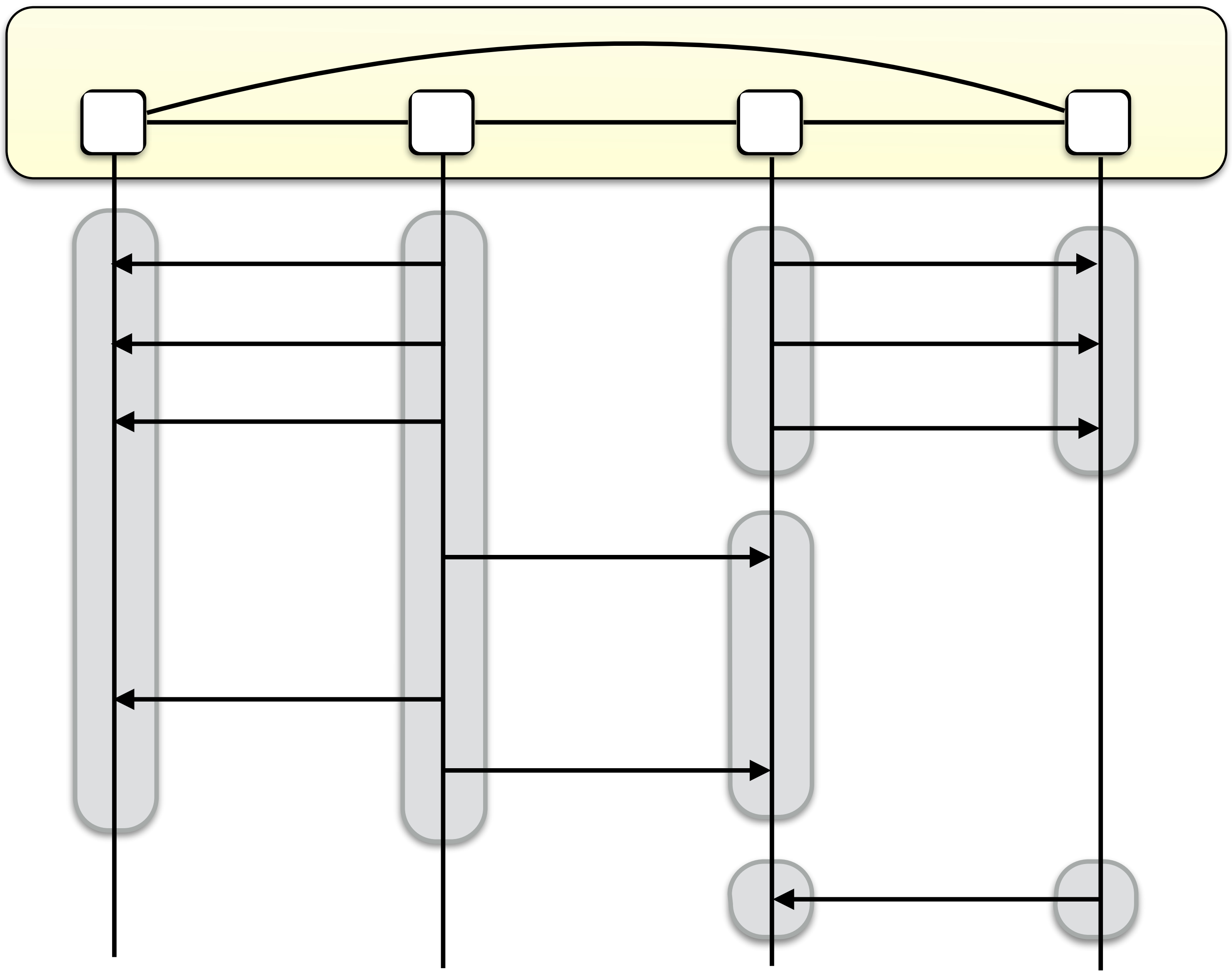

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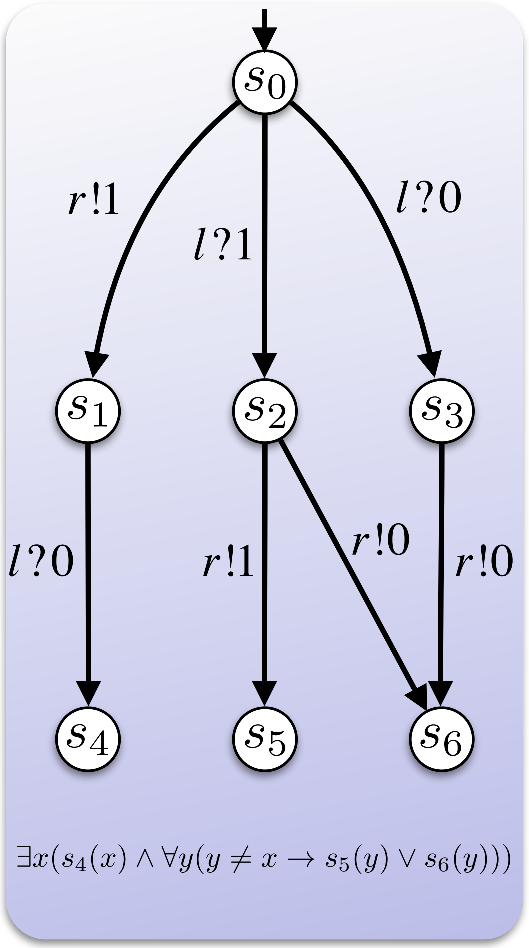
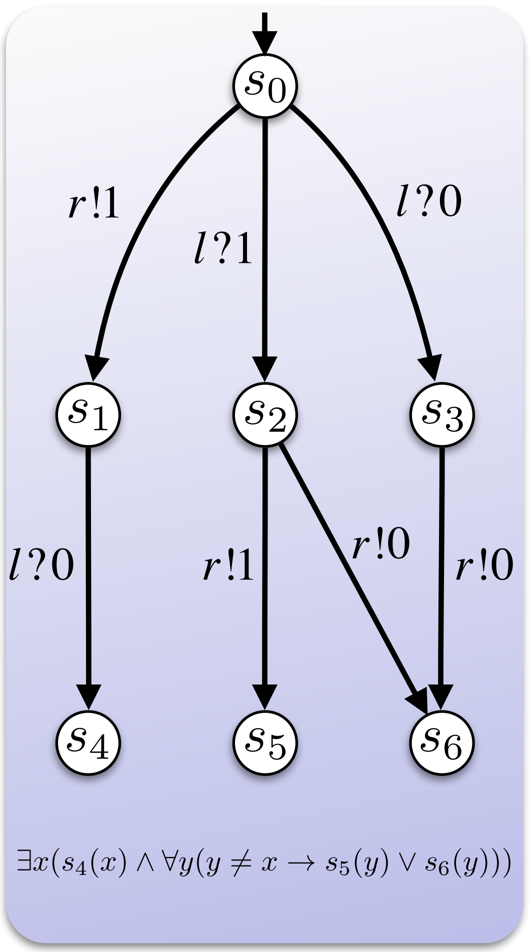
Context-Bounded PCAs



Definition: A PCA is k-bounded if the finite automaton restricts to k contexts.

Context-Bounded PCAs



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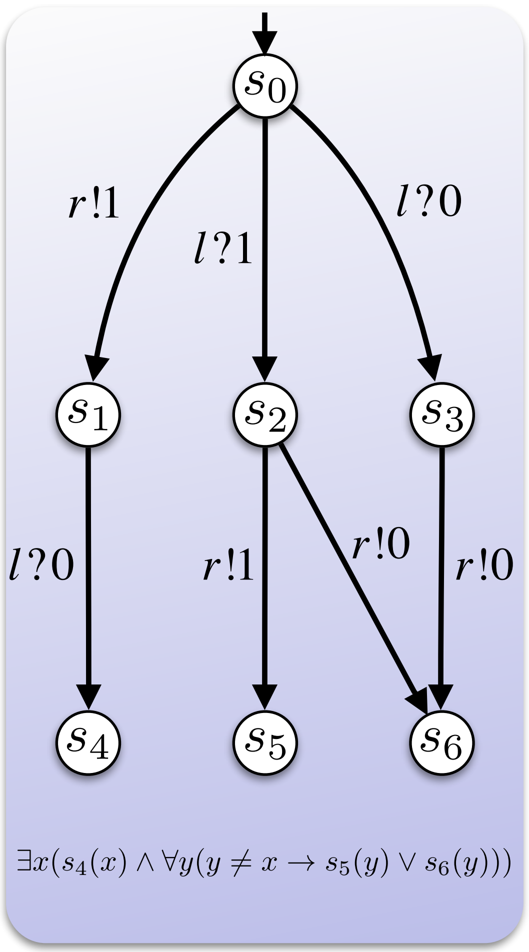

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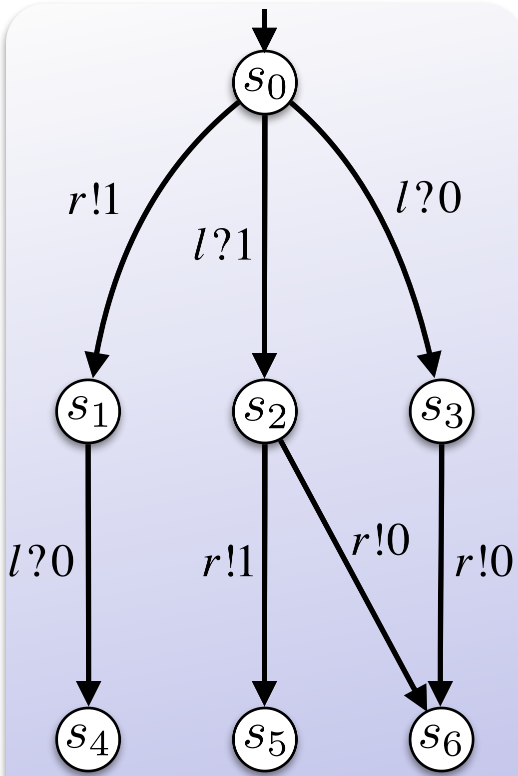
Theorem [B.-Gastin-Kumar; FSTTCS 2014]:
For every bounded PCA $\mathcal{A}$, there is a PCA $\mathcal{B}$ such that $L(\mathcal{B}) = \overline{L(\mathcal{A})}$.

Proof Outline

nondeterminism



$\exists x(s_4(x) \wedge \forall y(y \neq x \rightarrow s_5(y) \vee s_6(y)))$

k-bounded

disambiguation

every behavior has a unique run

complementation

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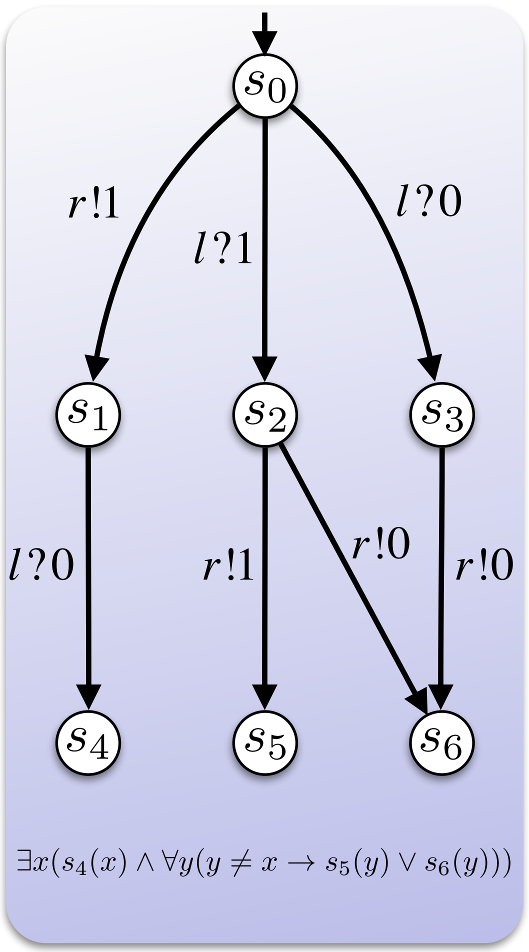

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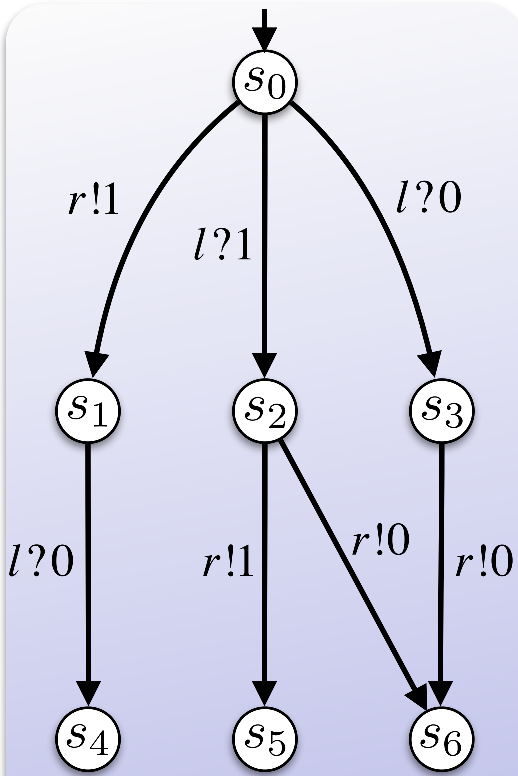

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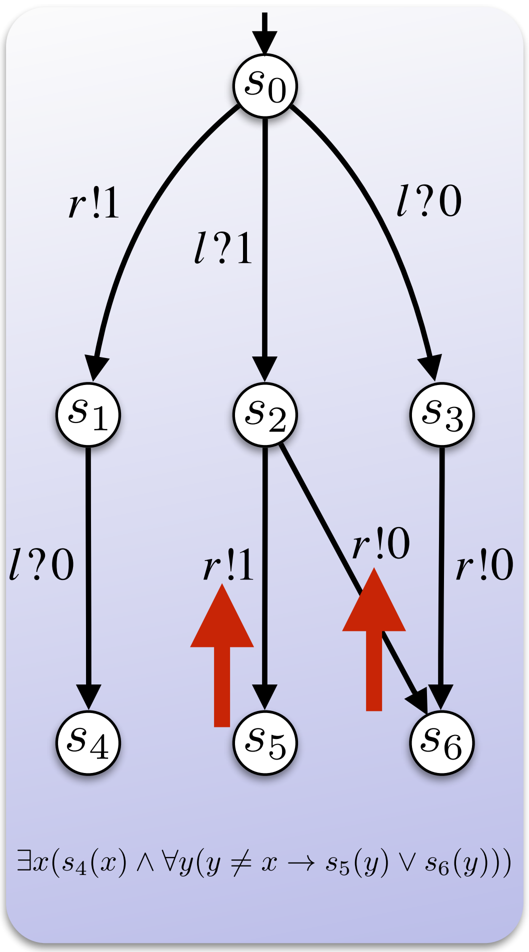

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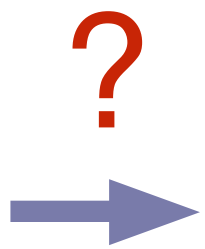

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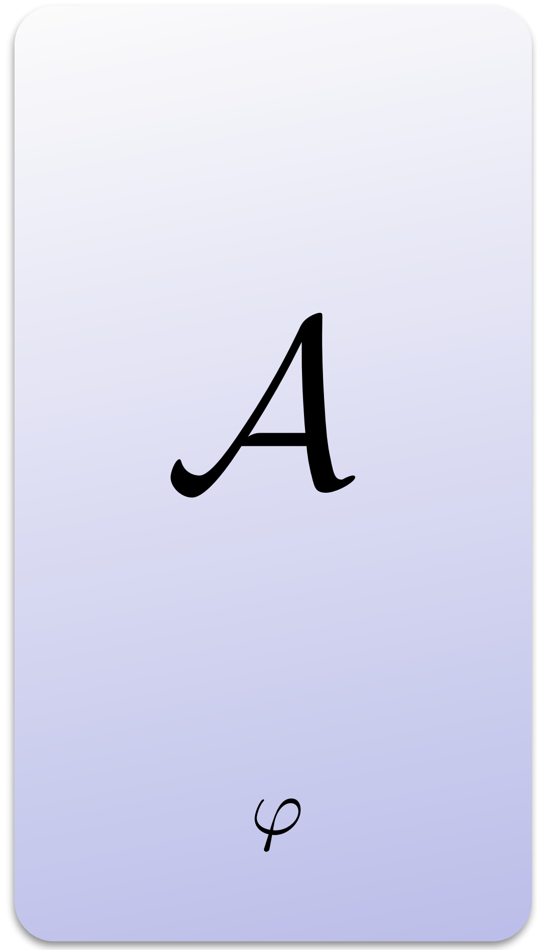

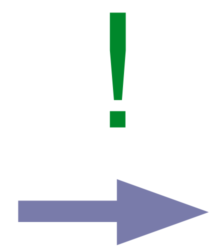

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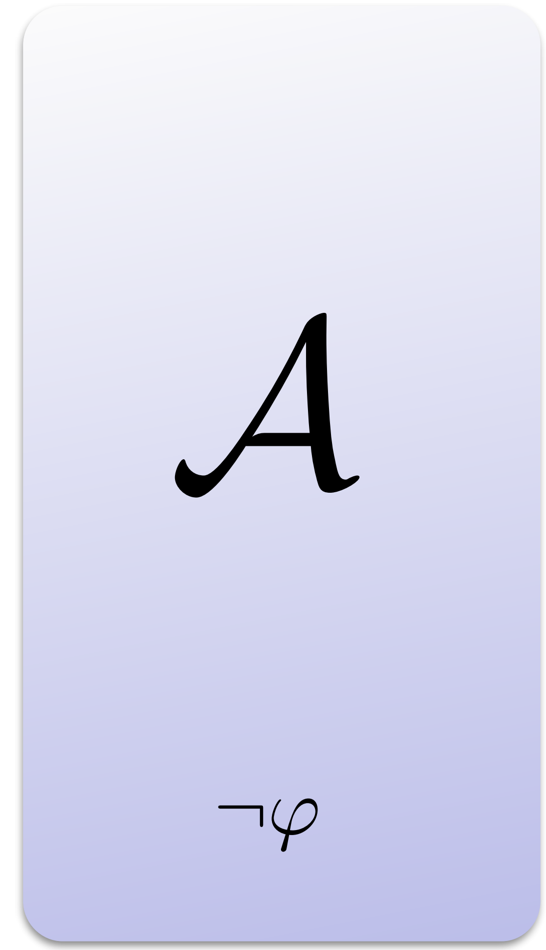

Powerset construction not applicable due to message contents.

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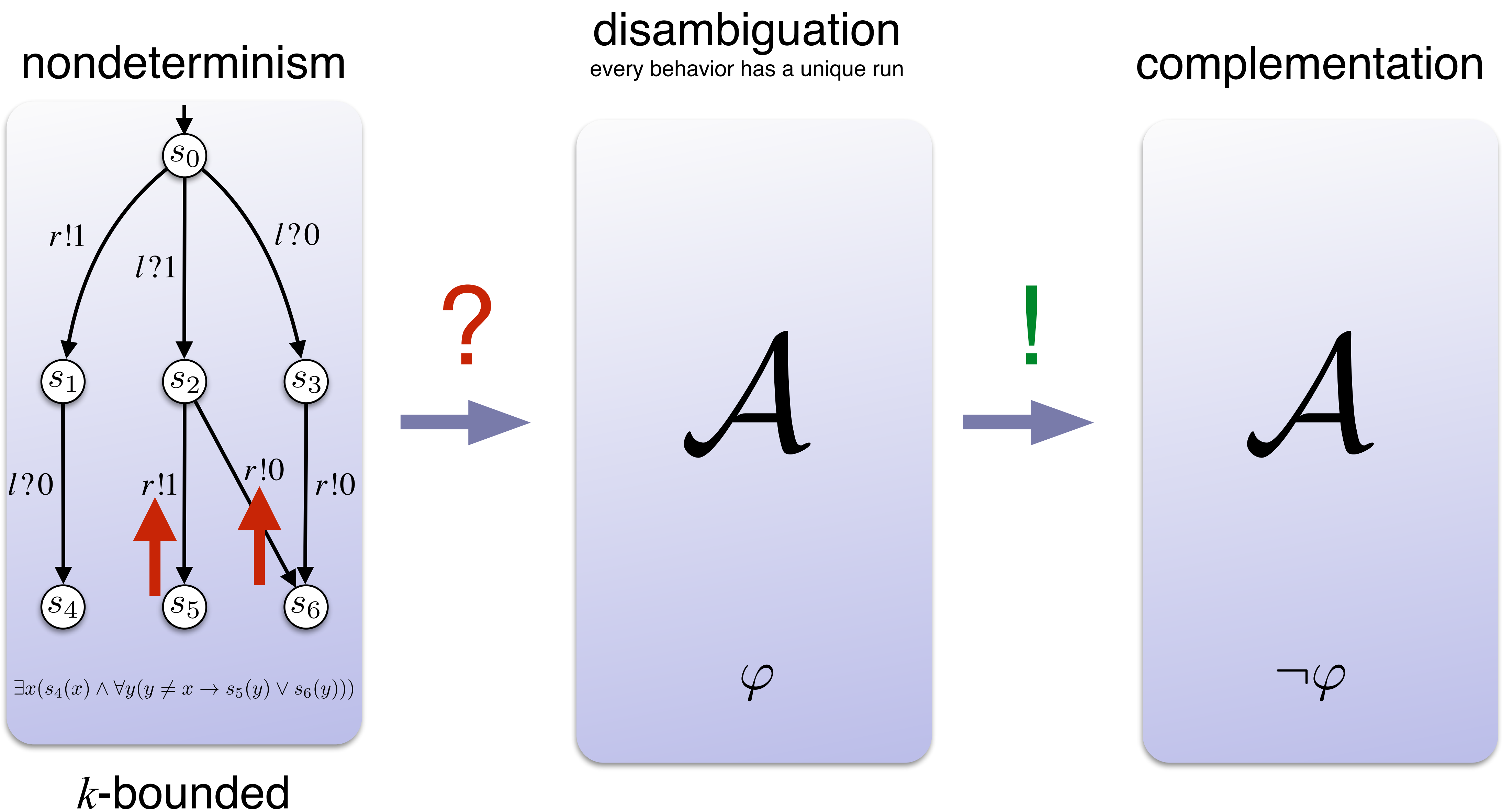

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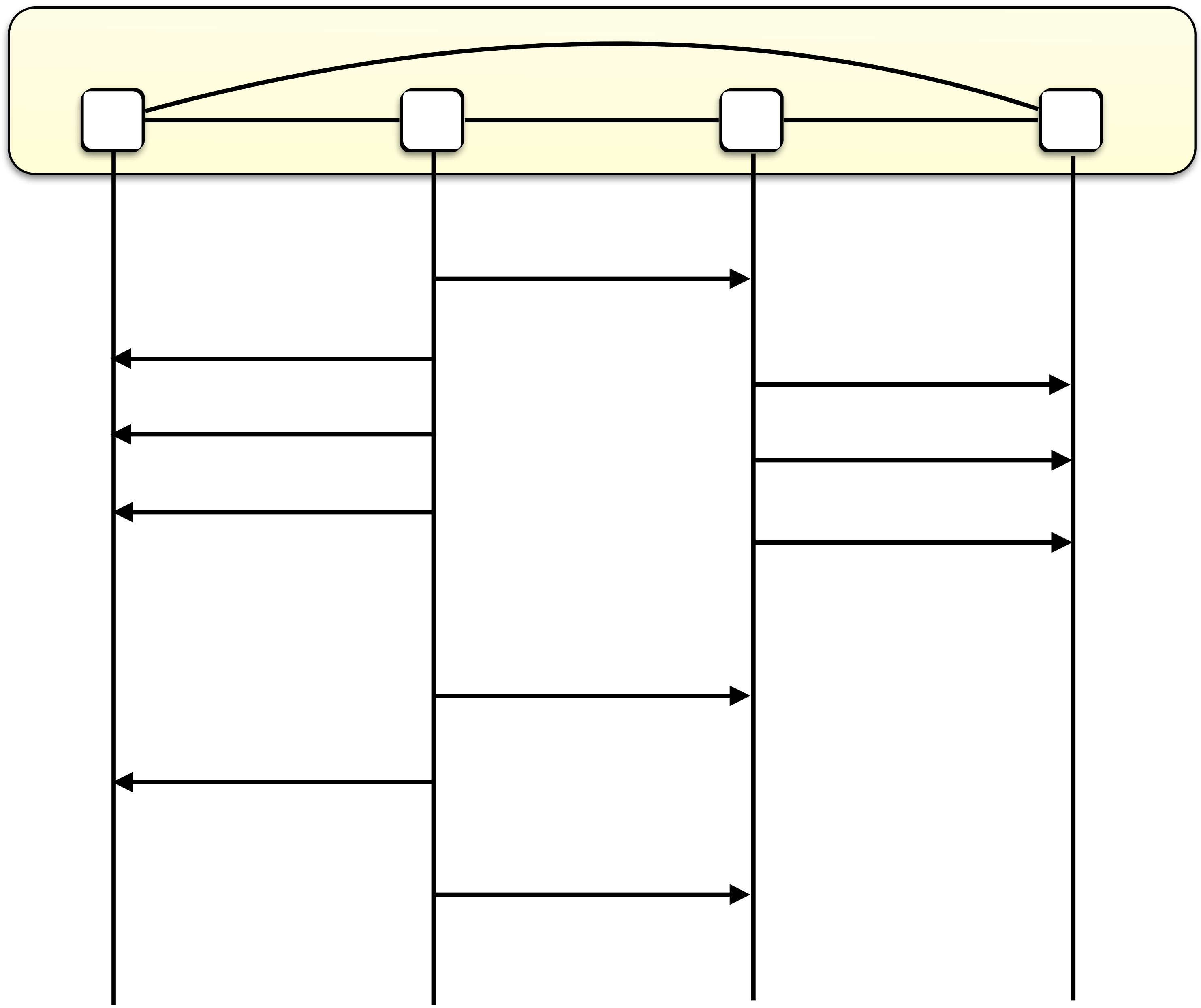
Disambiguation through summaries:

- Alur-Madhusudan: *Visibly pushdown languages.* STOC 2004.
- La Torre-Madhusudan-Parlato: *The language theory of bounded context switching.* LATIN 2010.
- La Torre-Napoli-Parlato: *Scope-bounded pushdown languages.* DLT 2014.

Disambiguation of context-bounded PCAs



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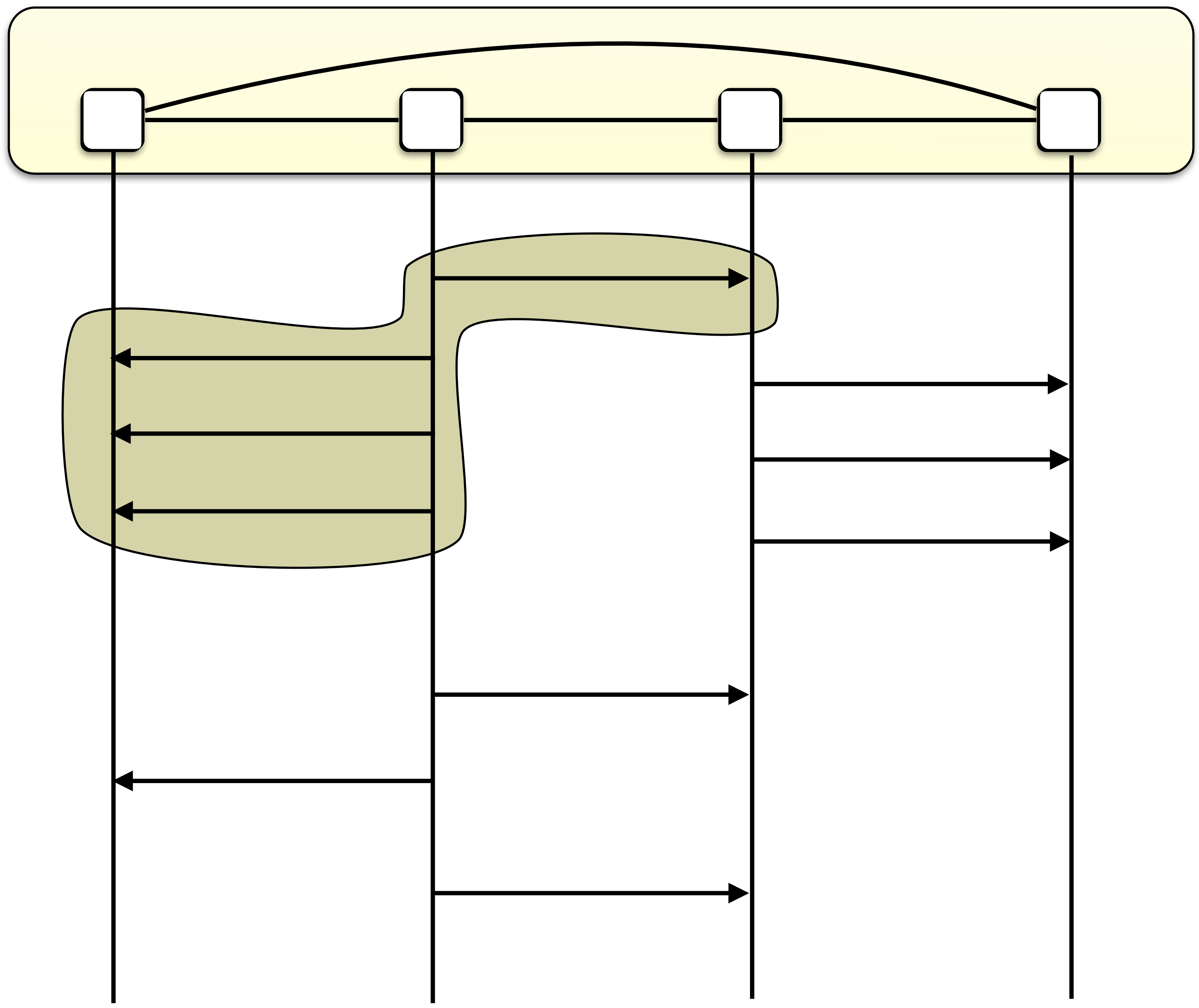

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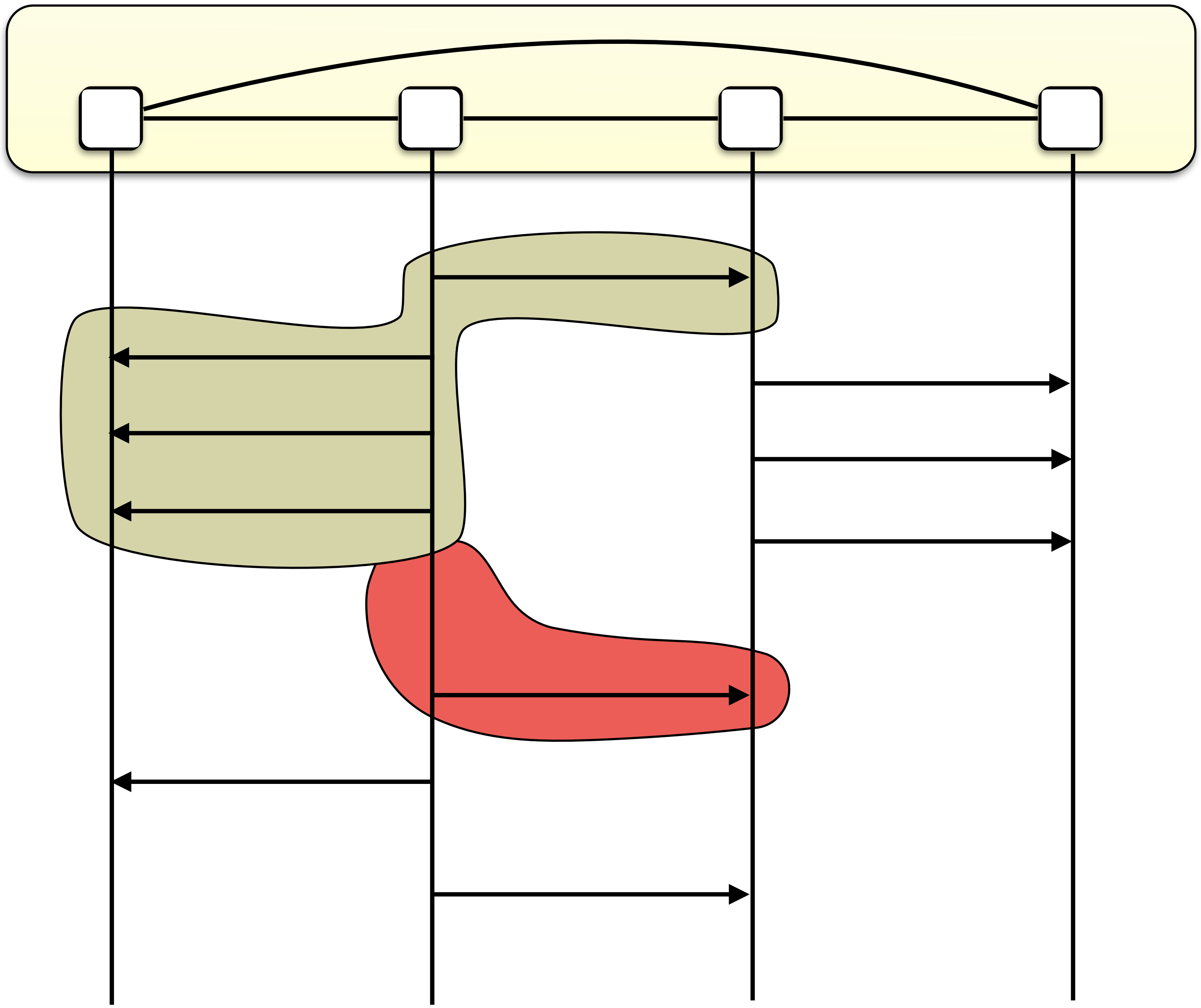

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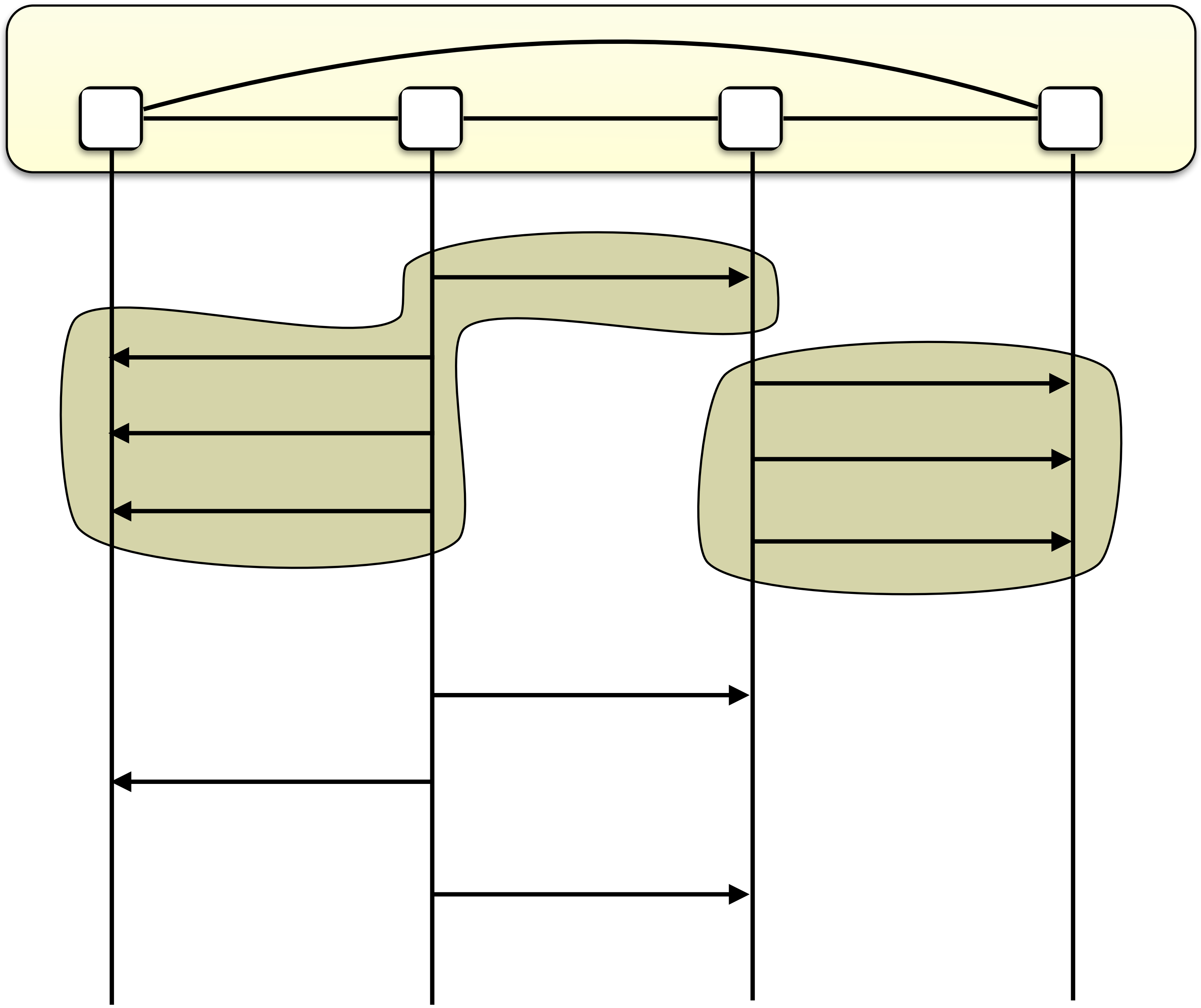

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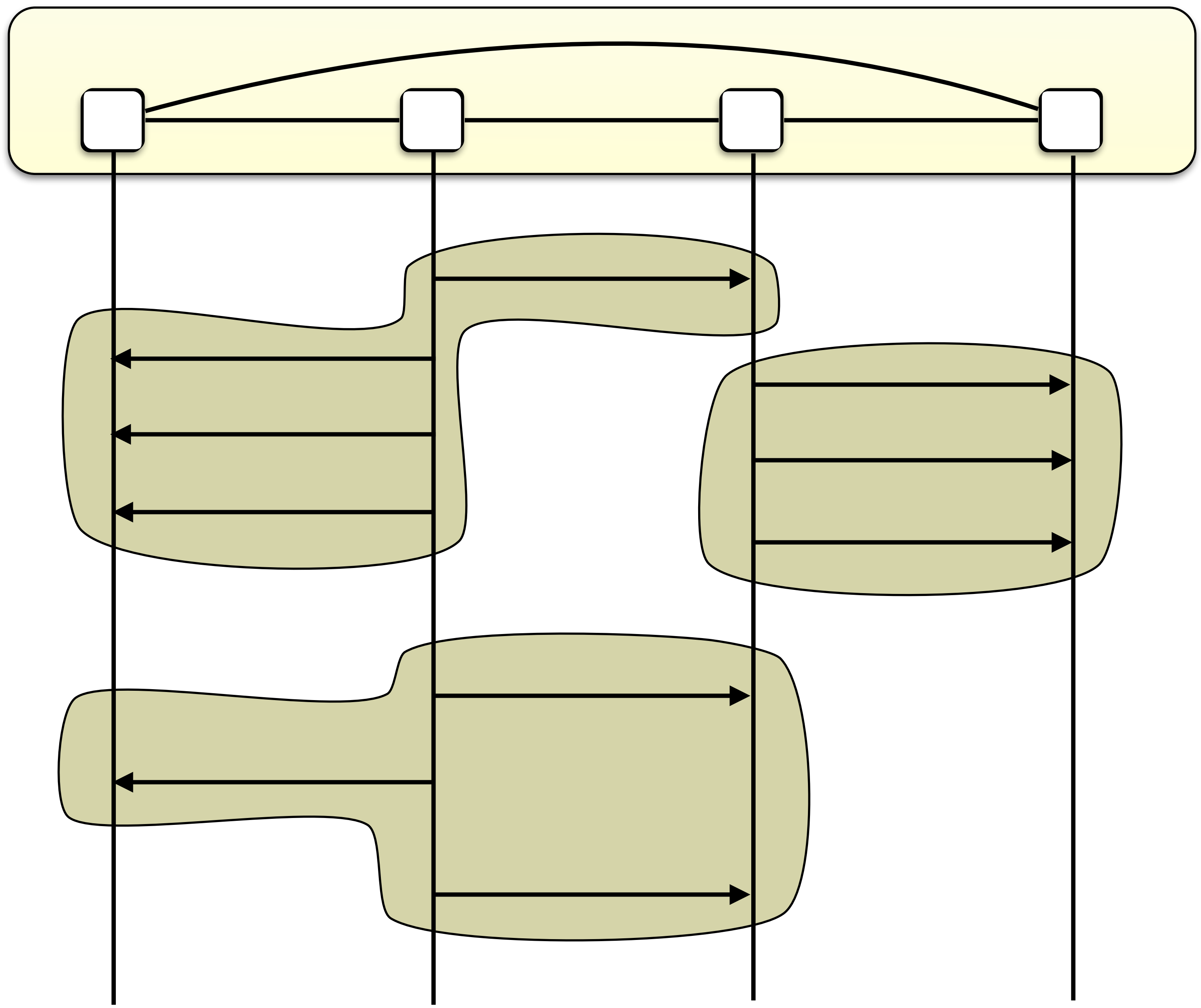

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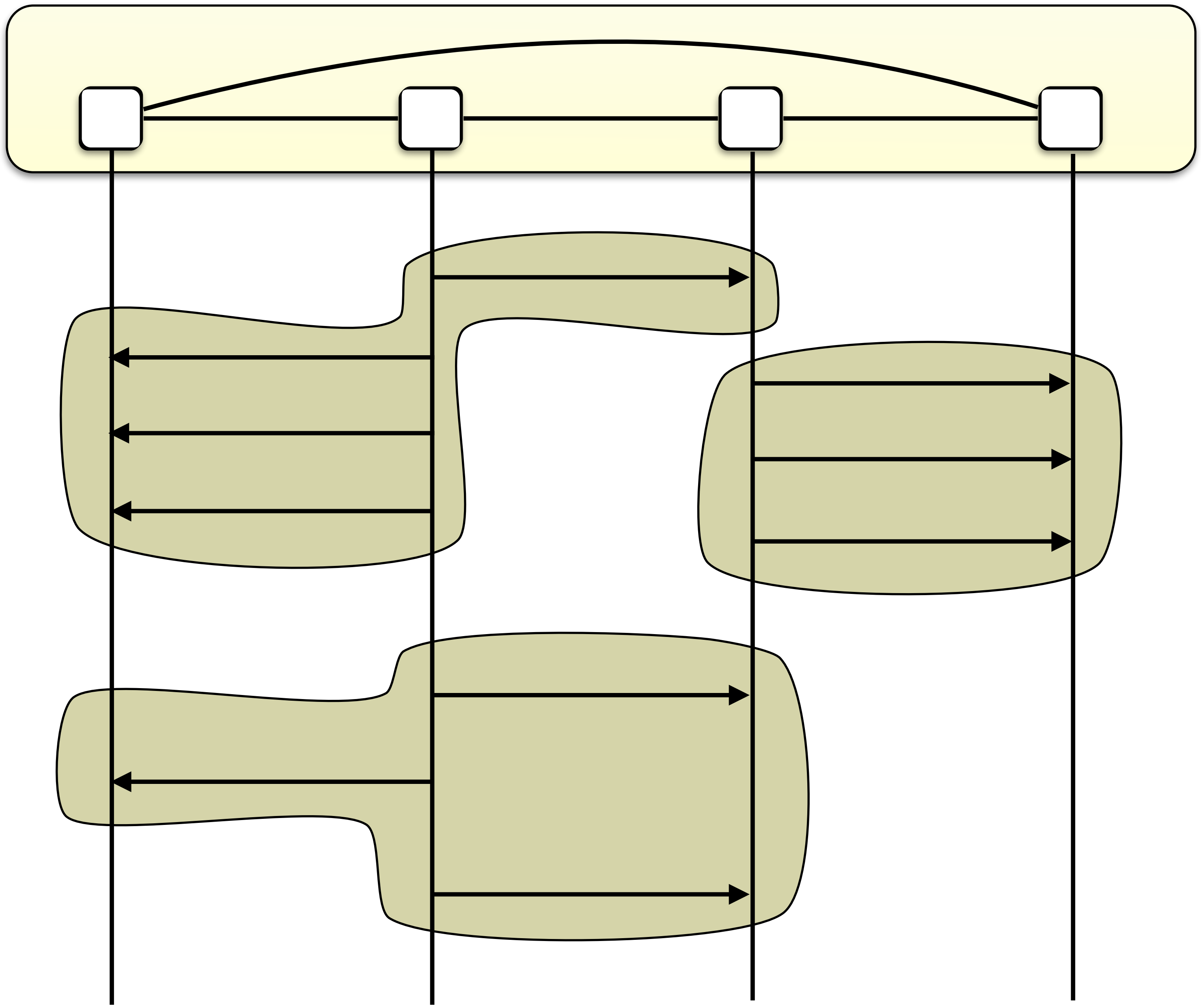

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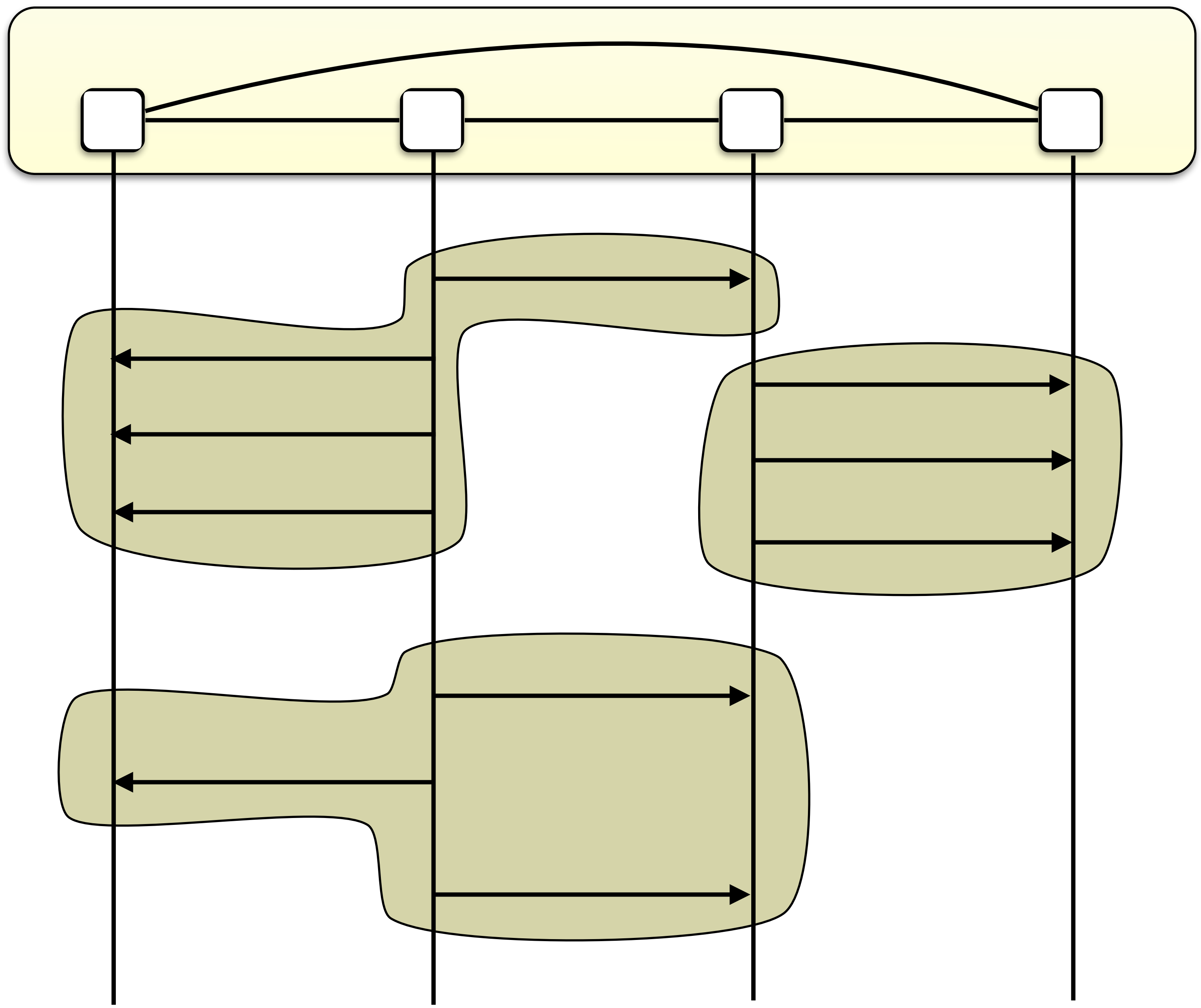

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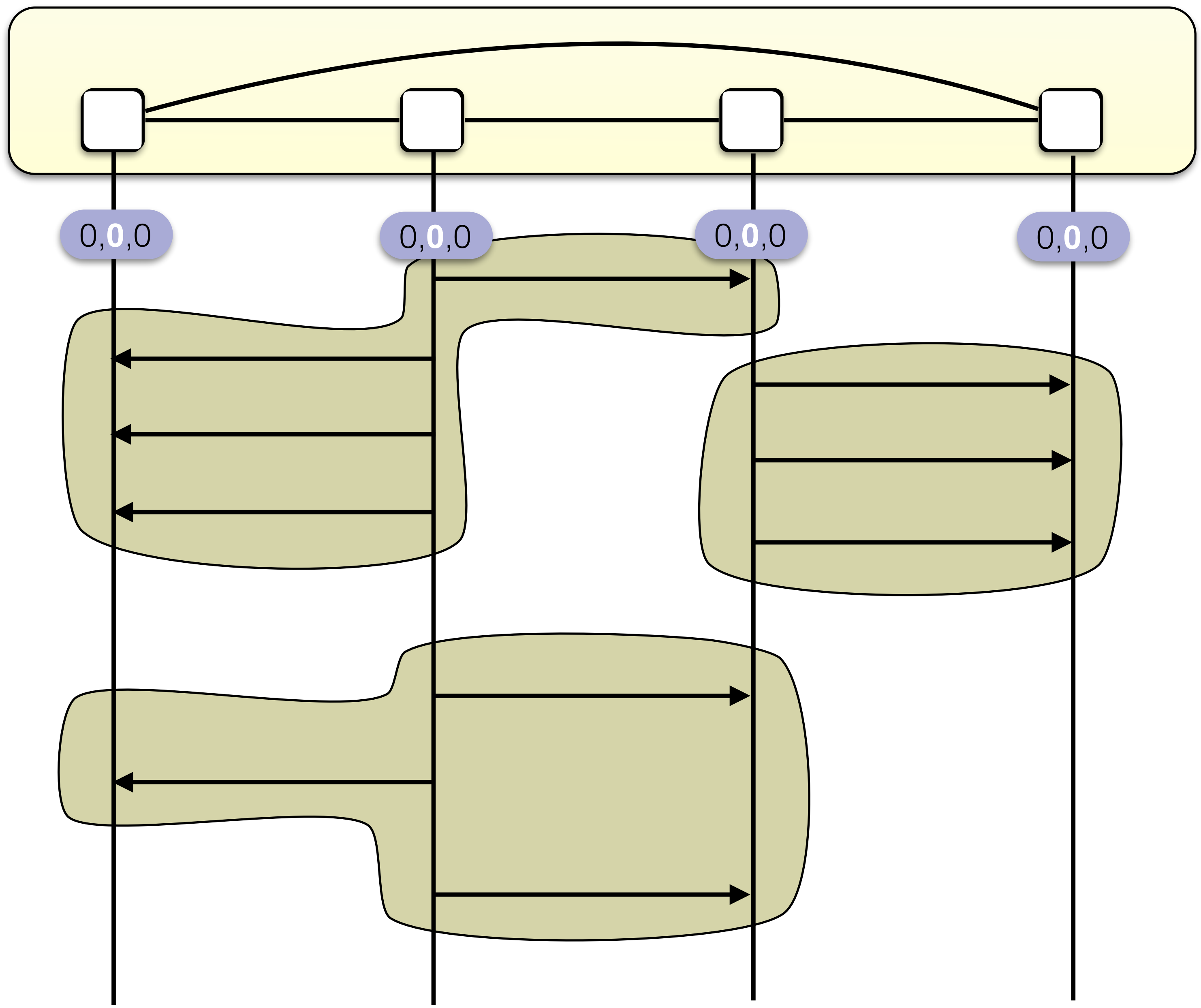

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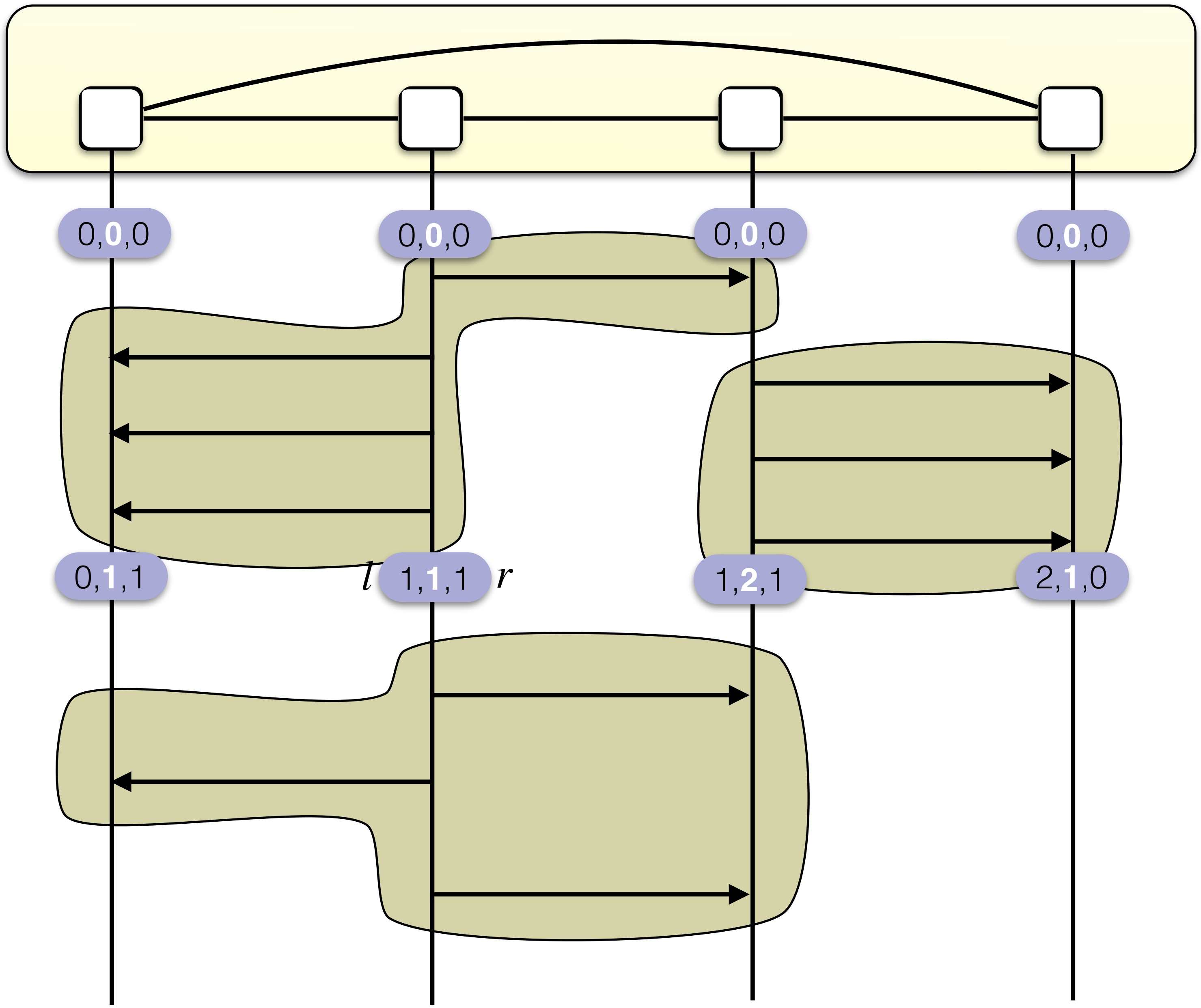

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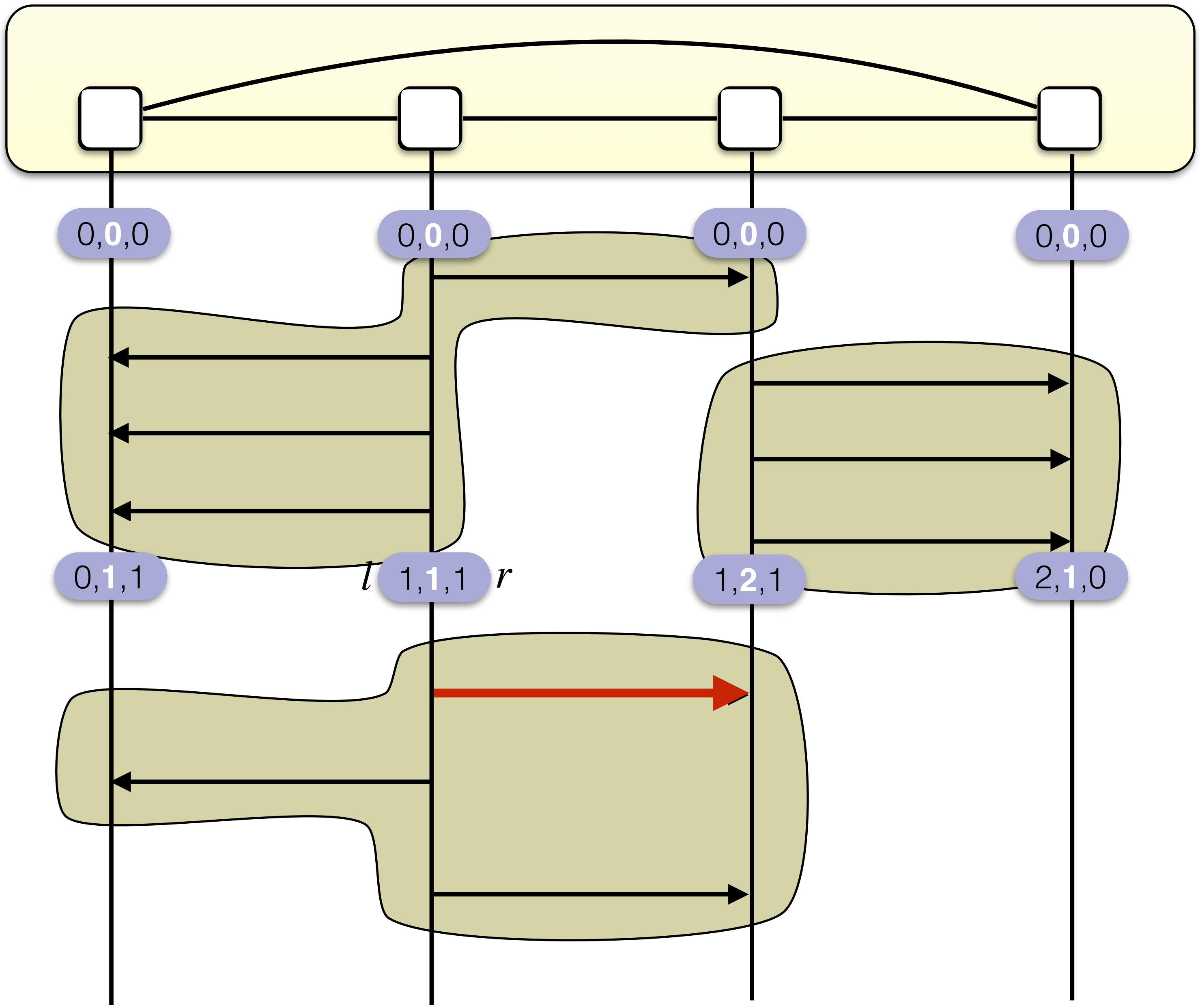

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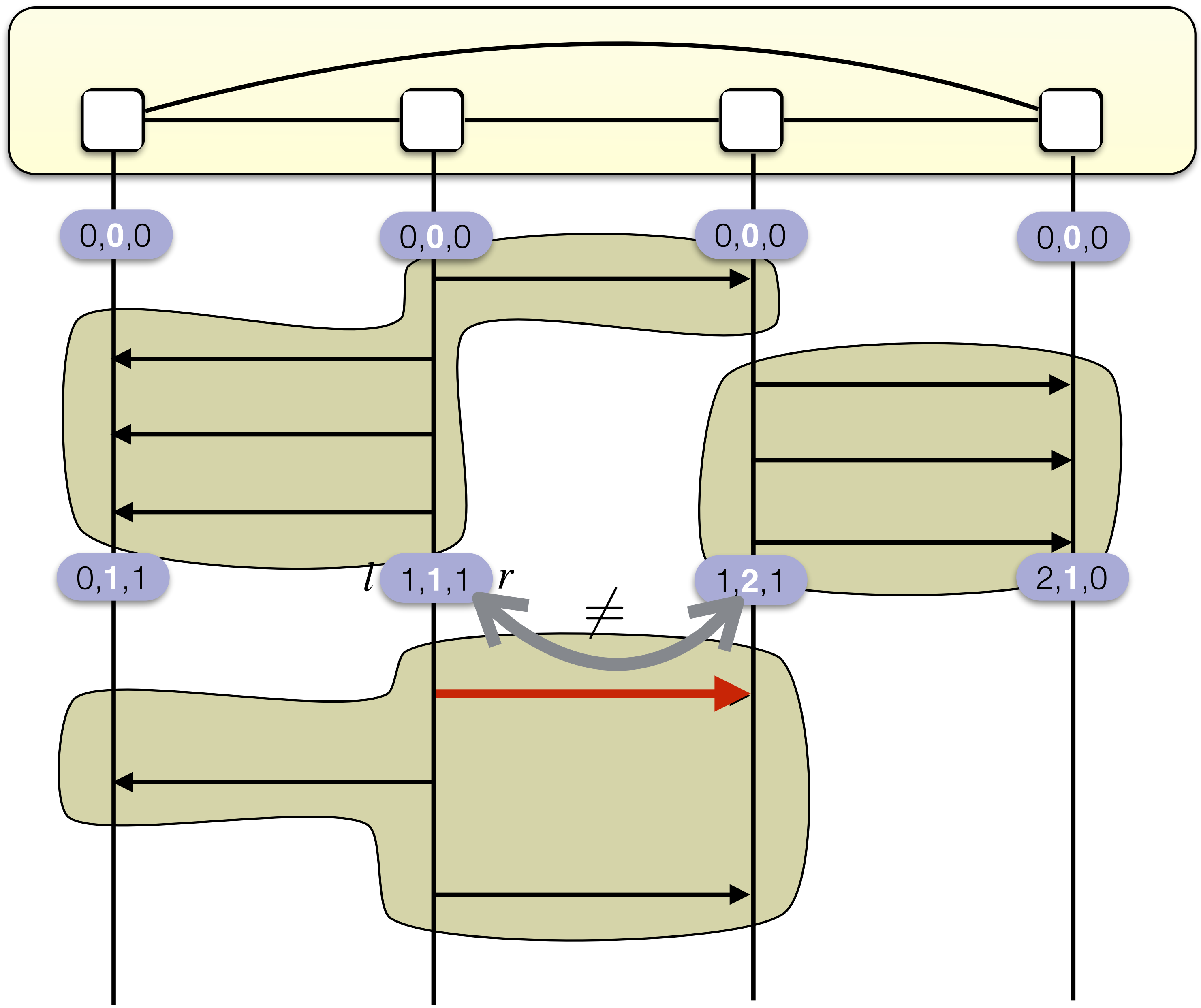

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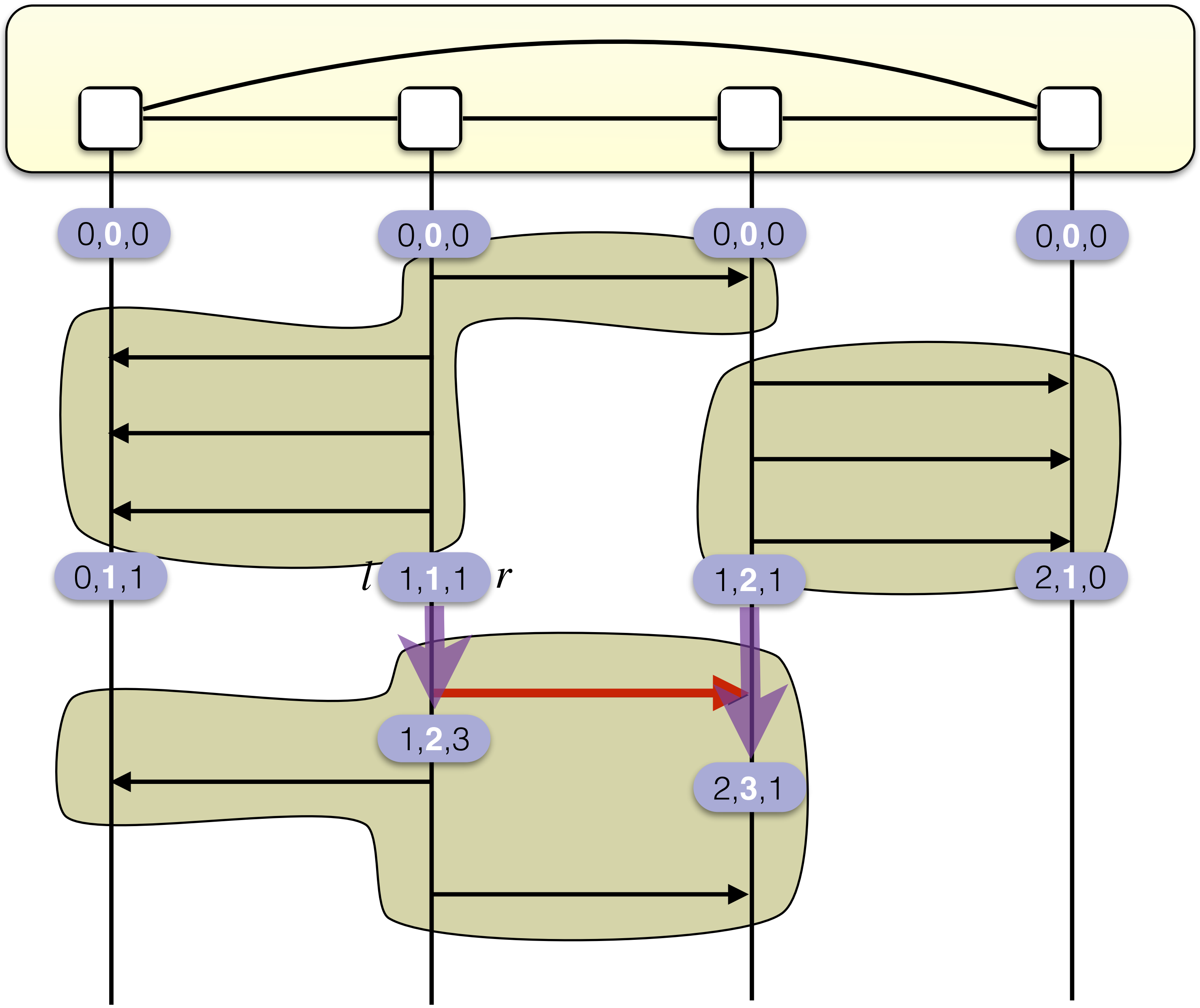

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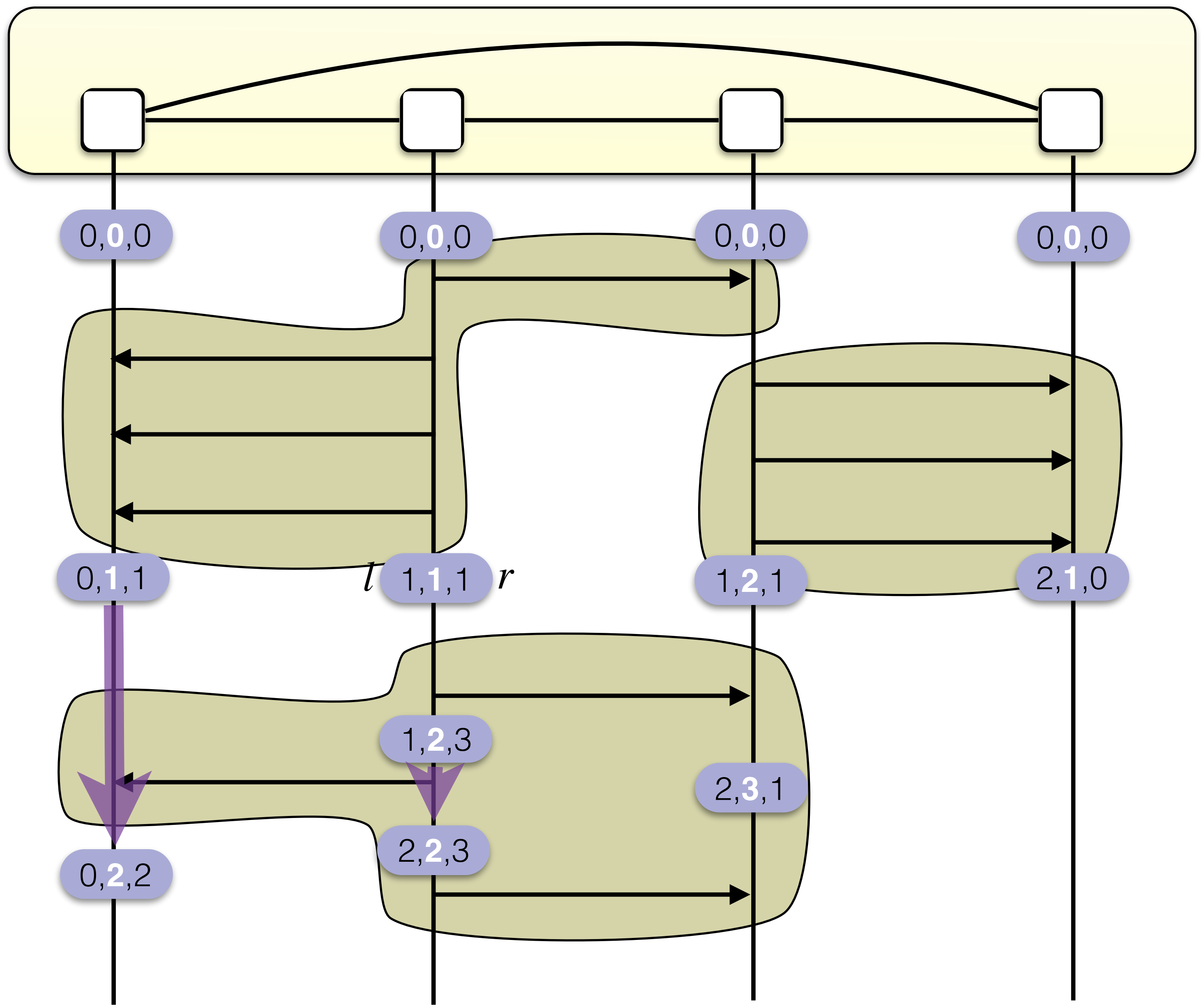

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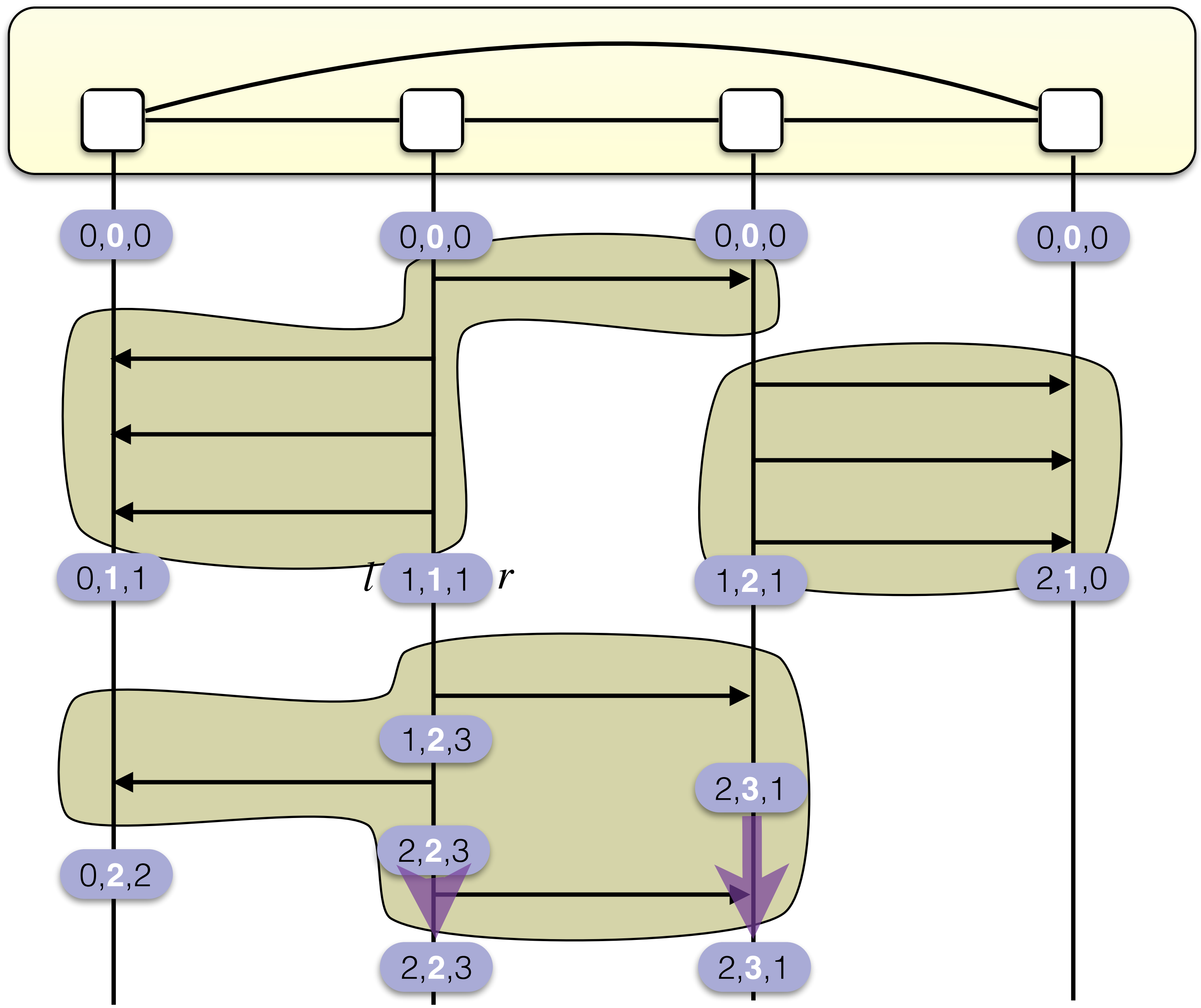

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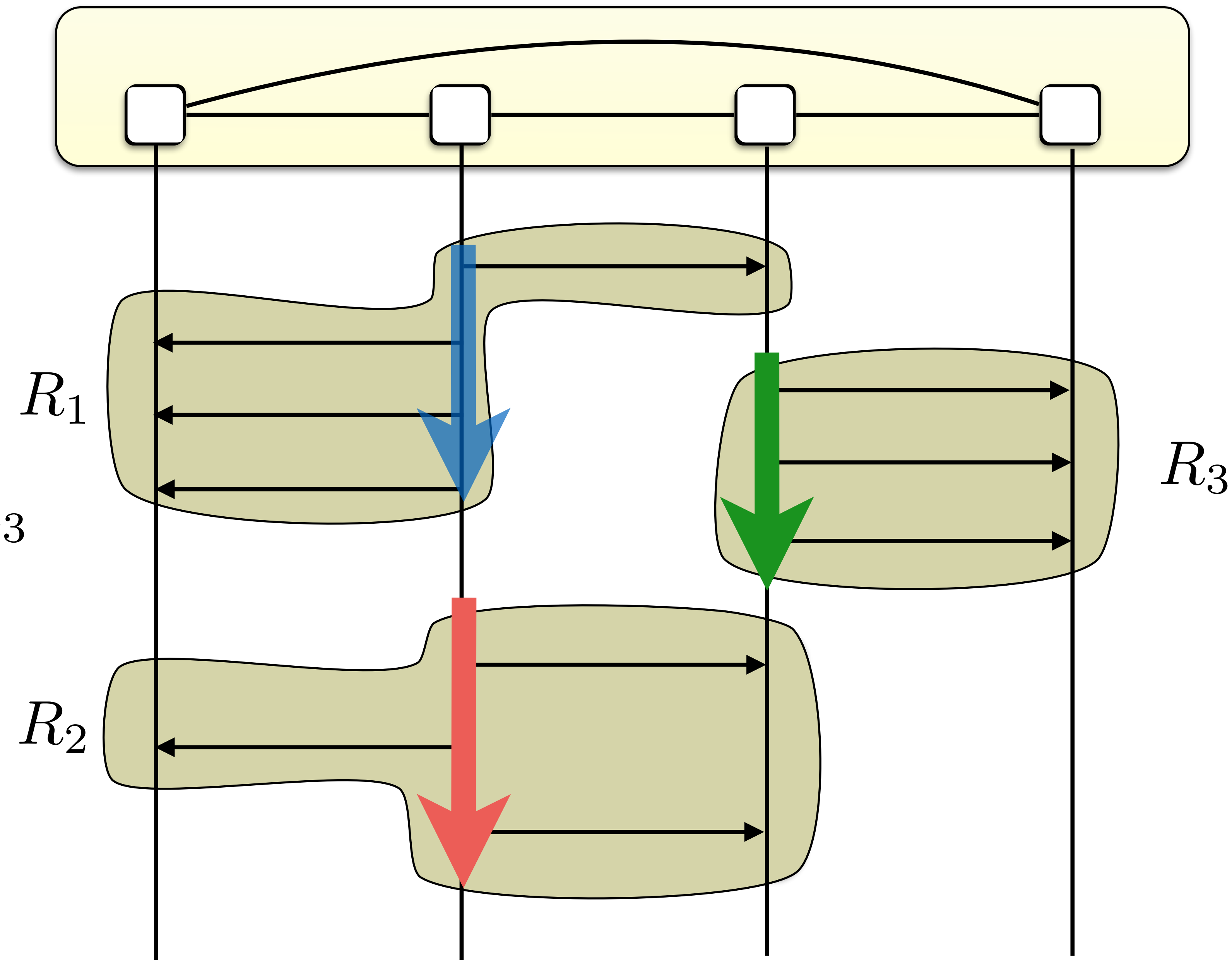

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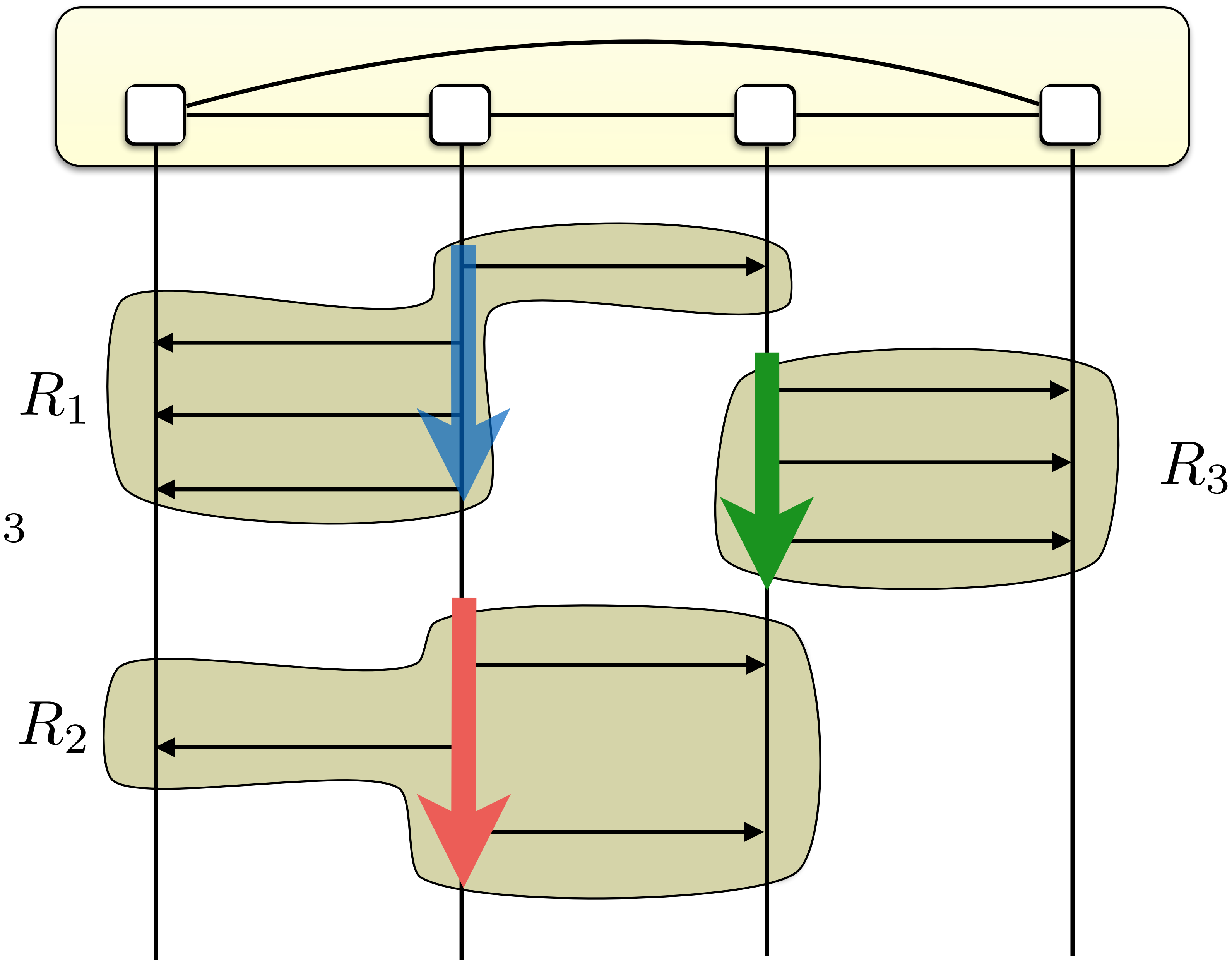

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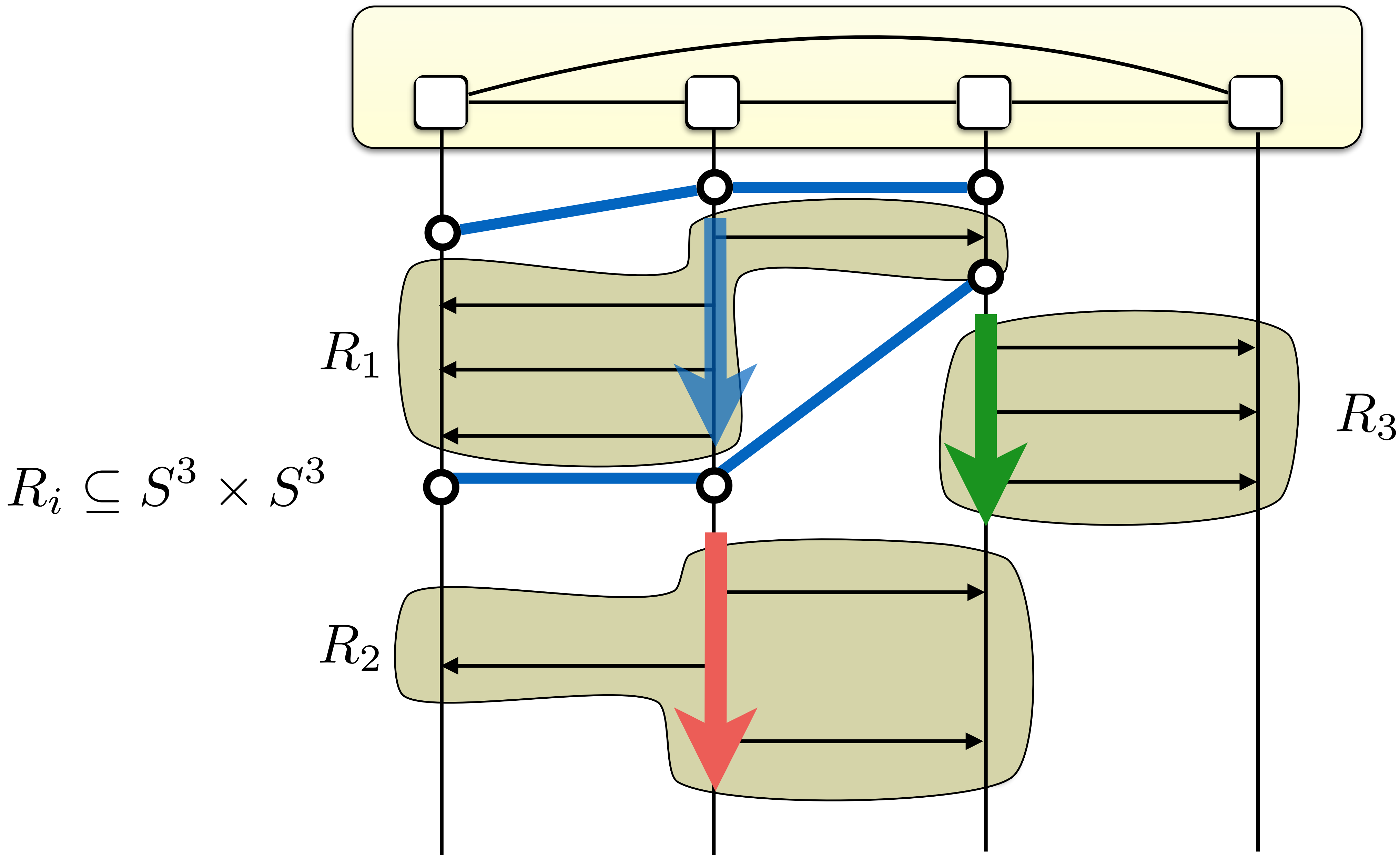

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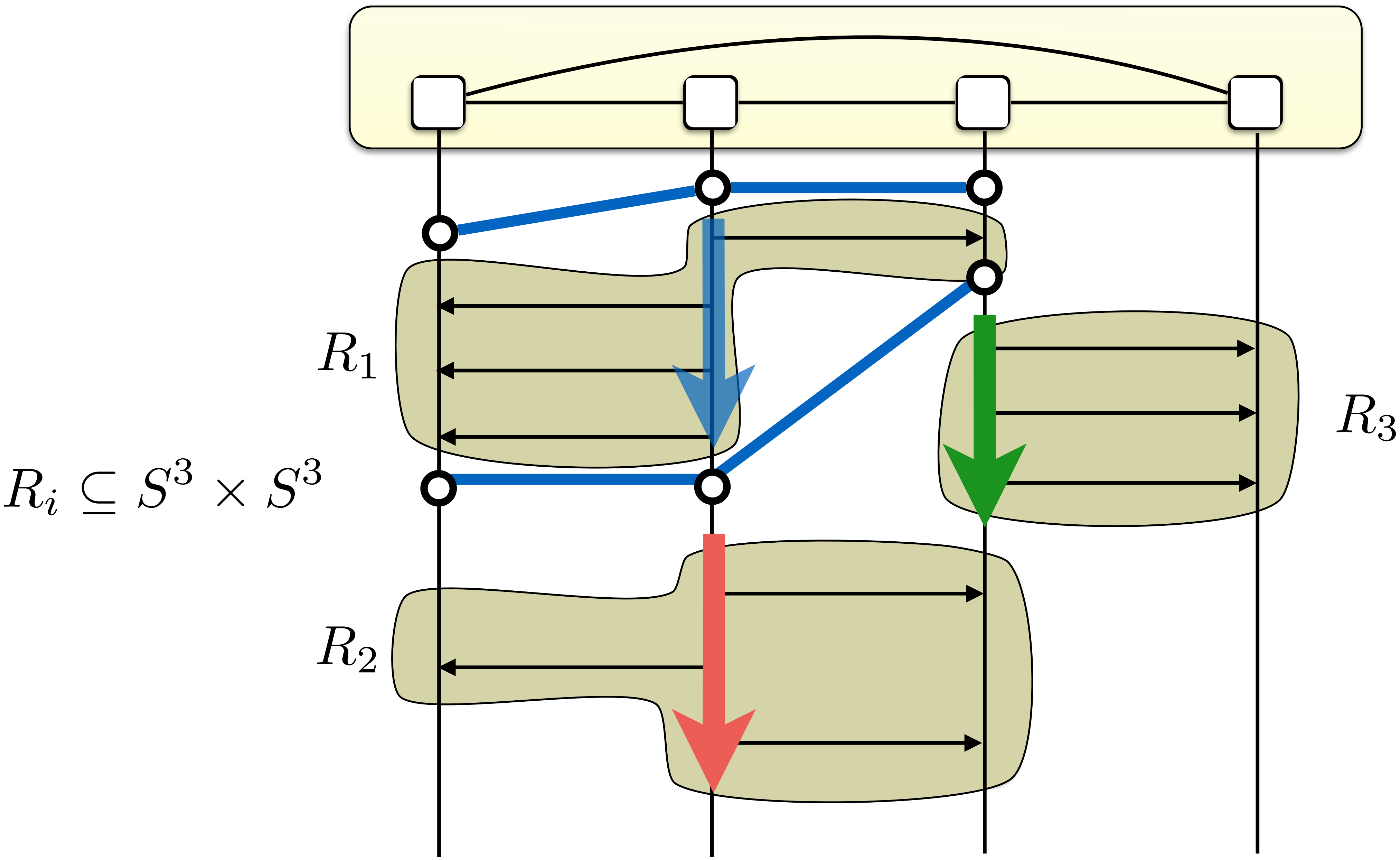

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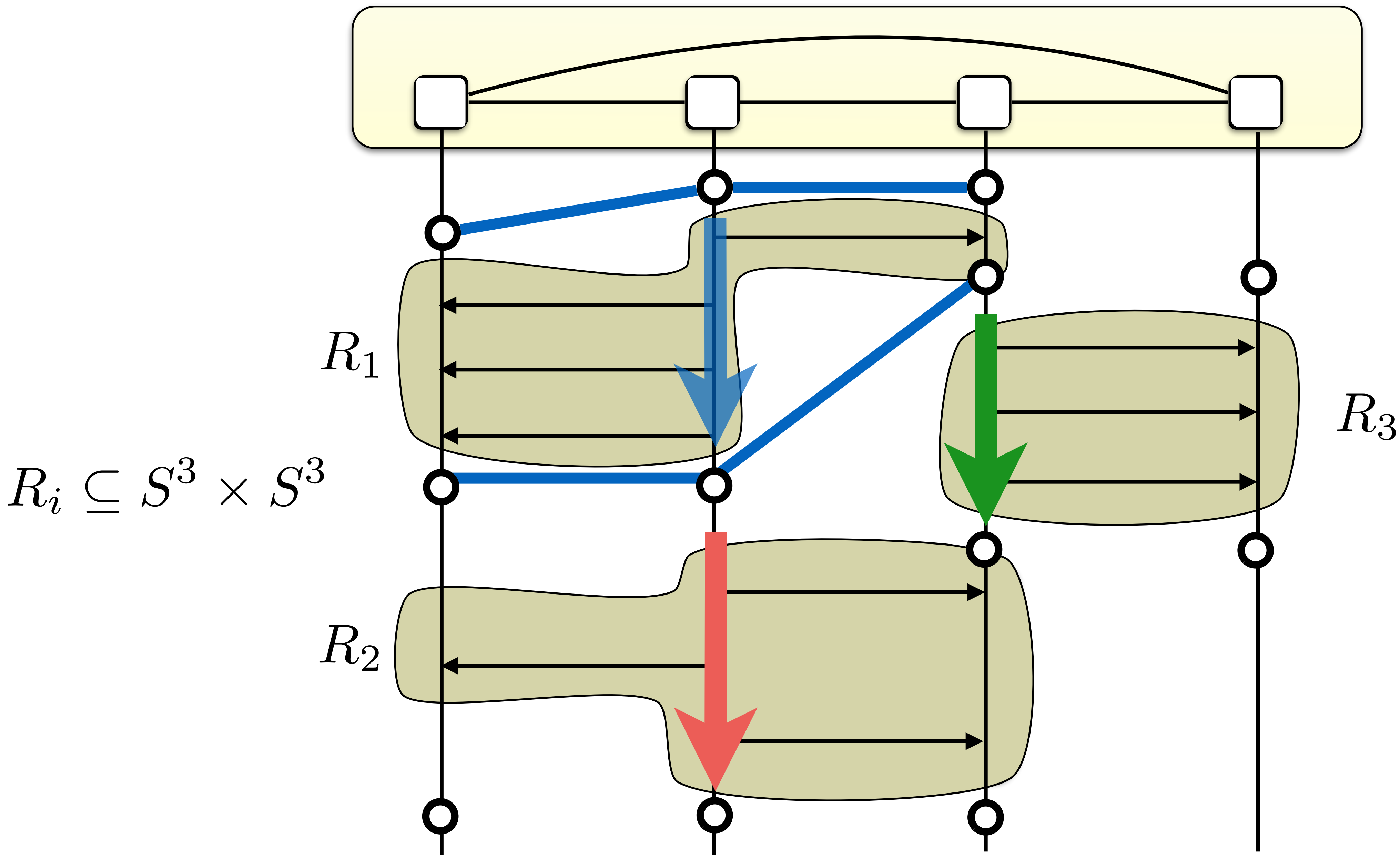

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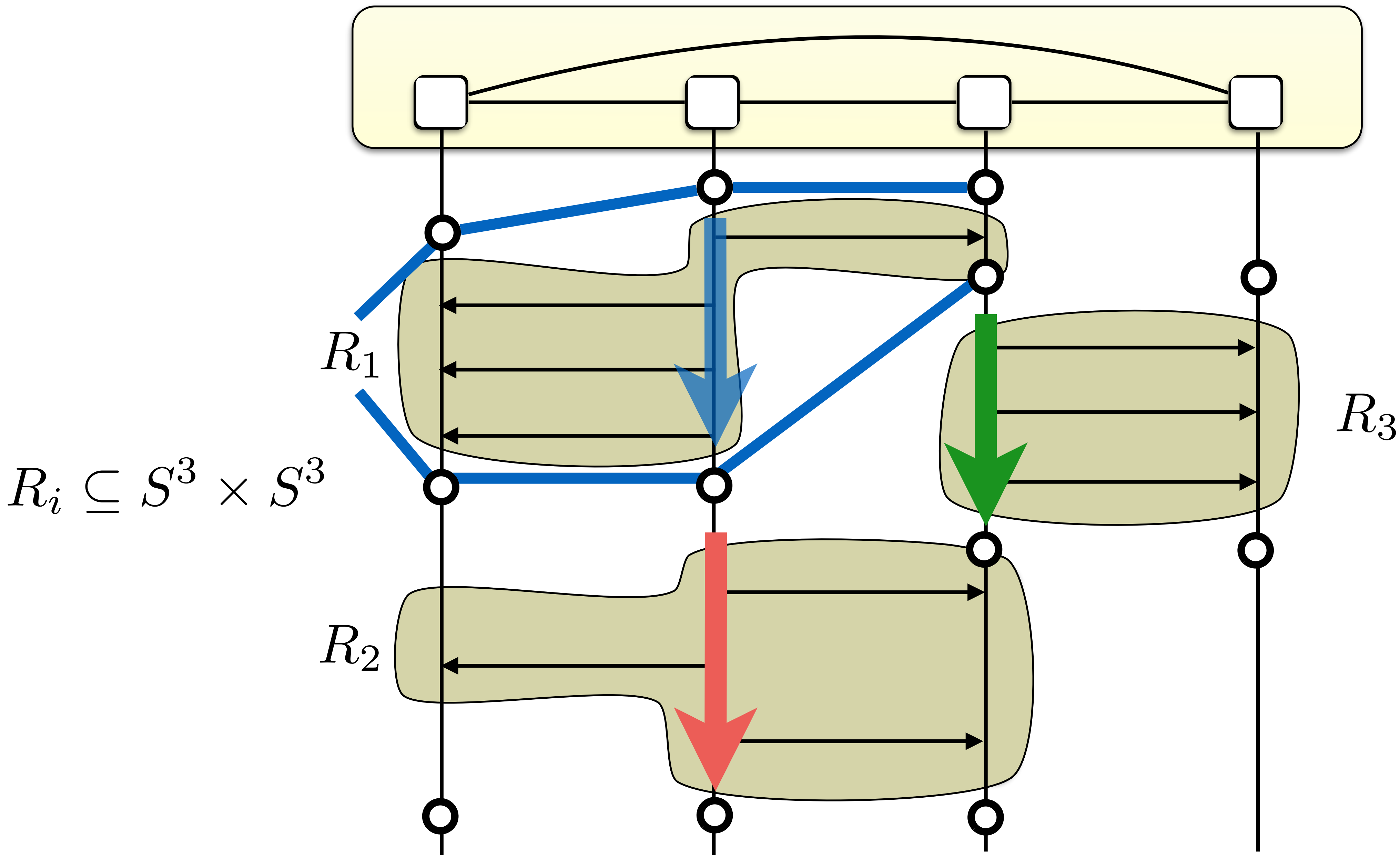

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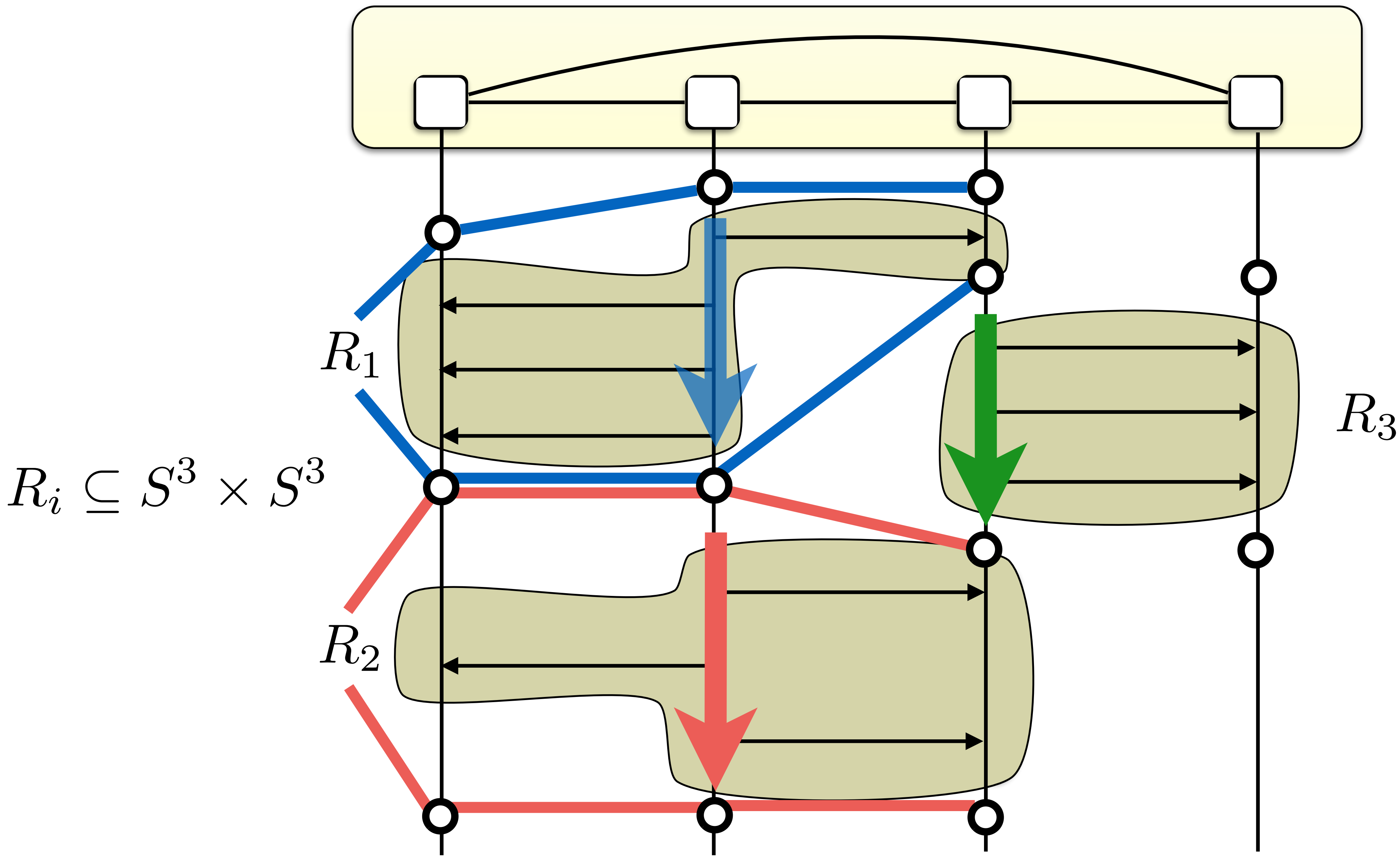

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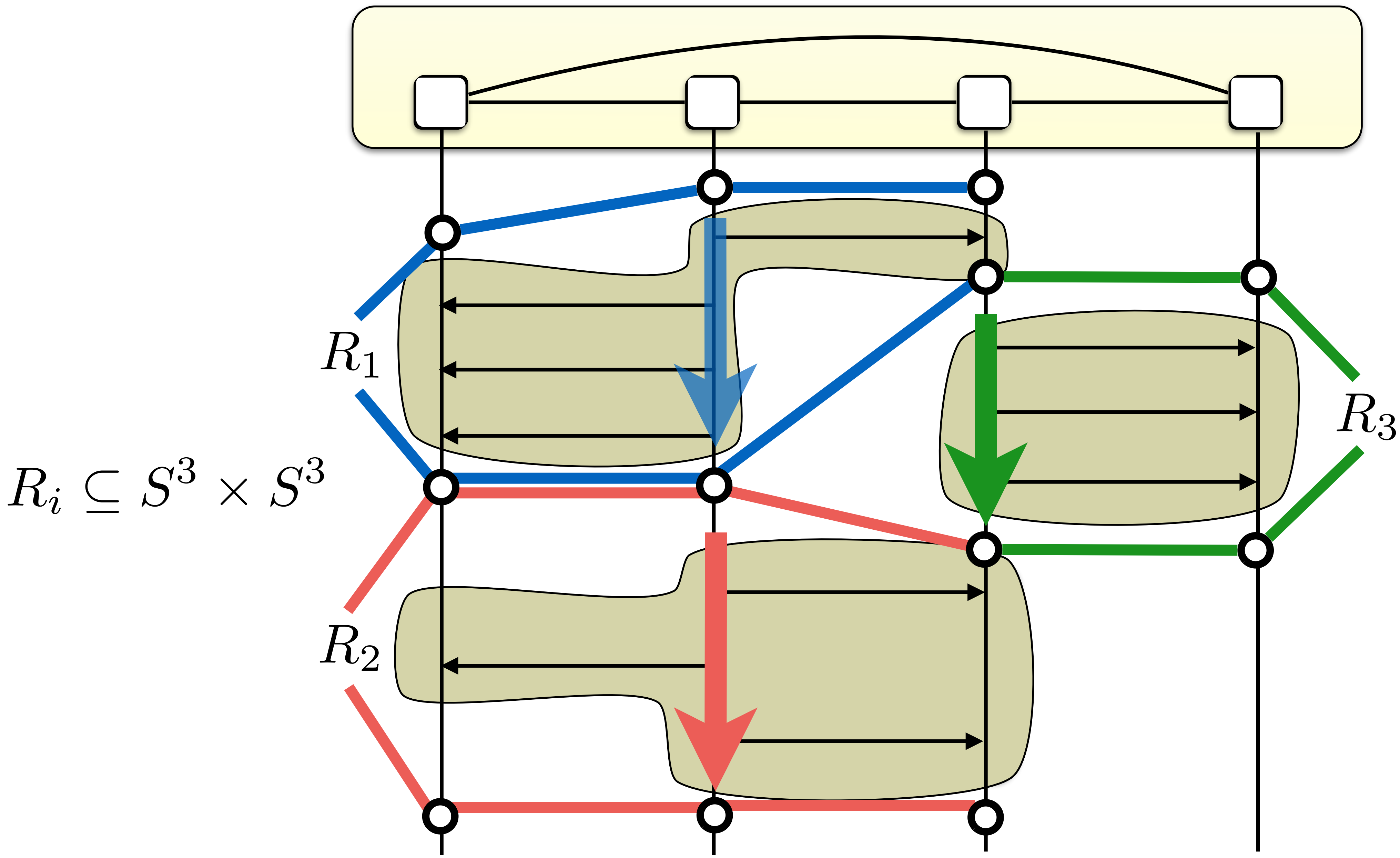

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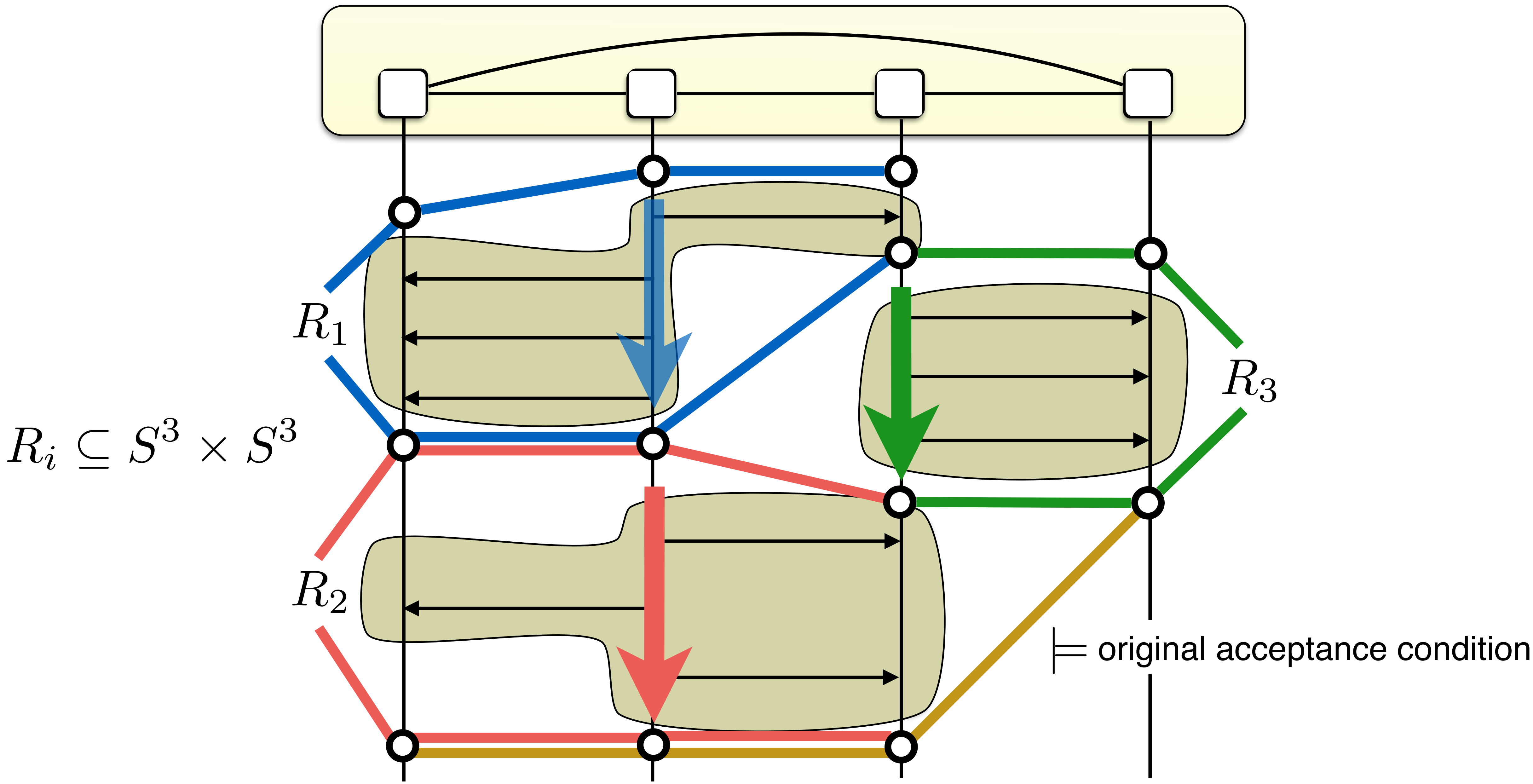

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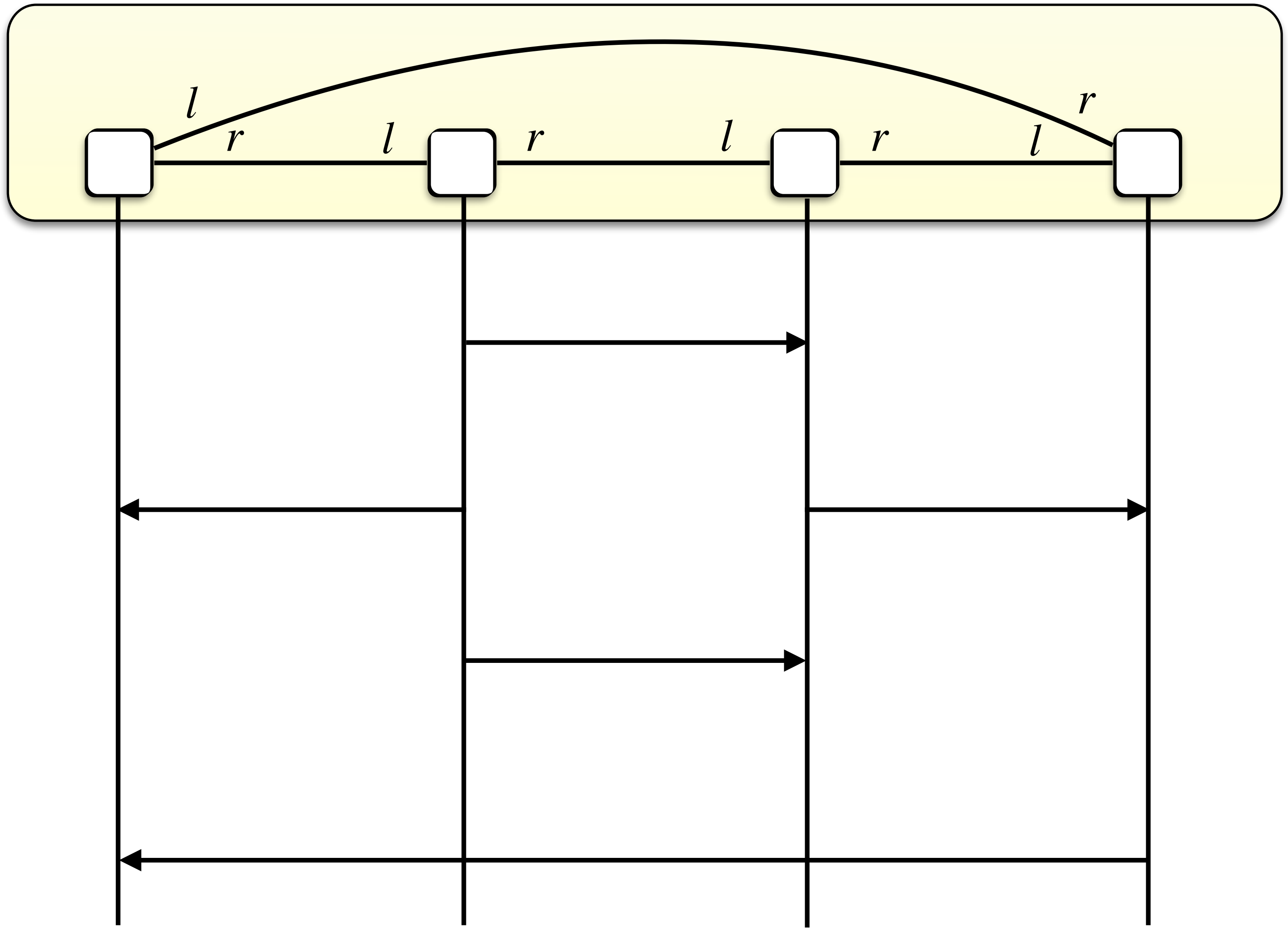
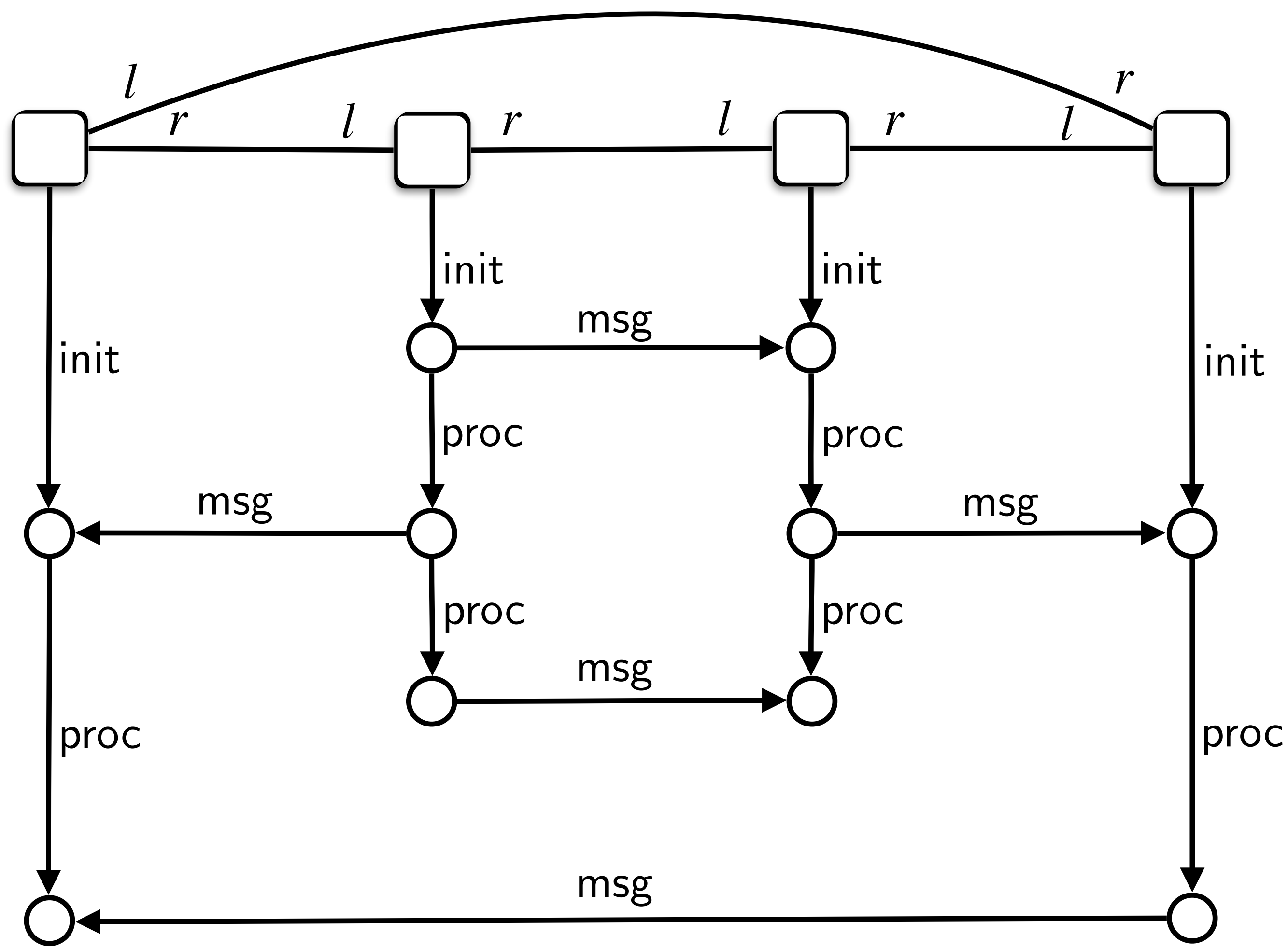
Logical Characterization of Context-Bounded PCAs



The Logic:

MSO logic over graphs, including process nodes and event nodes.

Logical Characterization of Context-Bounded PCAs



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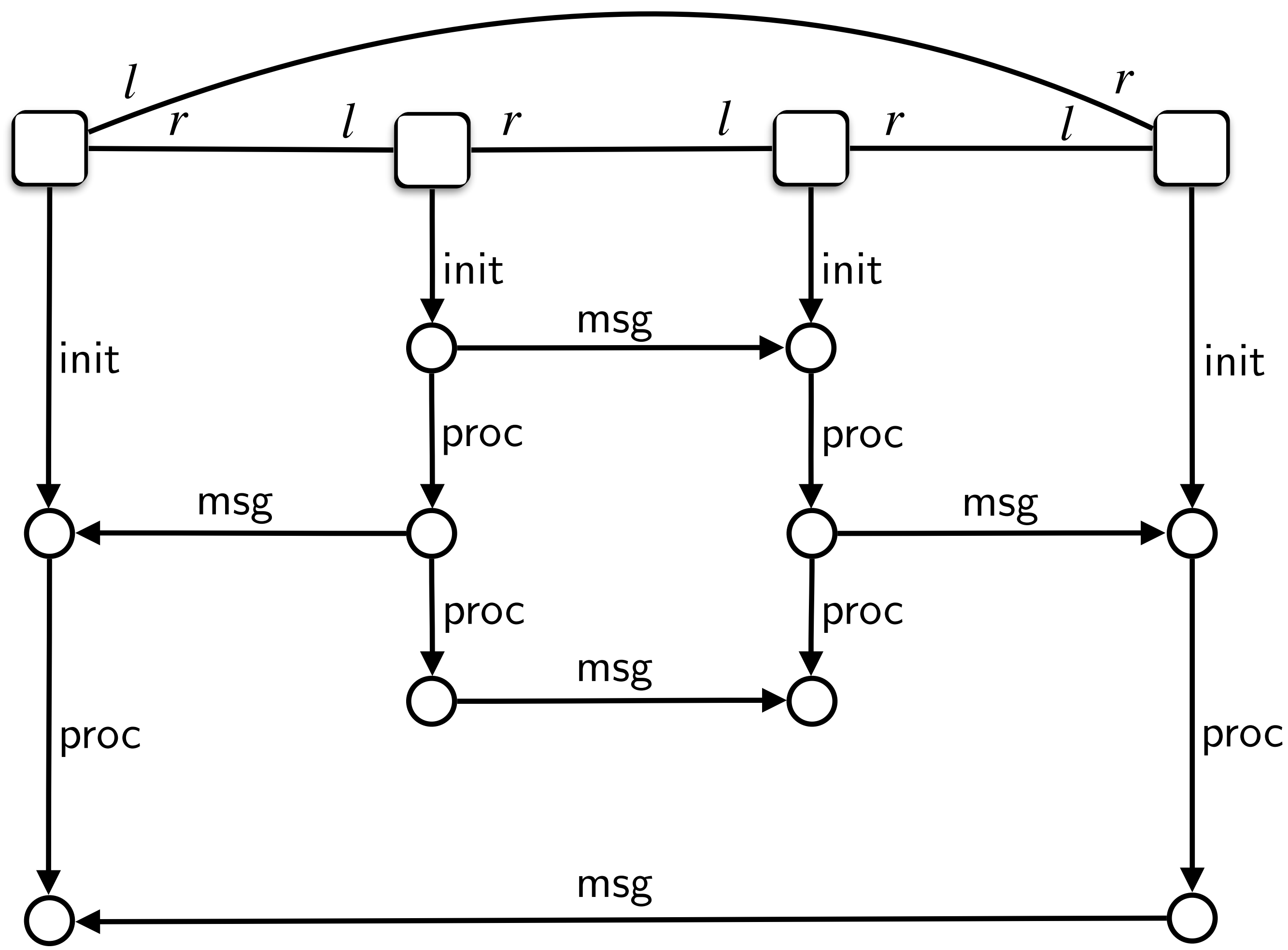

The Logic:
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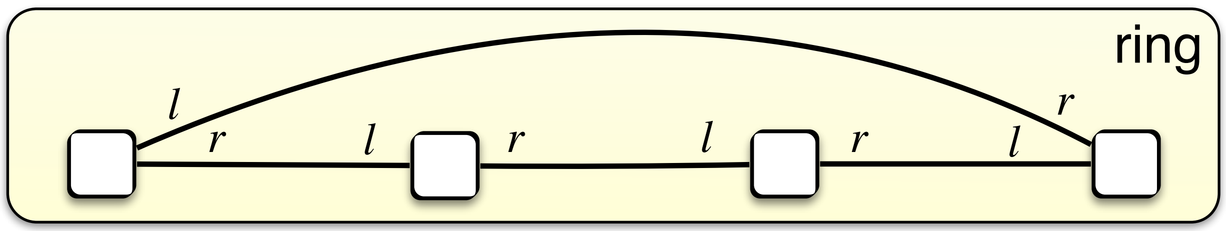
Corollary [B.-Gastin-Kumar; FSTTCS 2014]:
For every *bounded* set L of behaviors, the following are equivalent:

- L is recognized by some PCA.
- L is definable in MSO logic.

Topologies of Bounded Degree

Complementation and MSO characterization hold wrt. the class of **all topologies** over a fixed set of ports. With 4 ports, this captures **rings, binary trees, and grids**.



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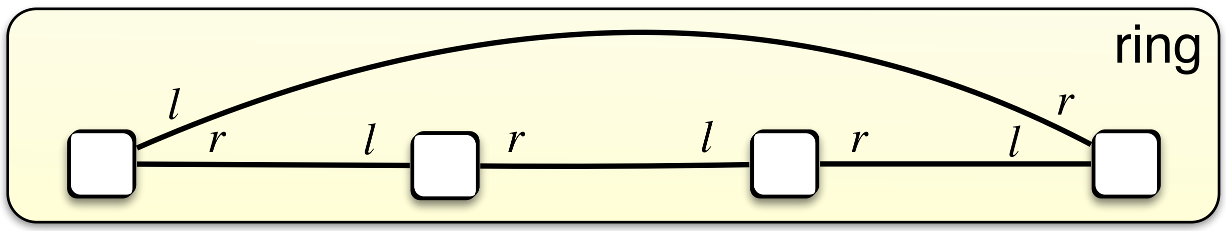

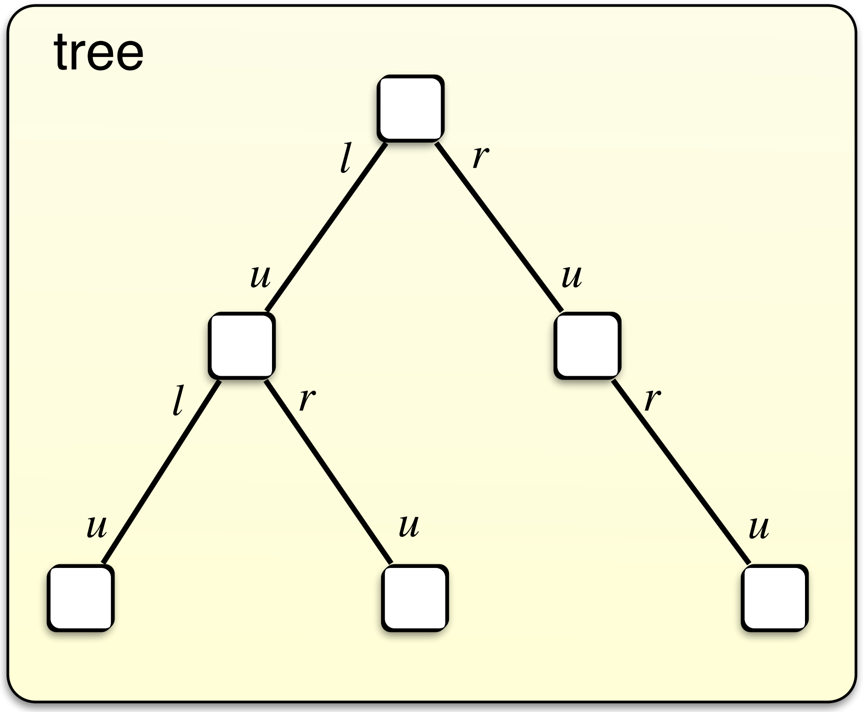

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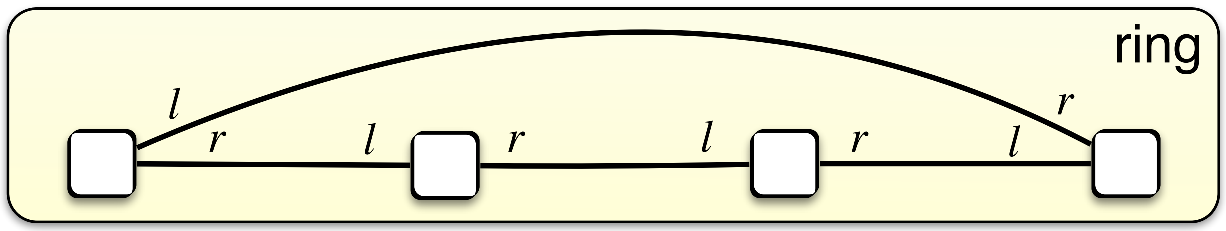

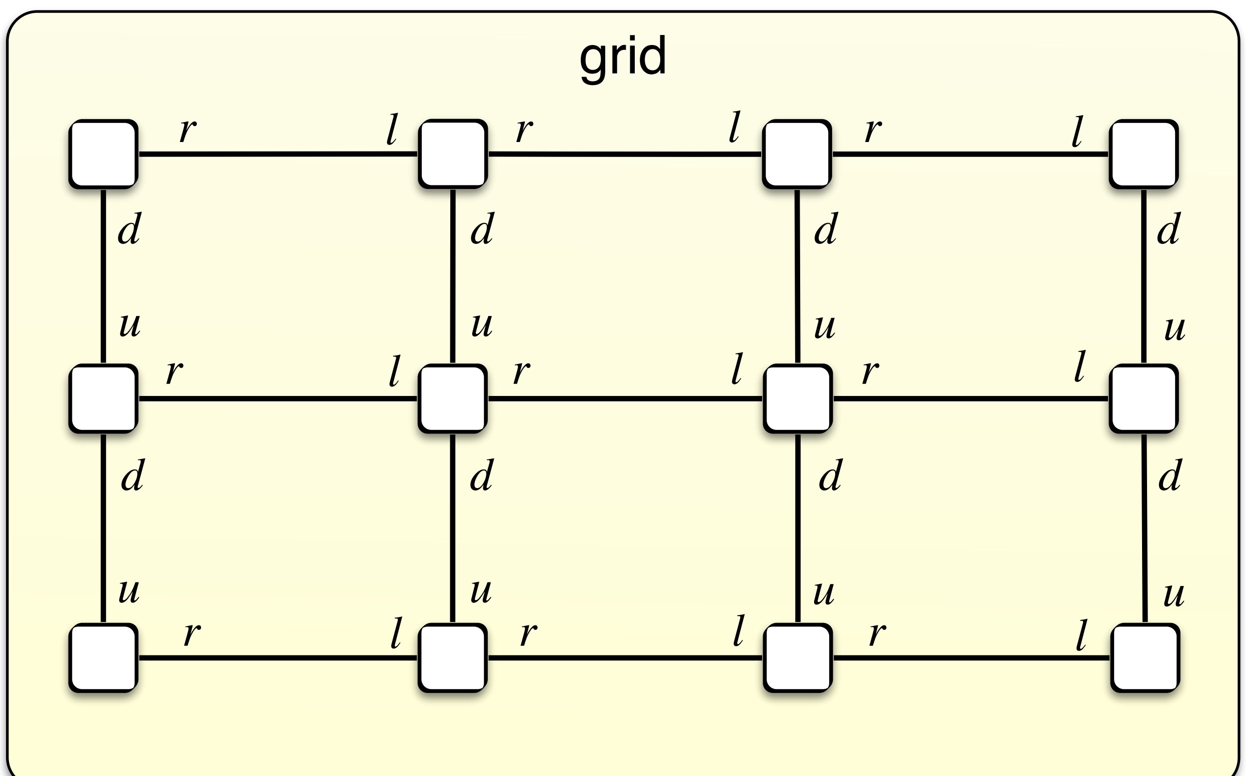

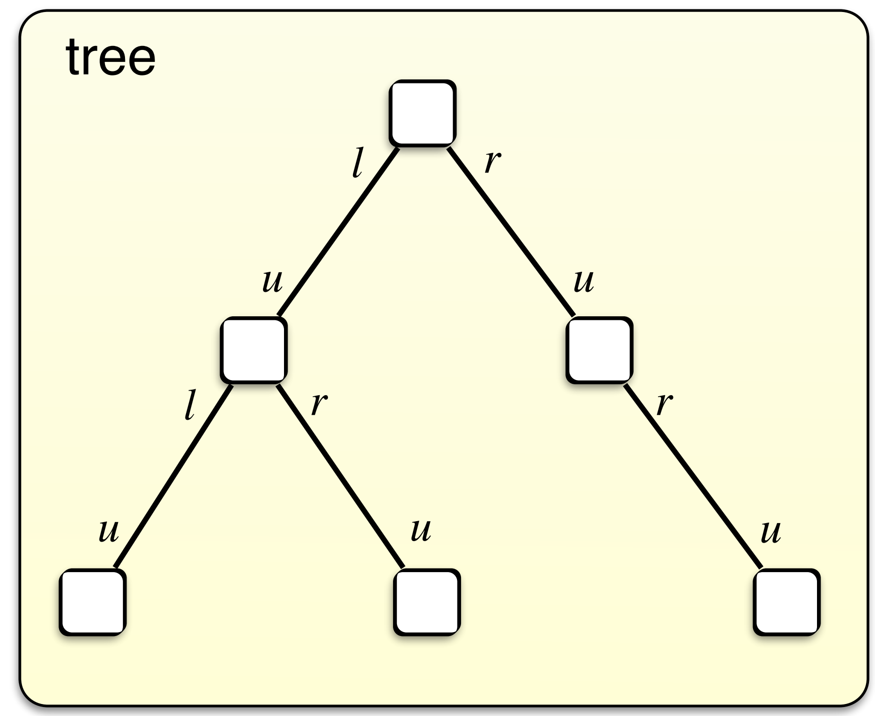

Context-Bounded Model Checking

Theorem [B.-Gastin-Kumar; FSTTCS 2014]:
Context-bounded MSO model checking is decidable over rings.

Input: PCA $\mathcal{A}$; $k \in \mathbb{N}$; MSO formula φ

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Context-bounded nonemptiness checking over rings is PSPACE-complete when the acceptance condition is presented as a finite automaton.

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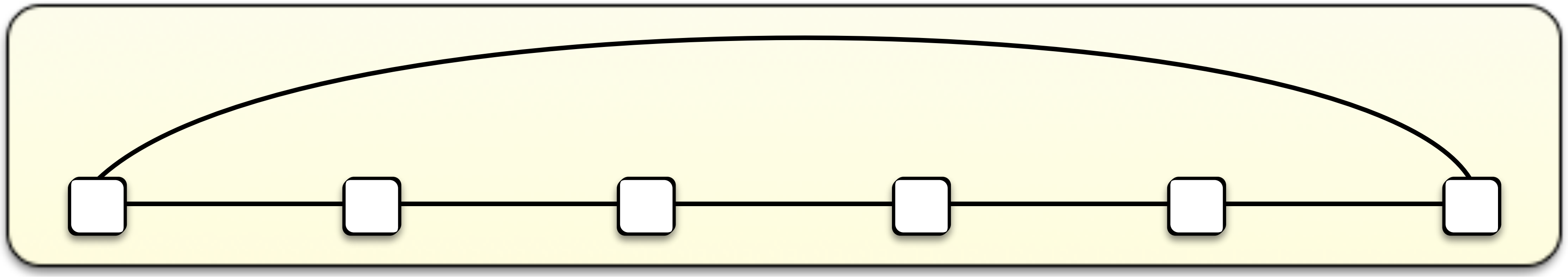

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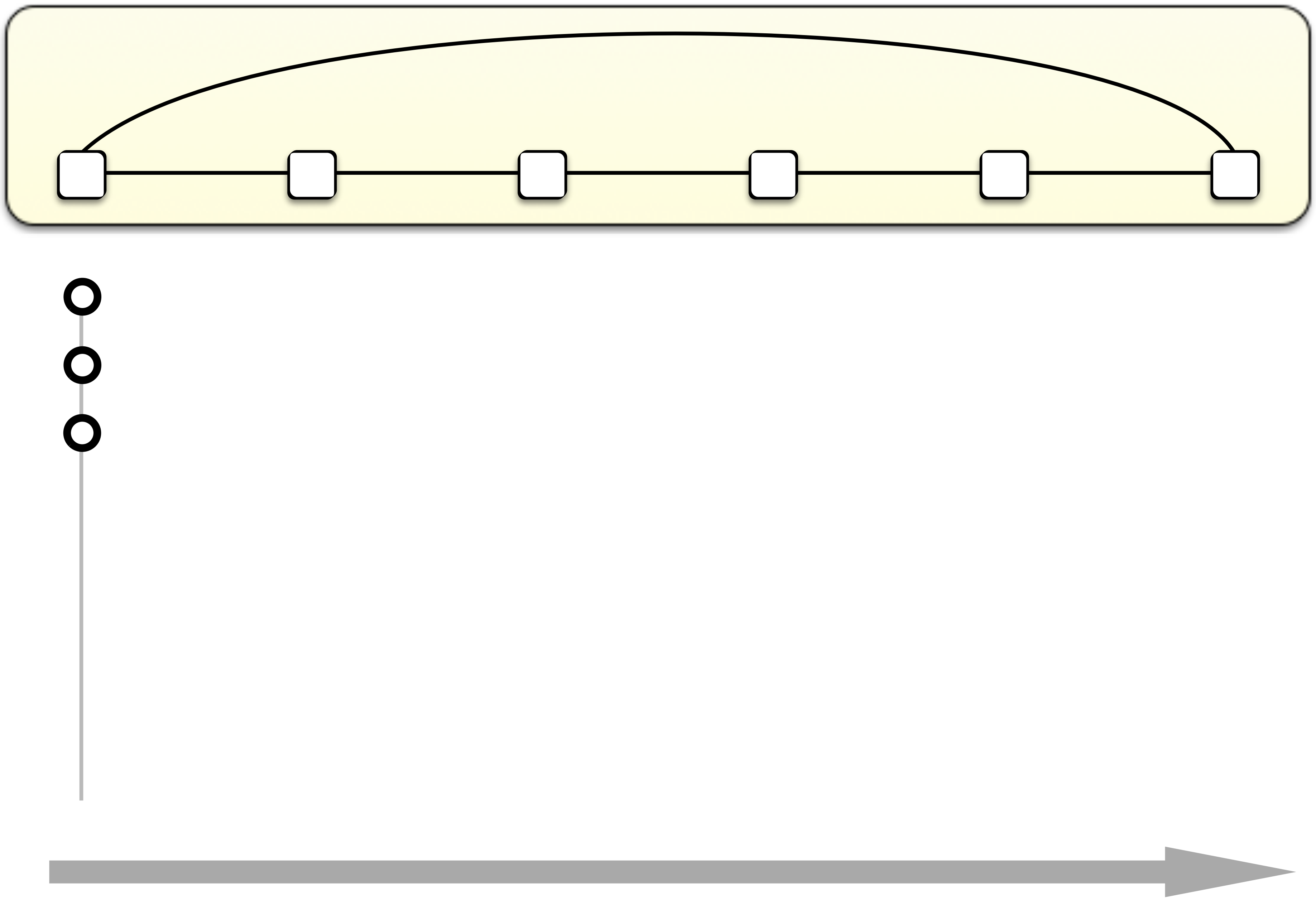

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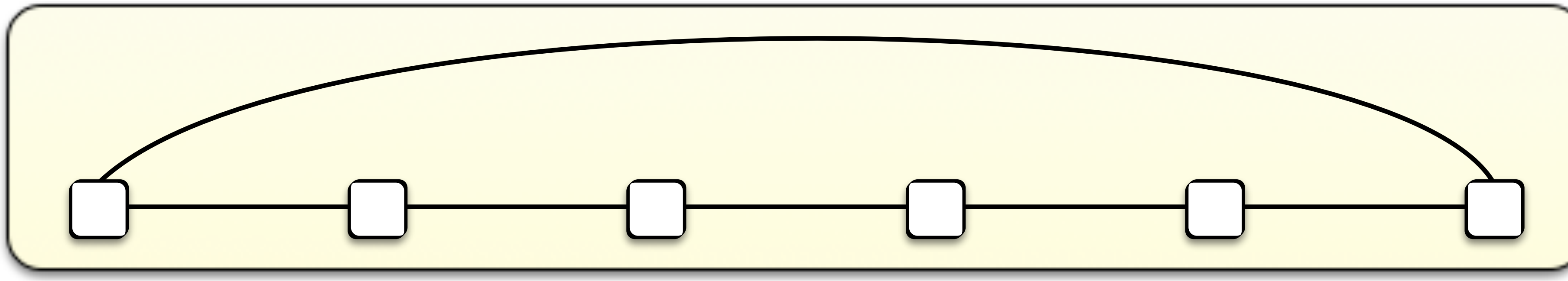

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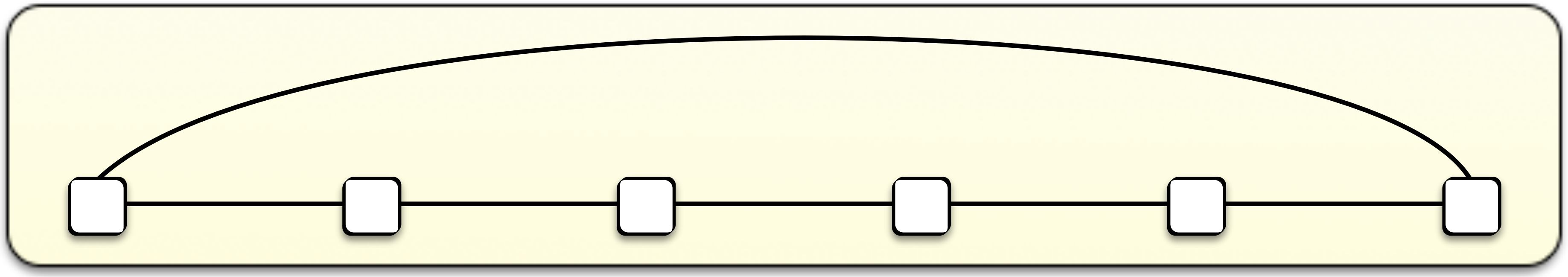

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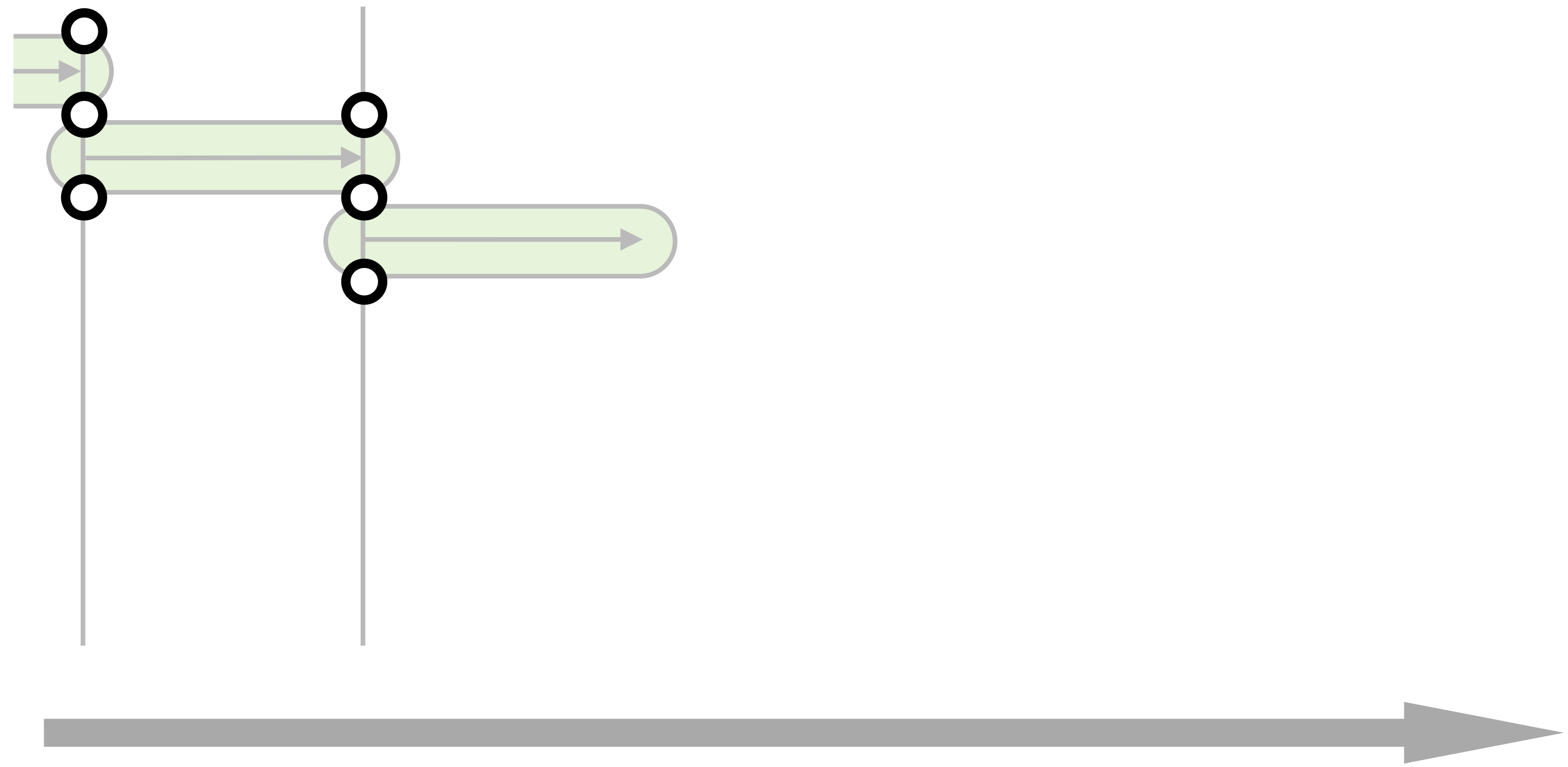

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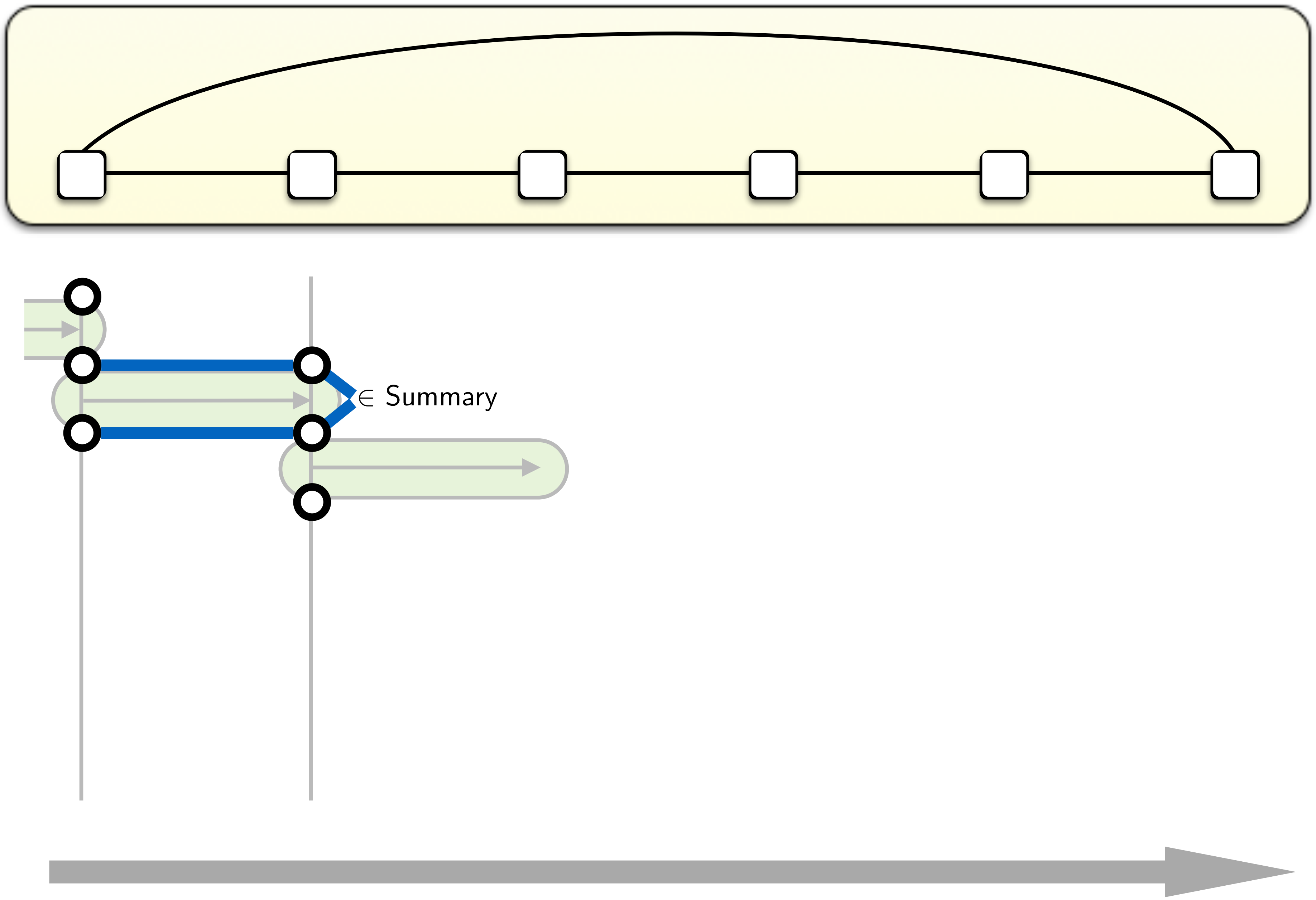

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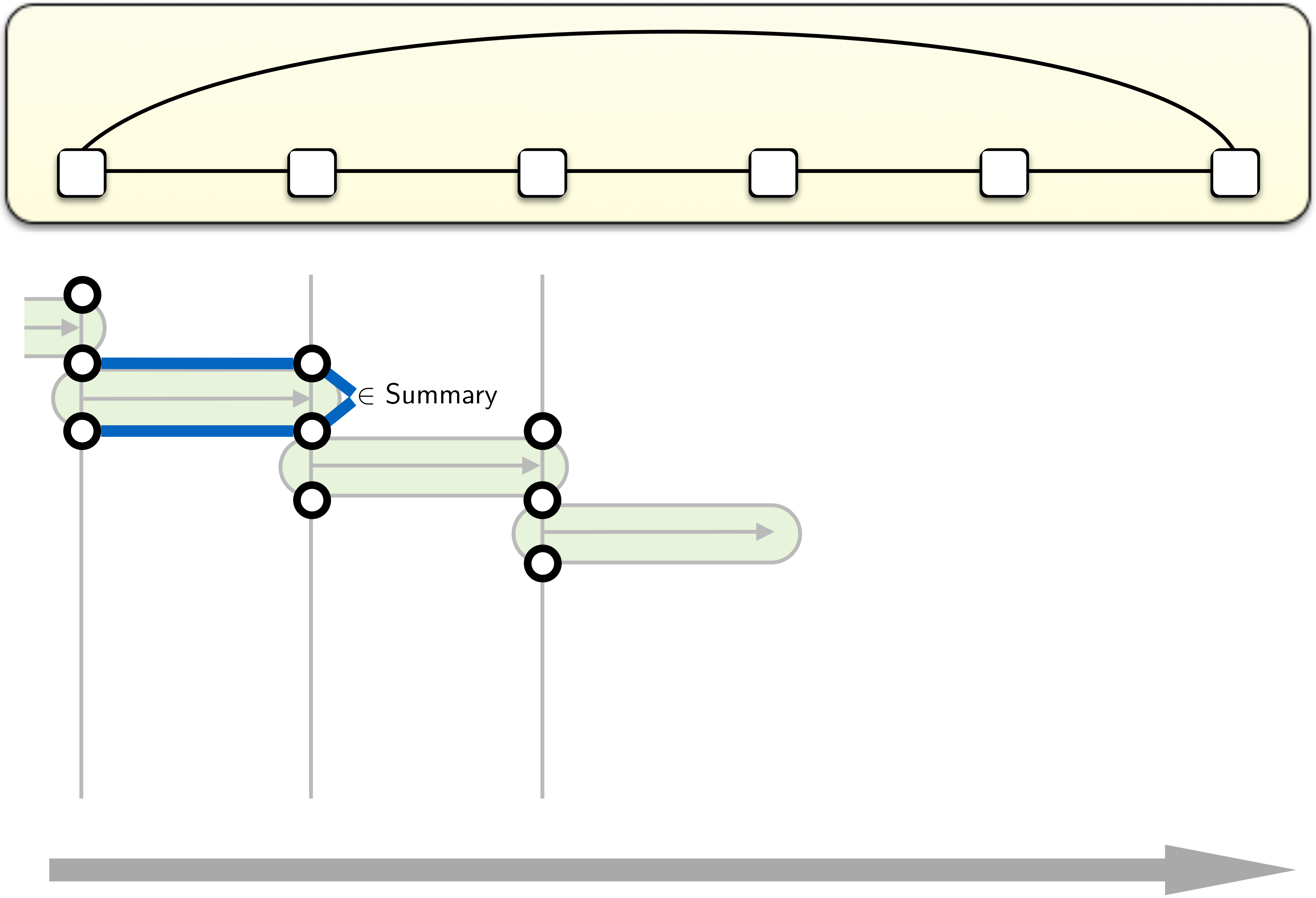

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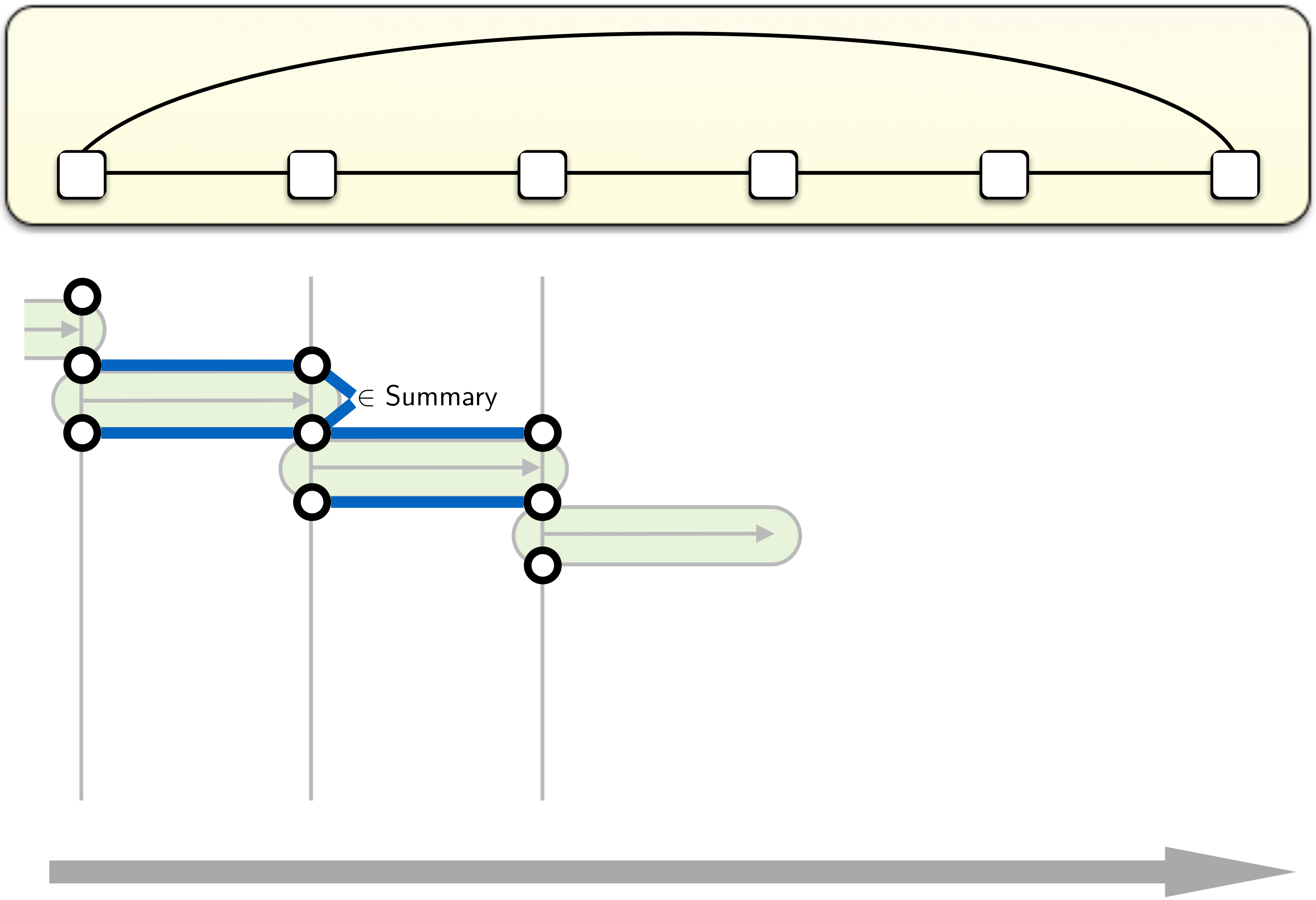

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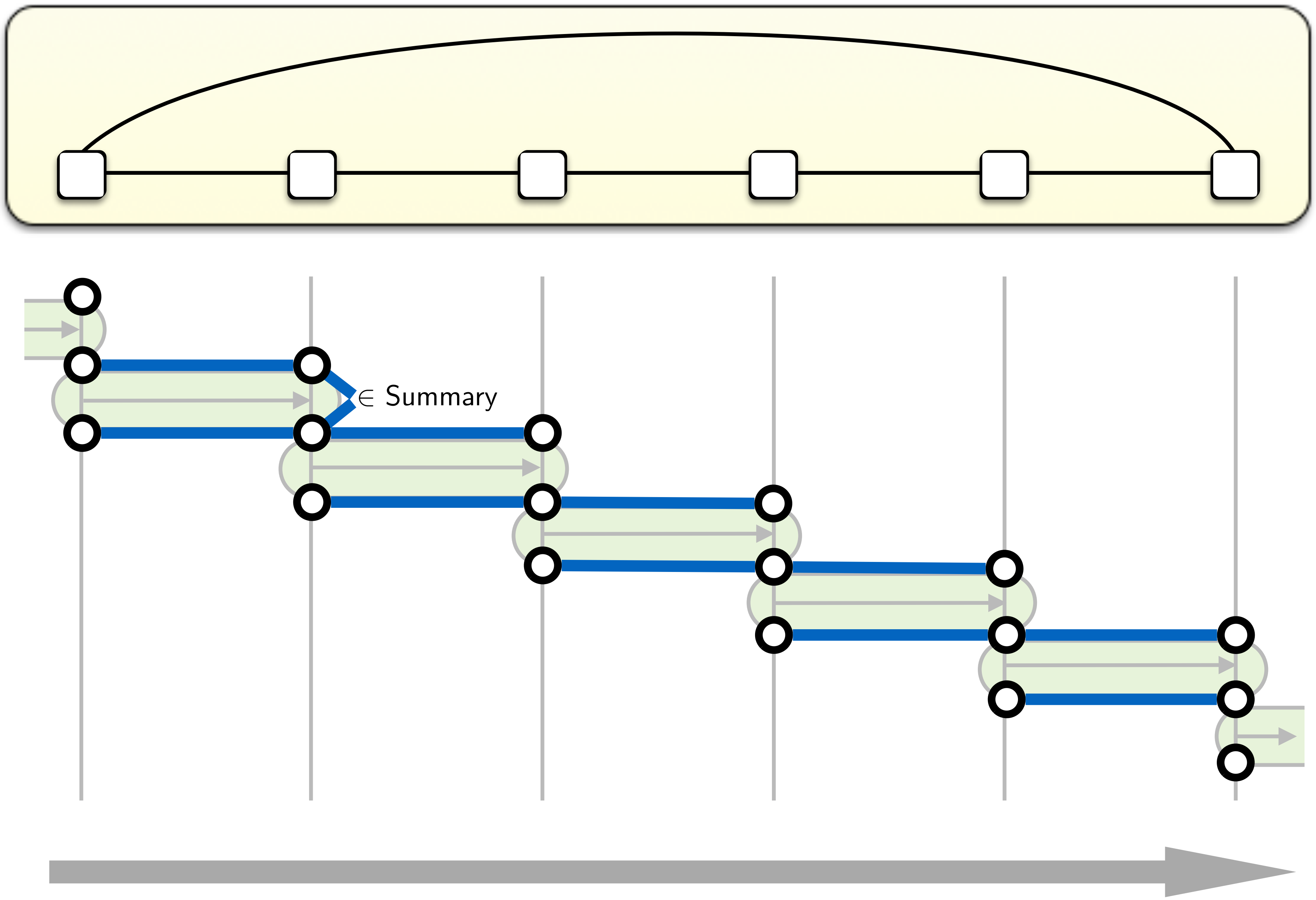

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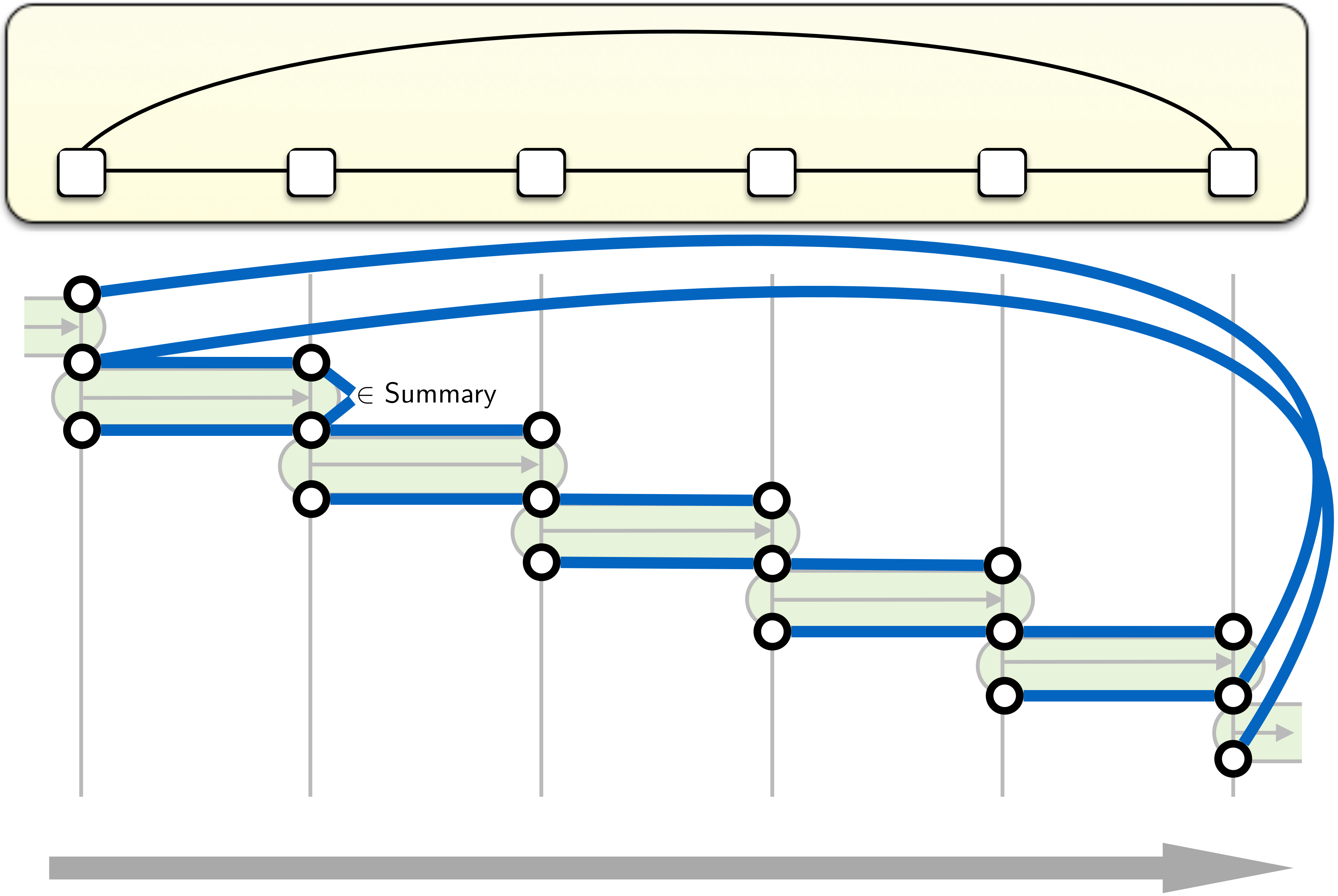

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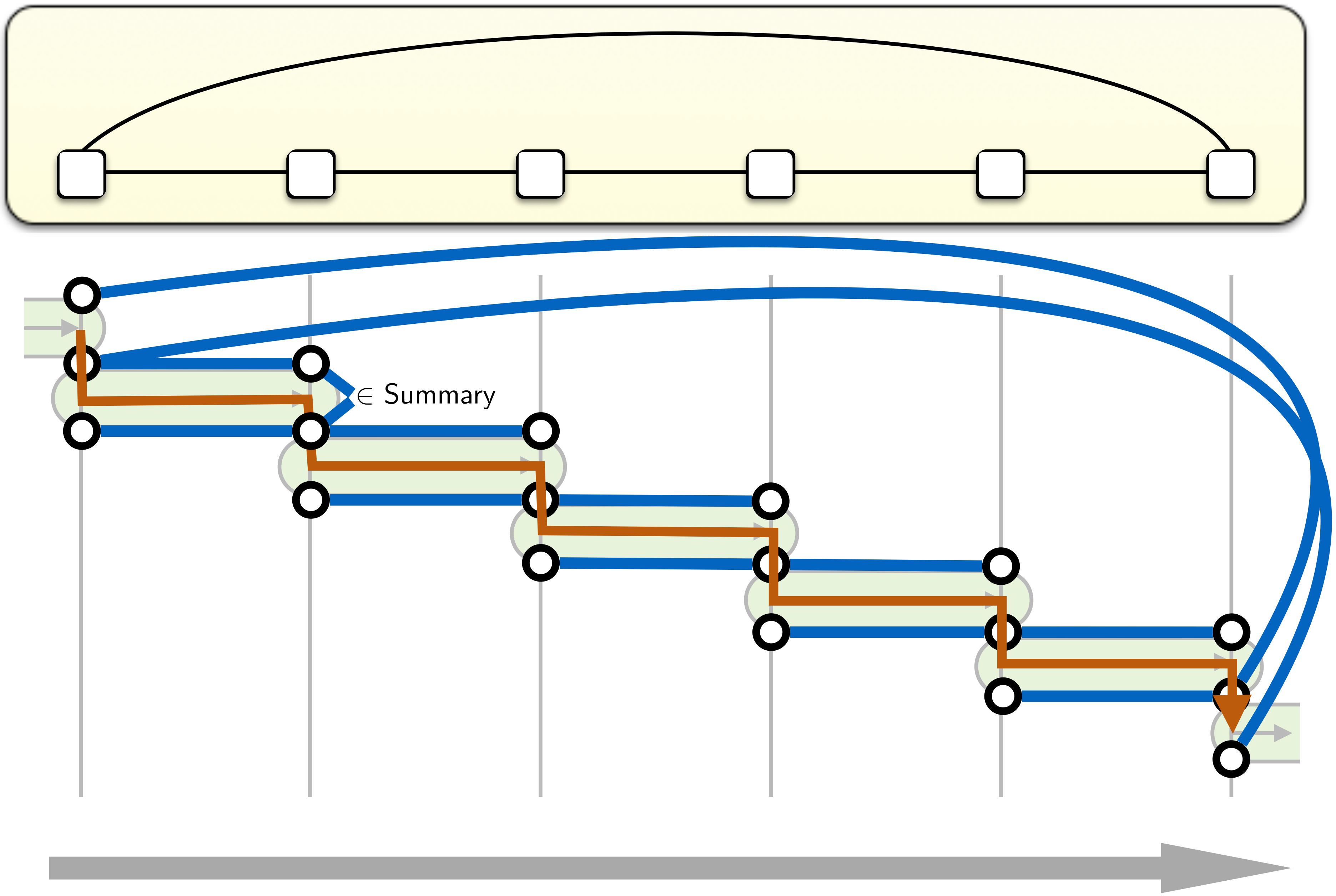

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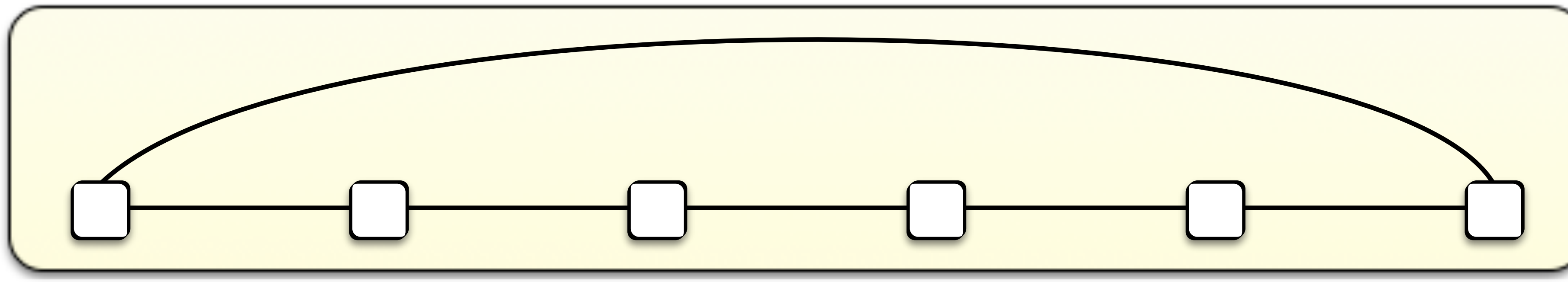

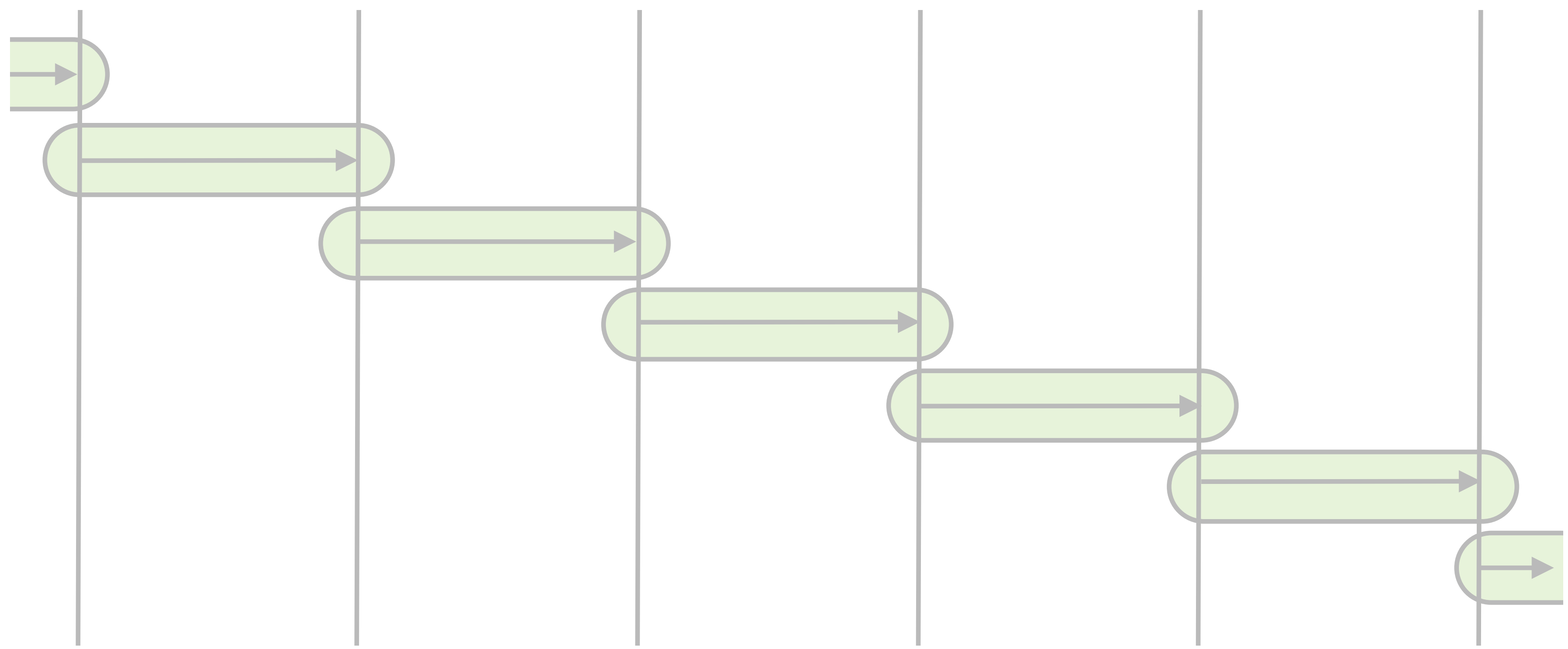

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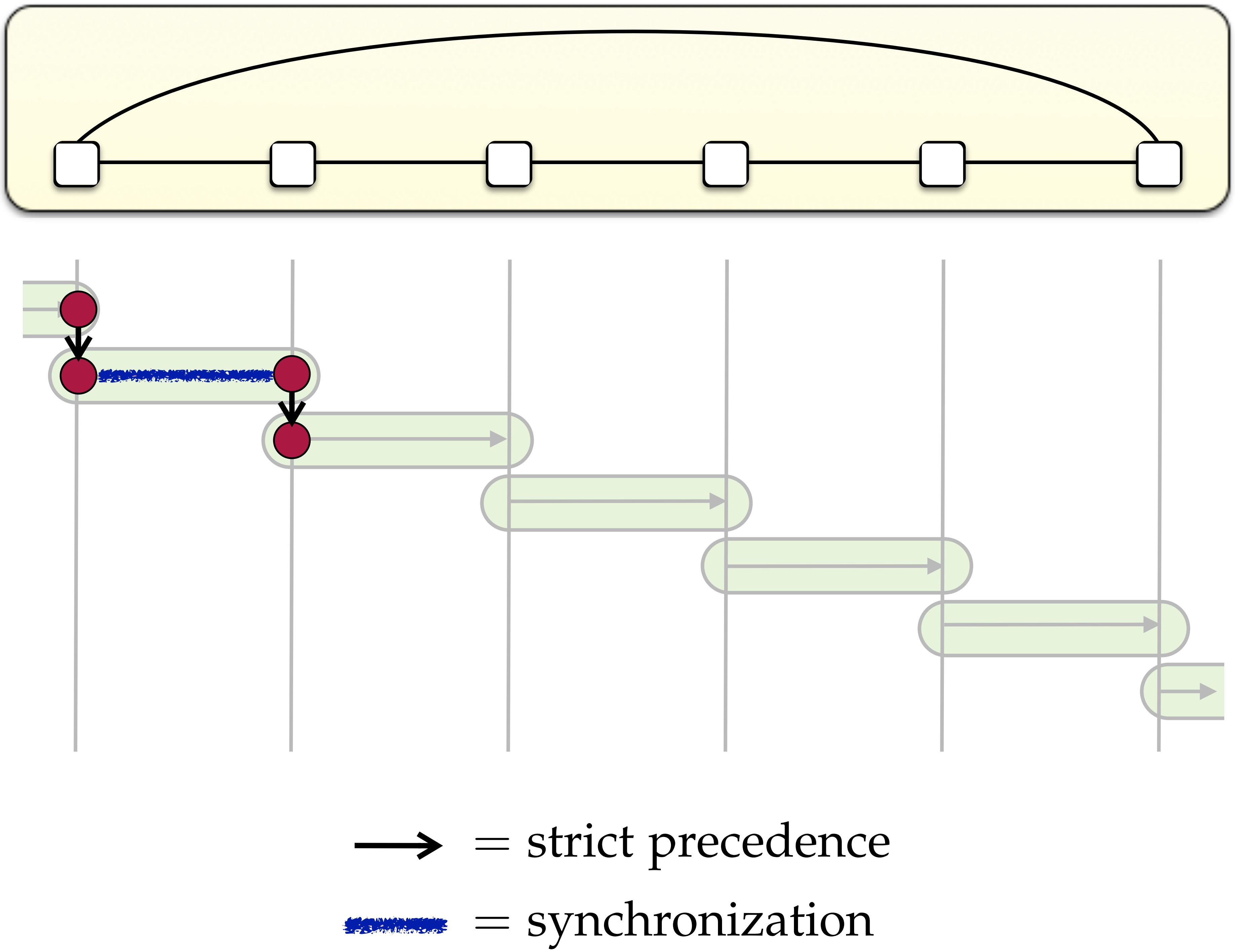

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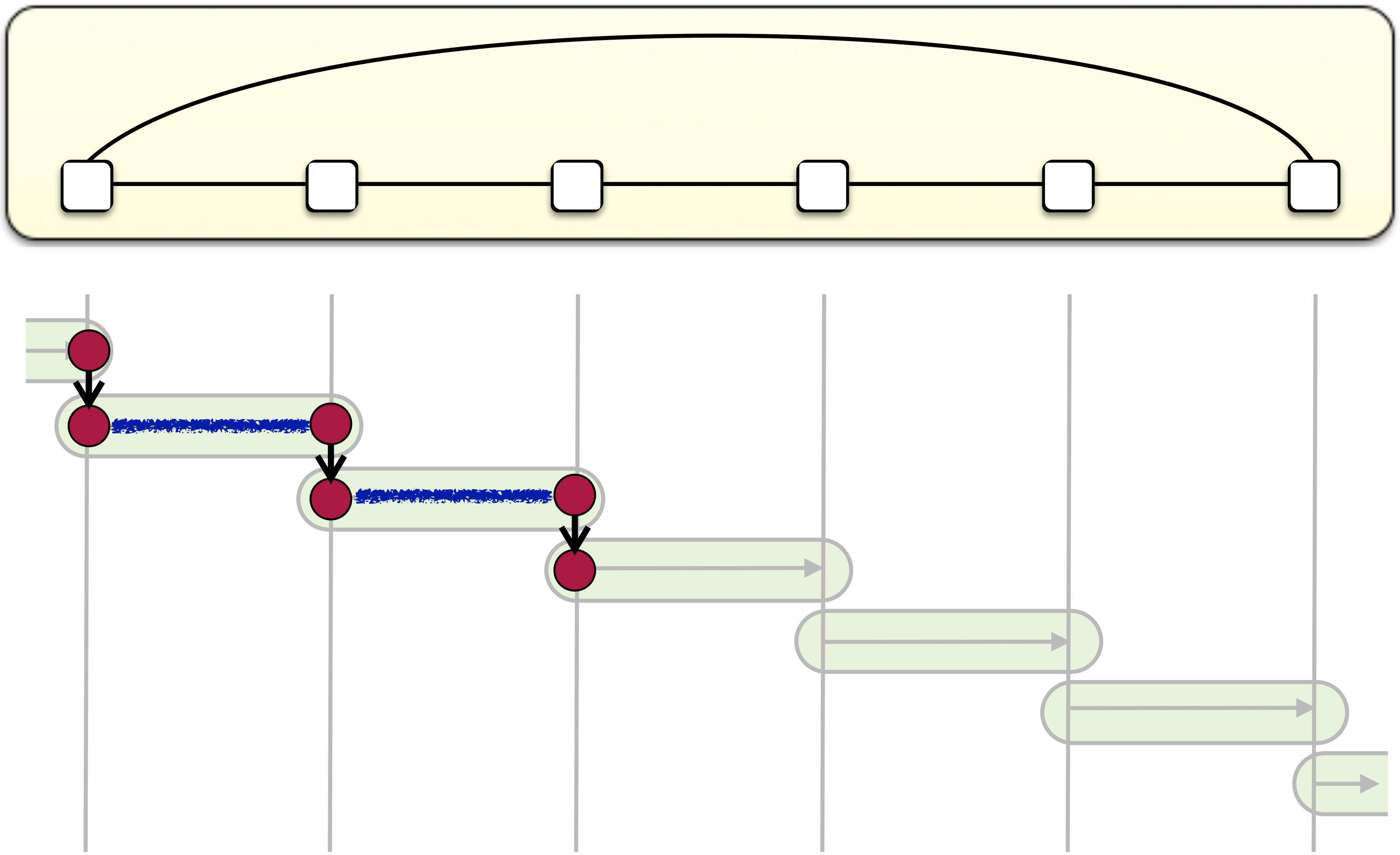

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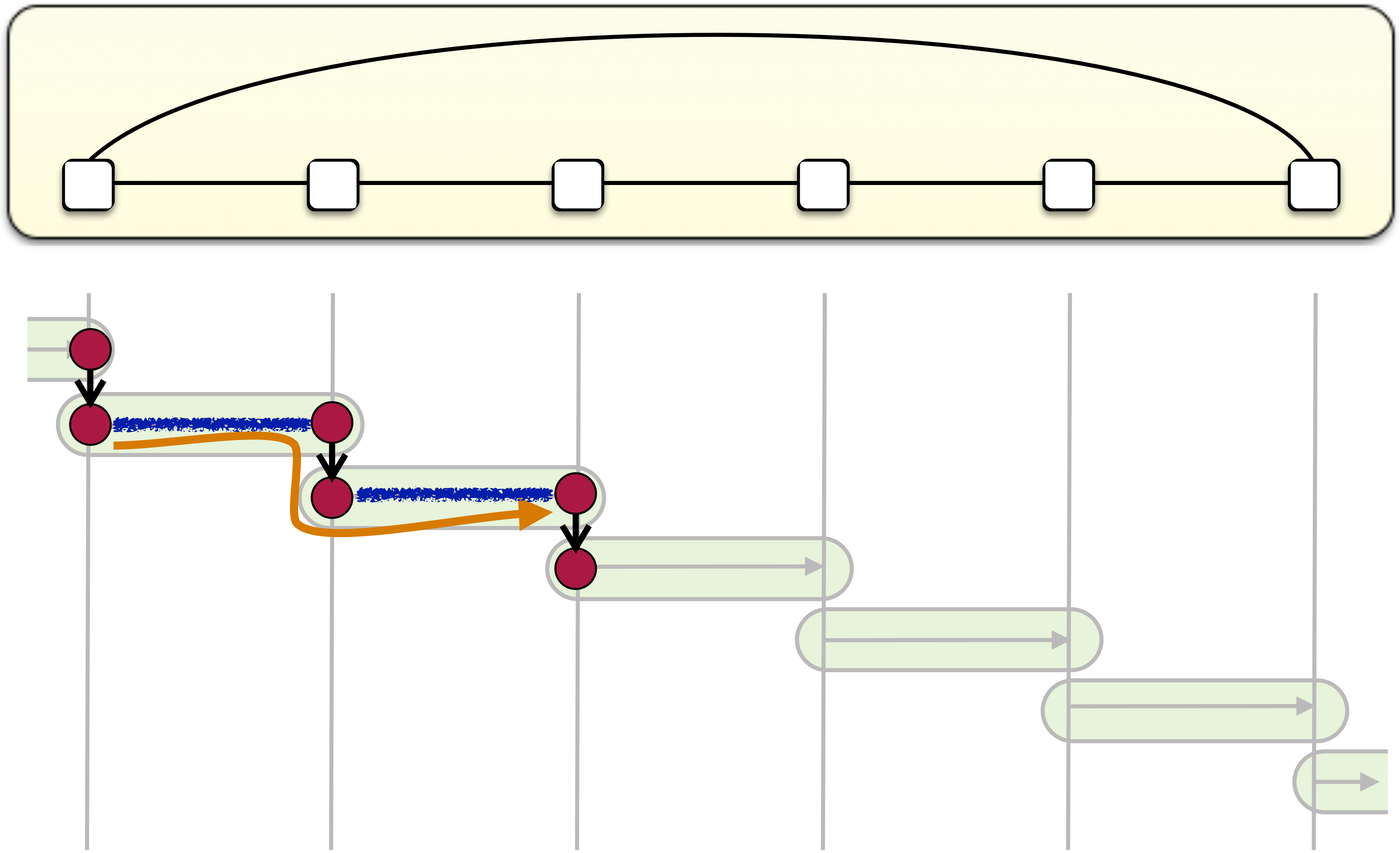

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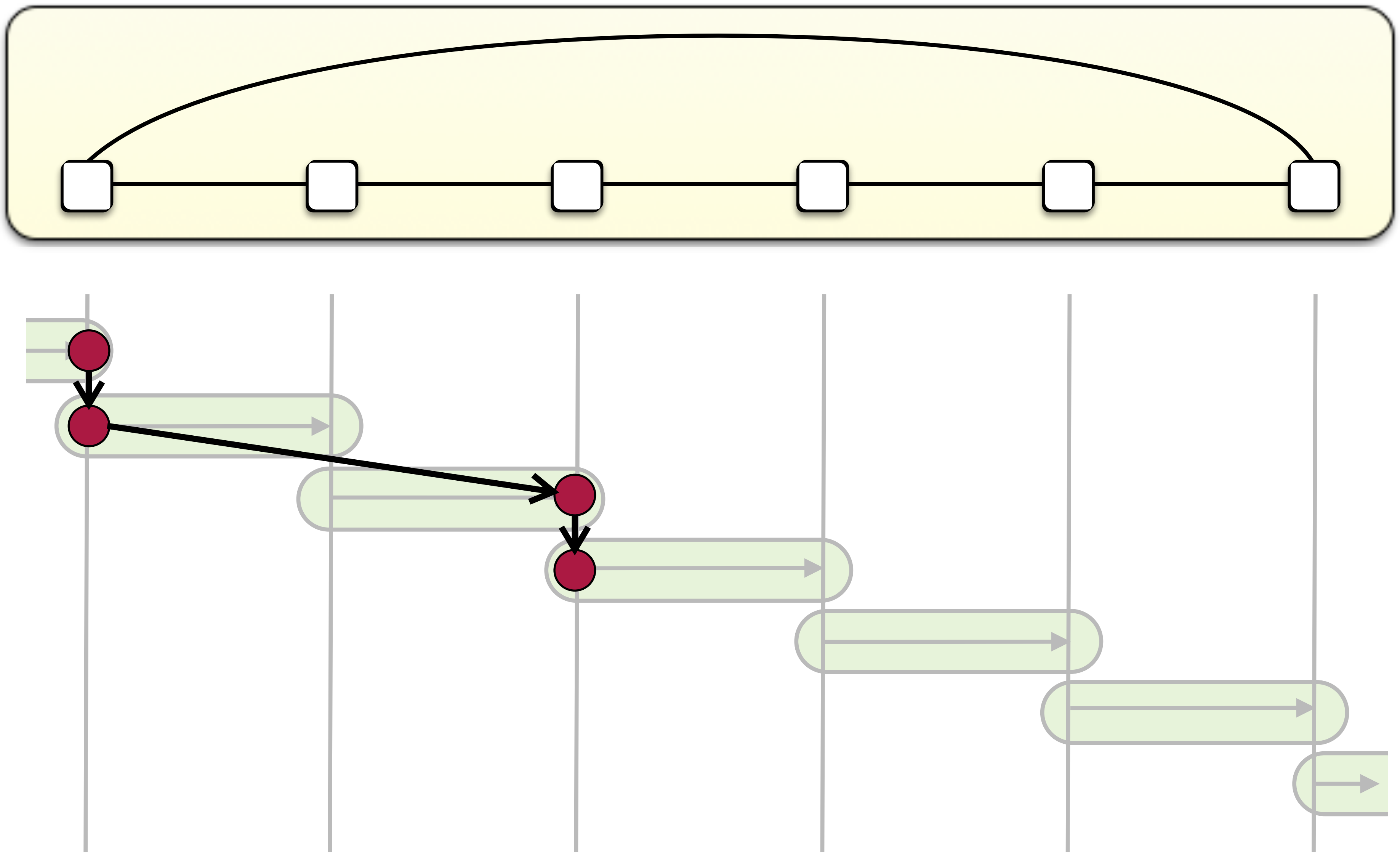

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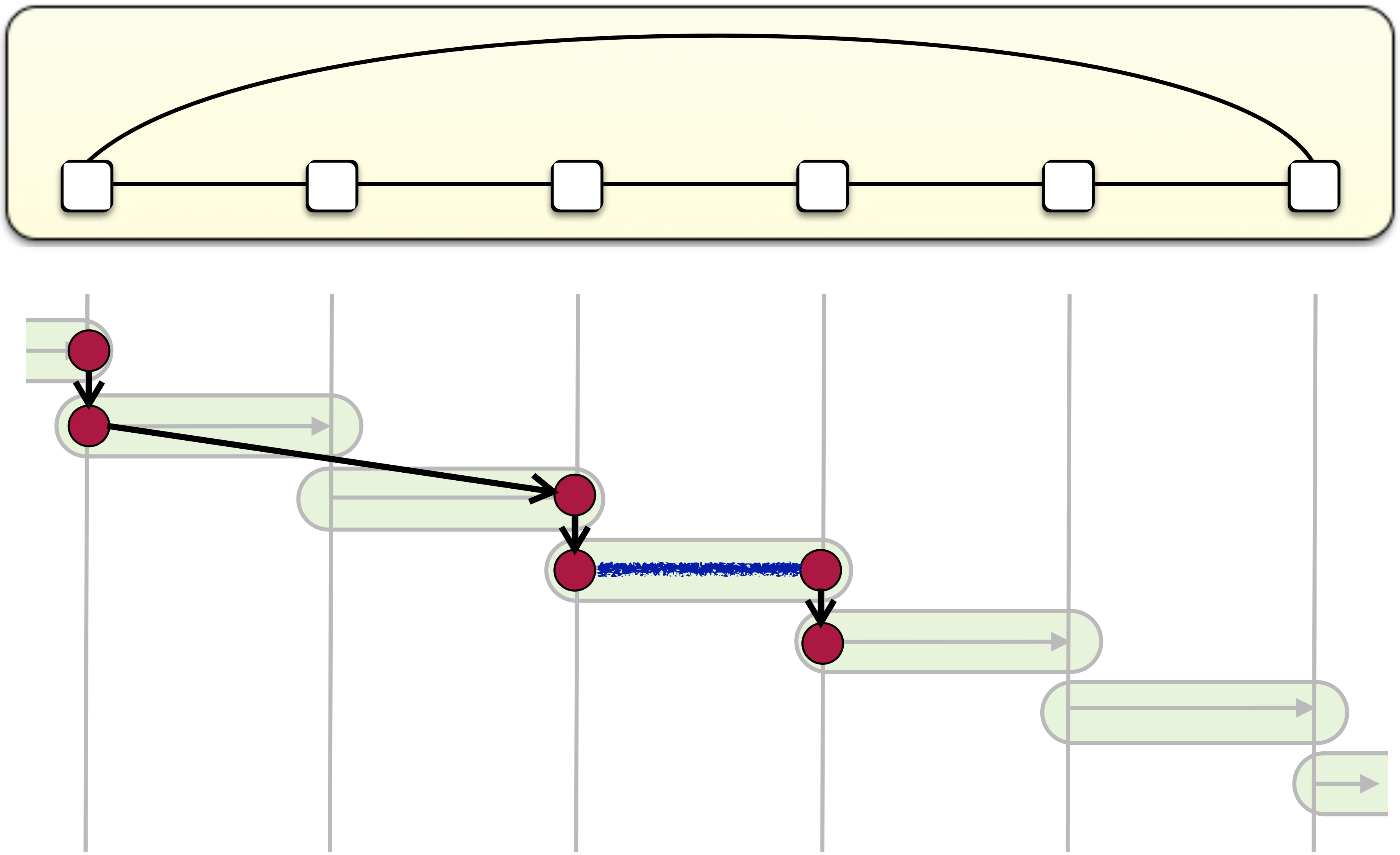

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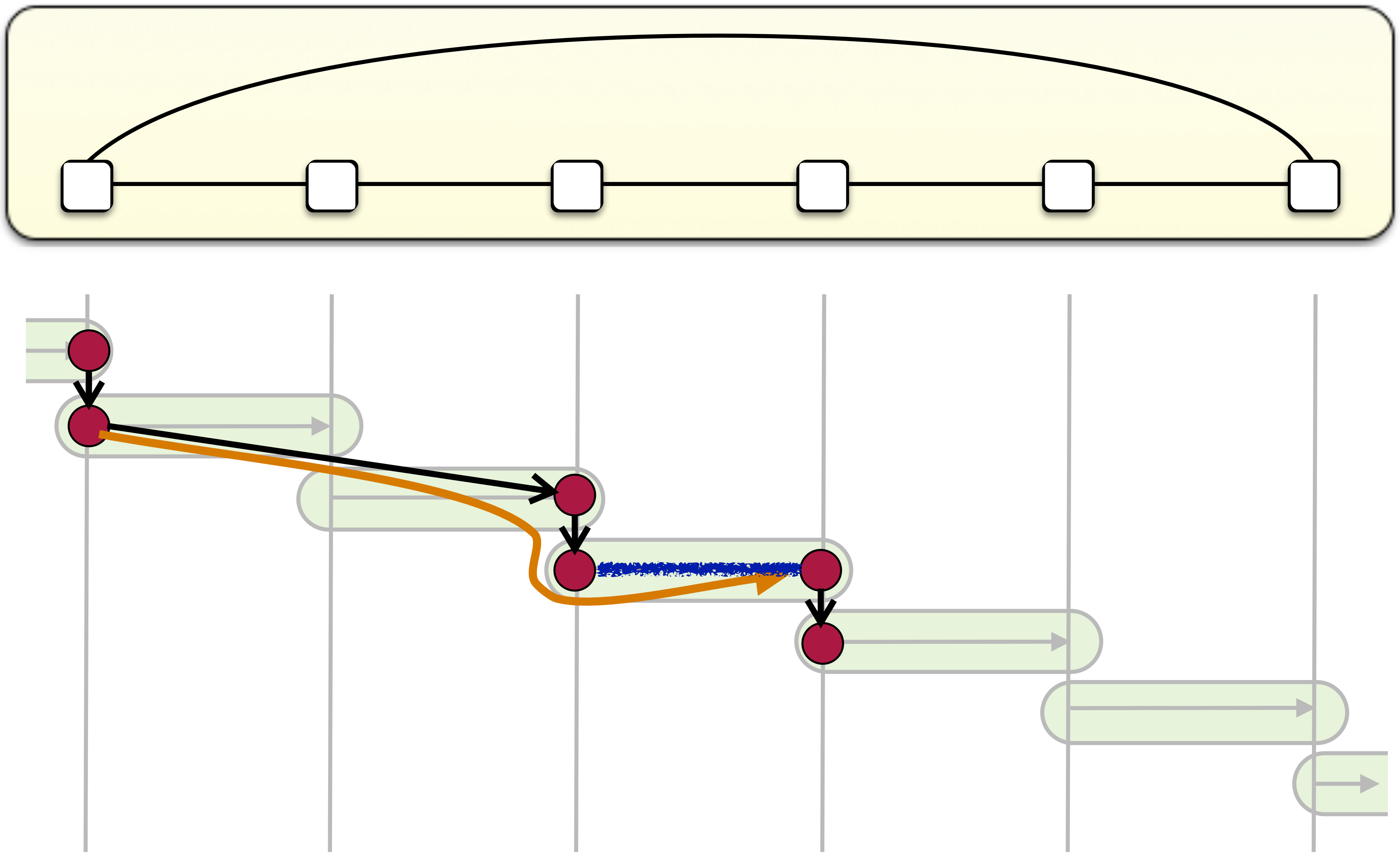

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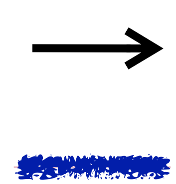

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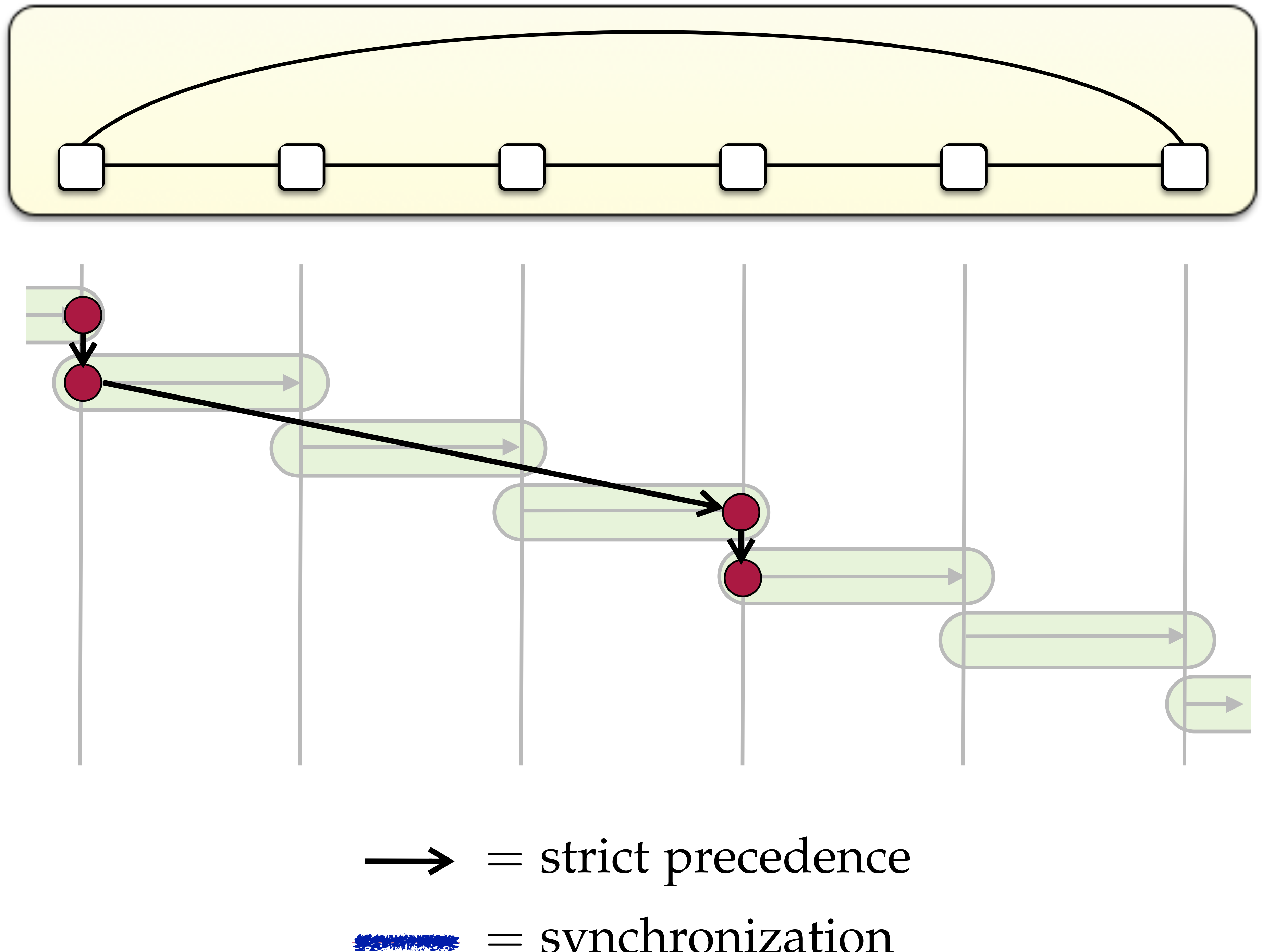

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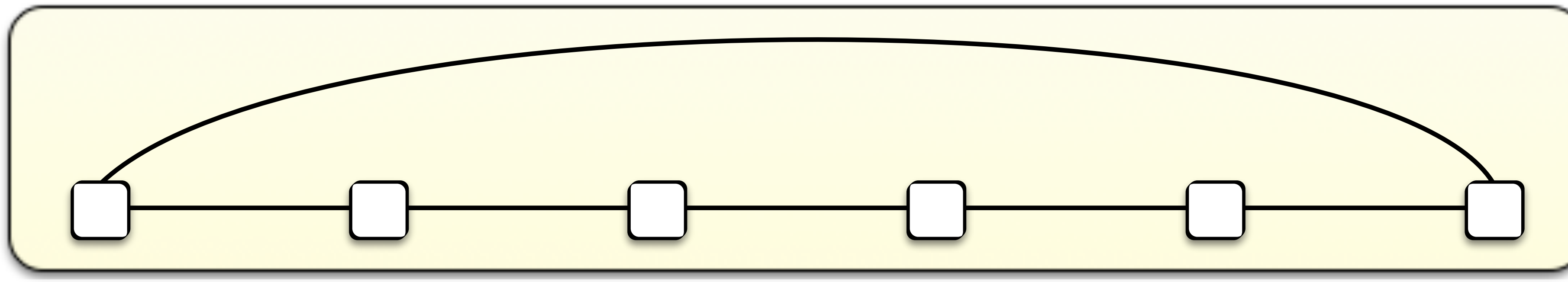

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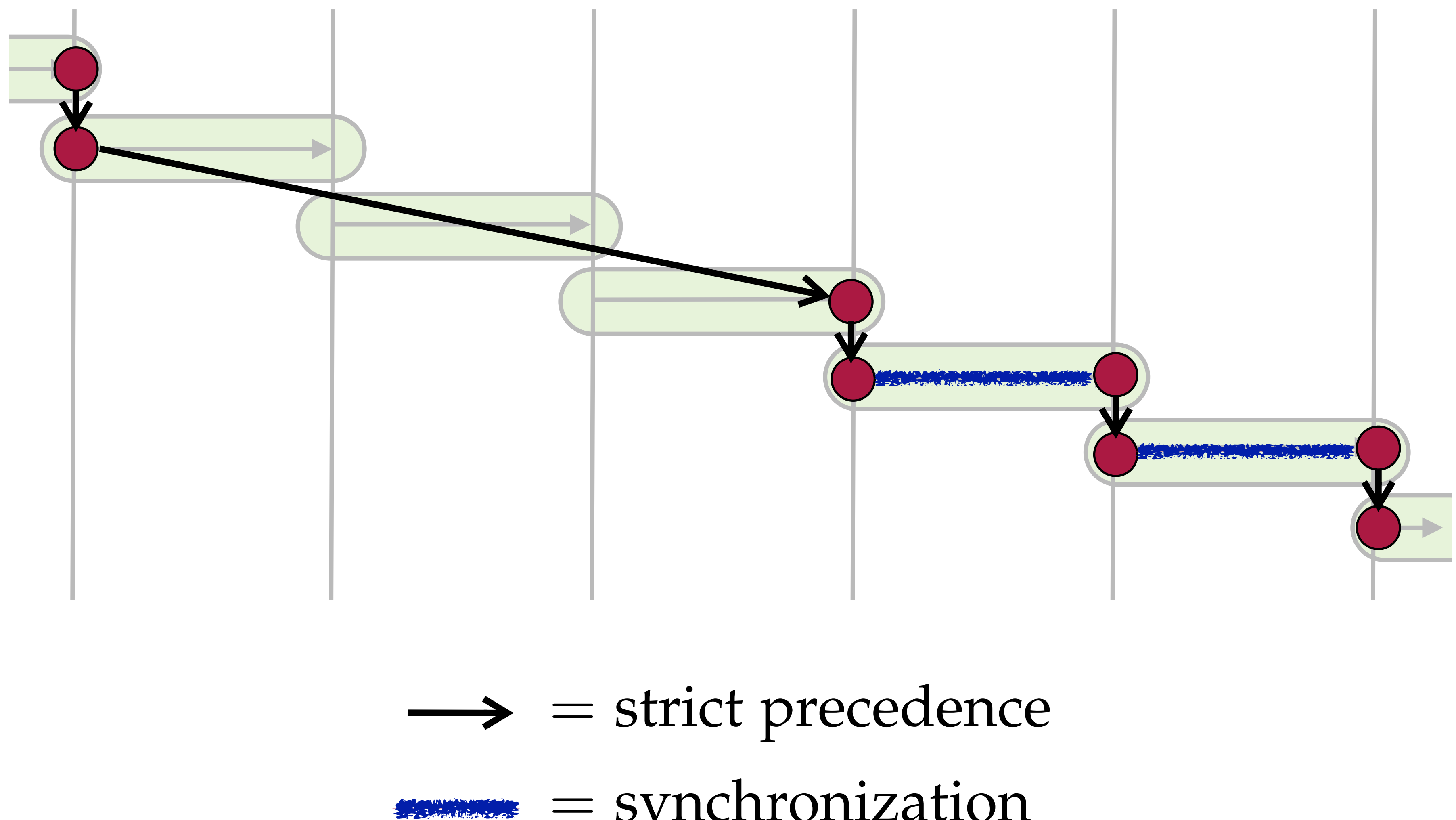

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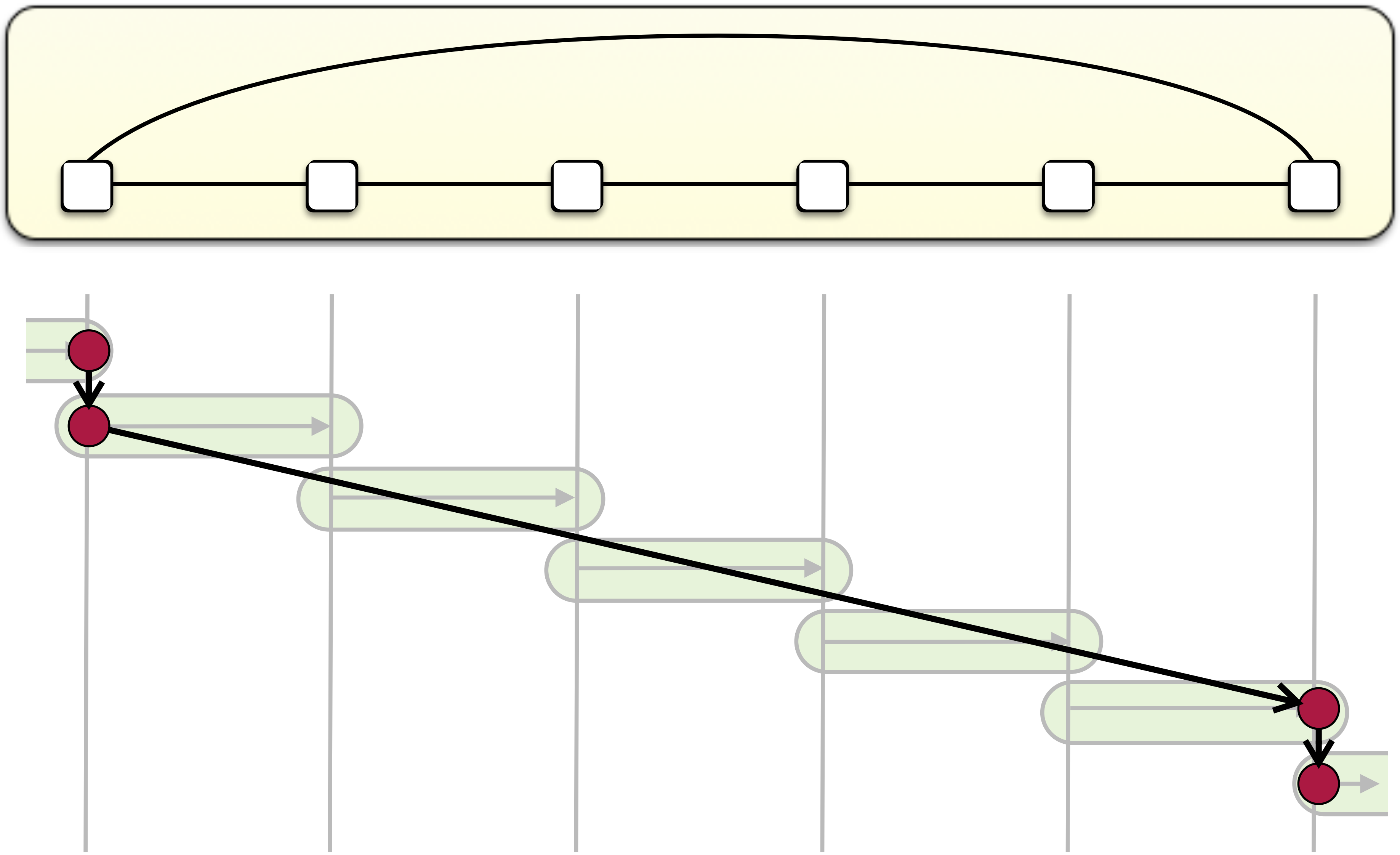

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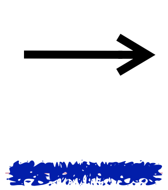

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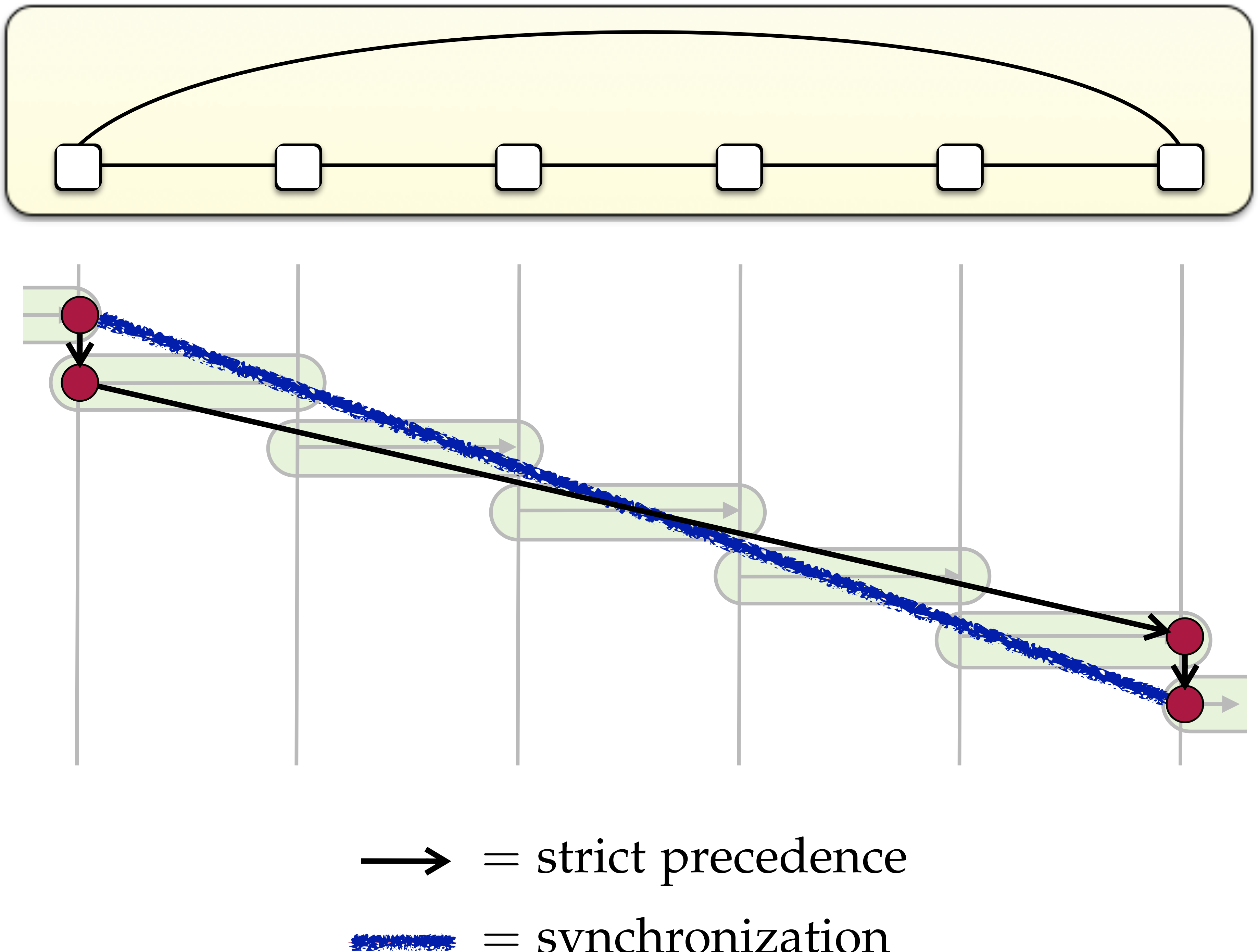

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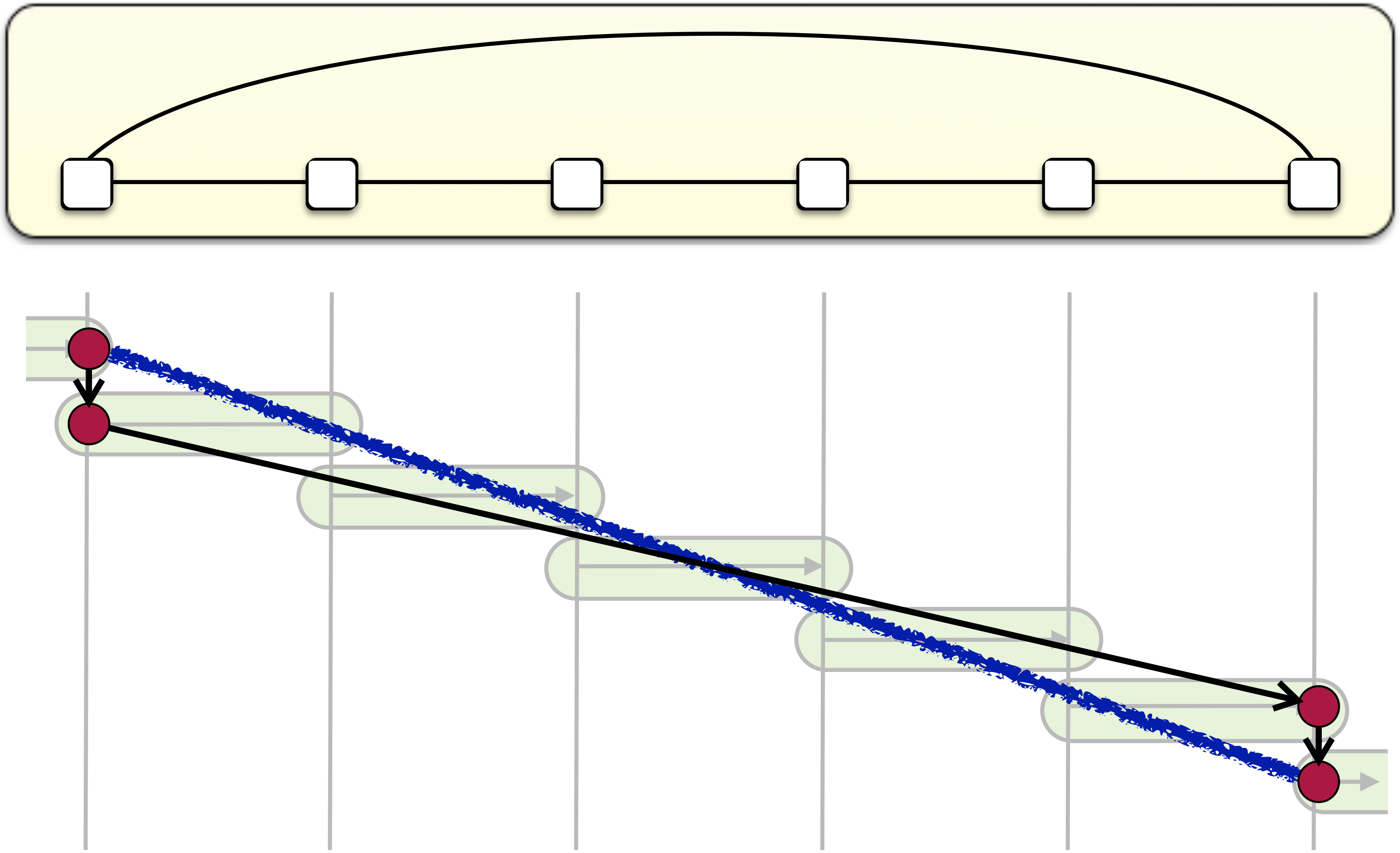

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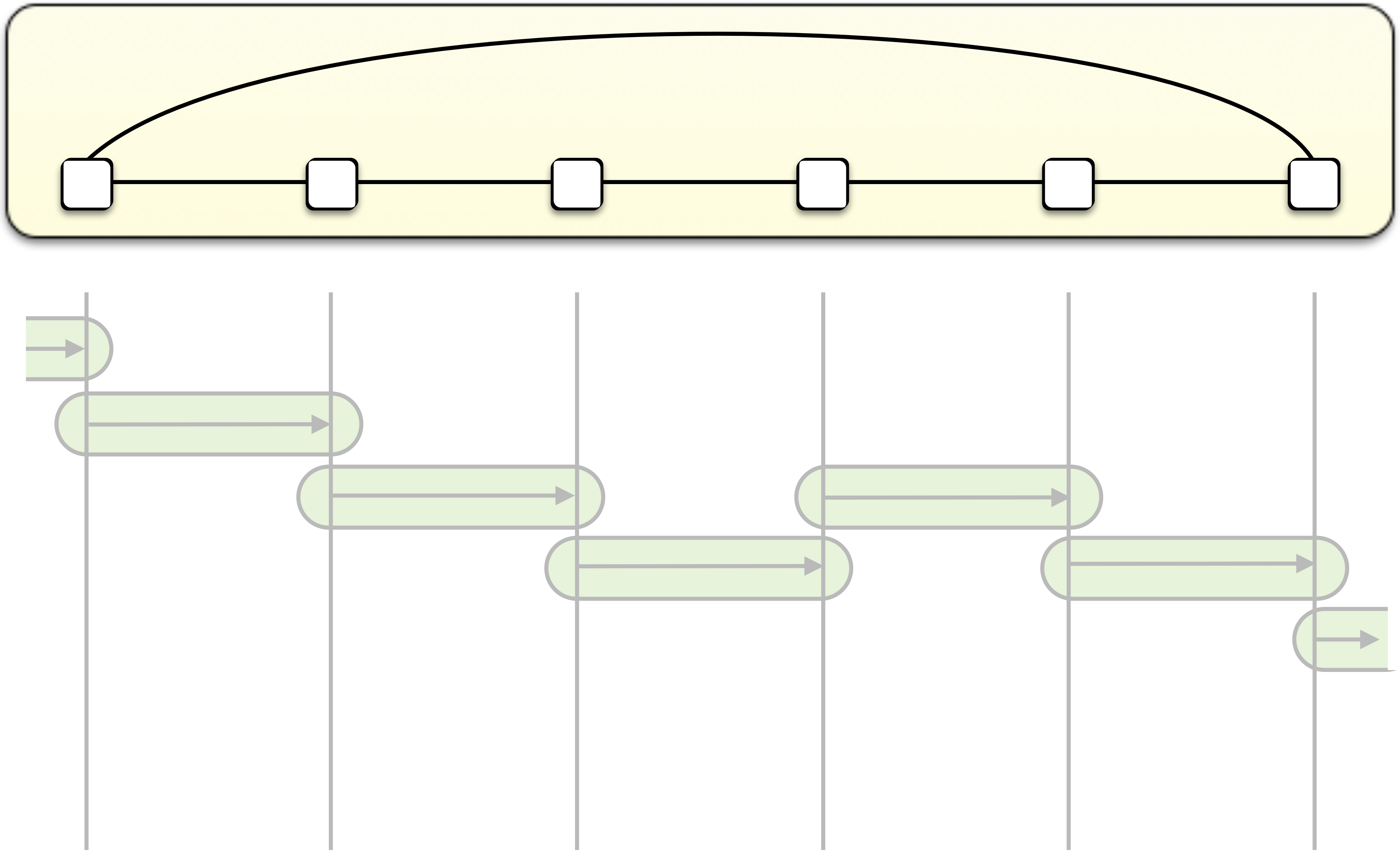

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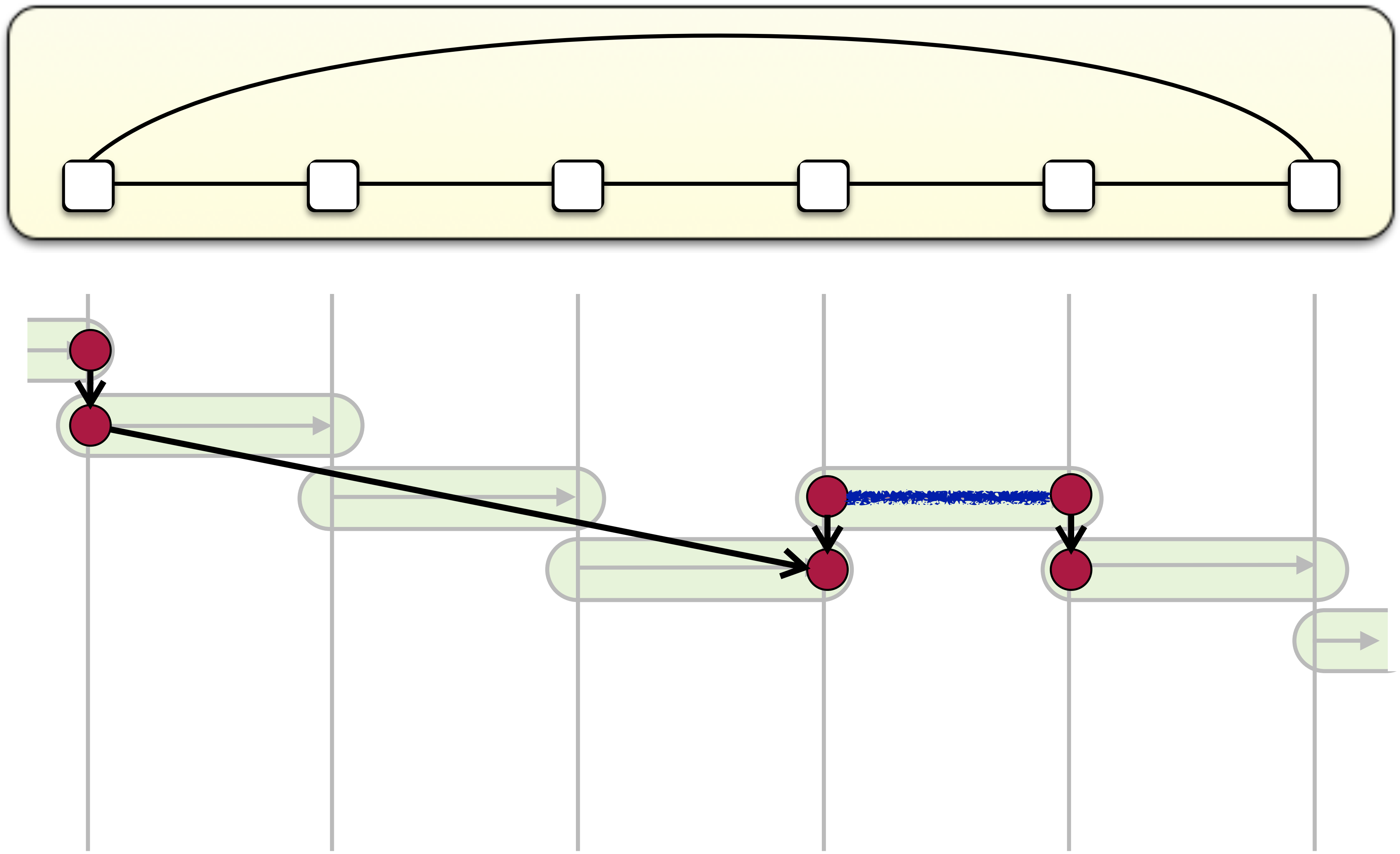

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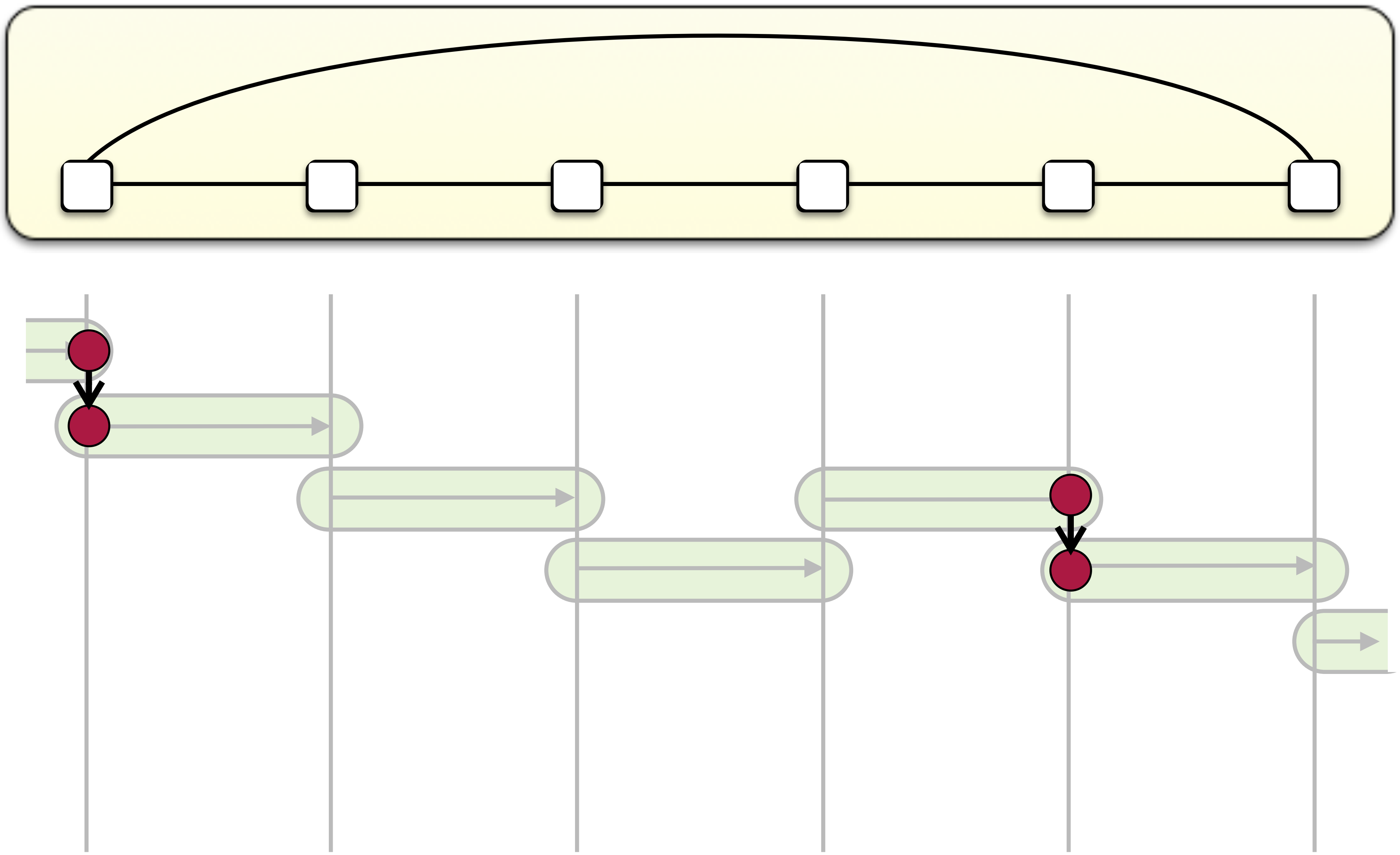

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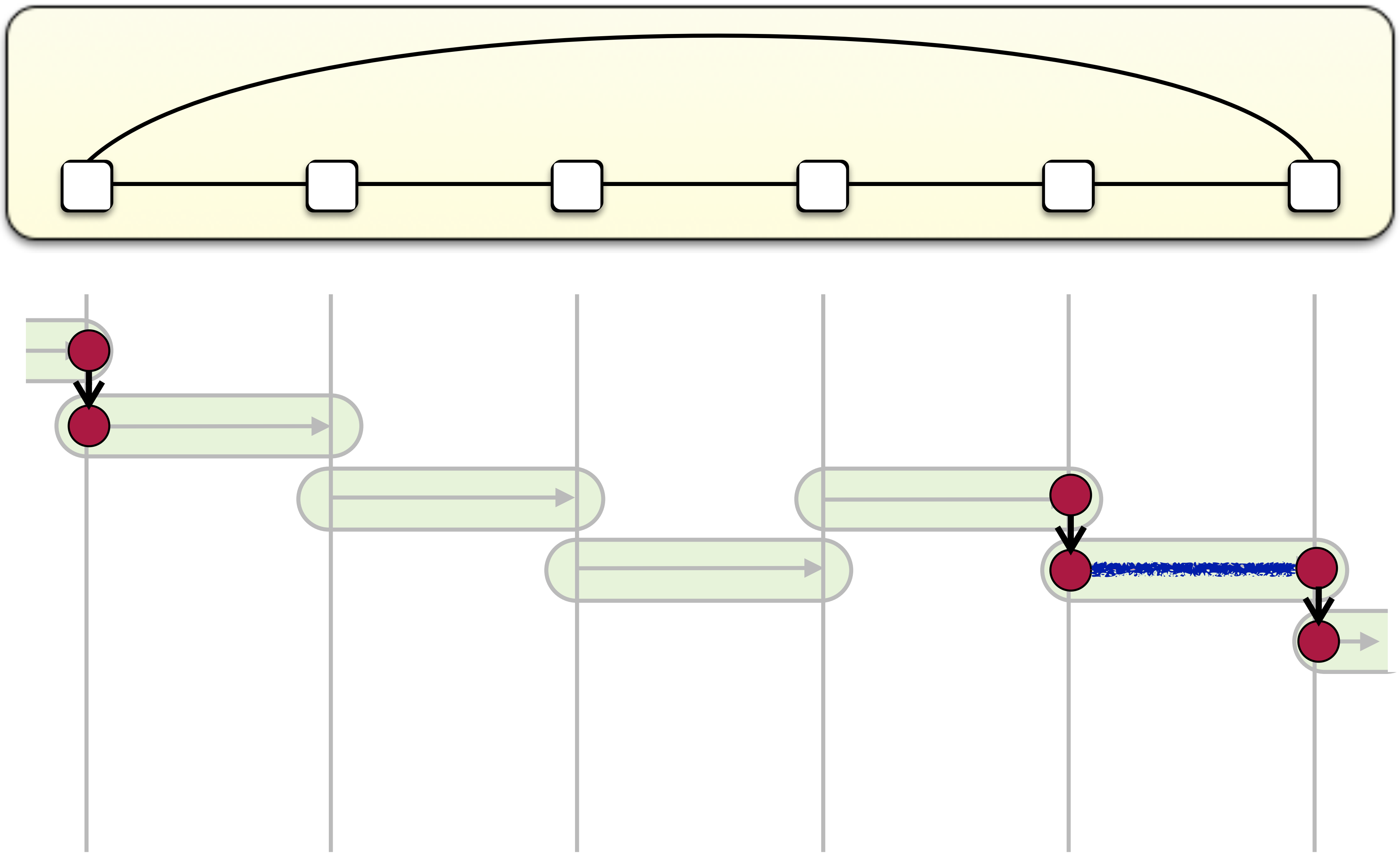

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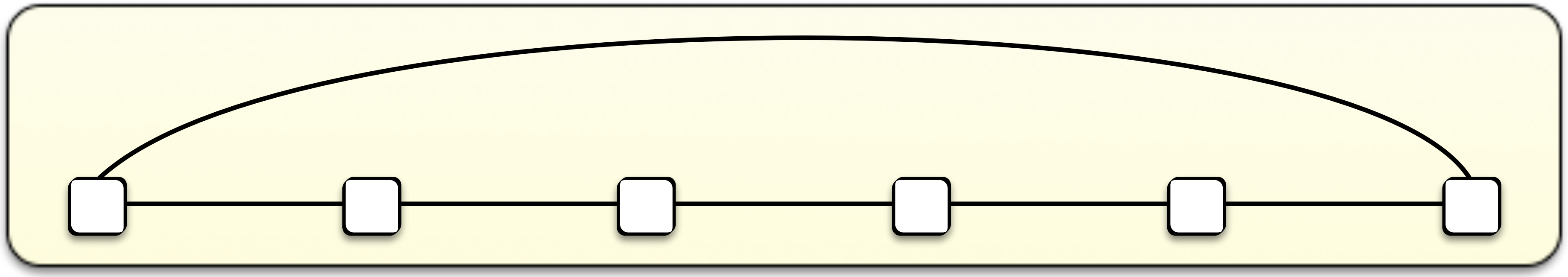

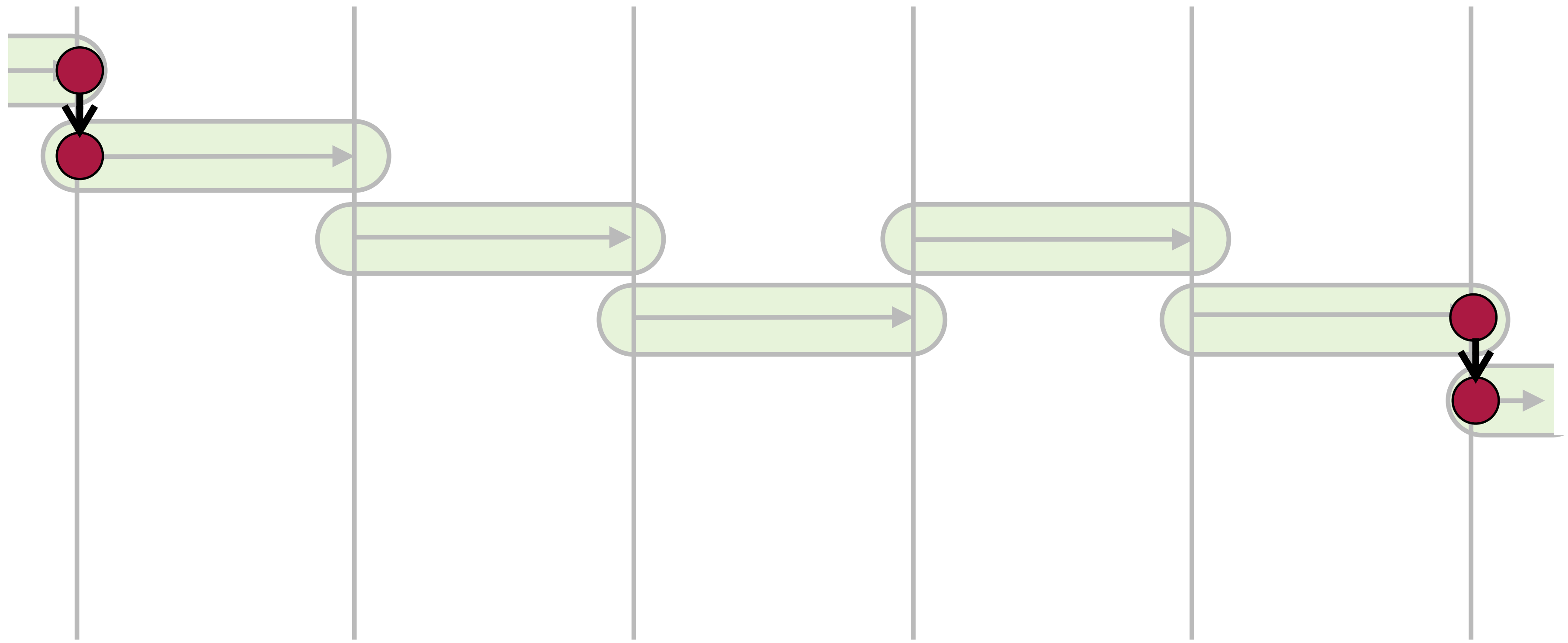

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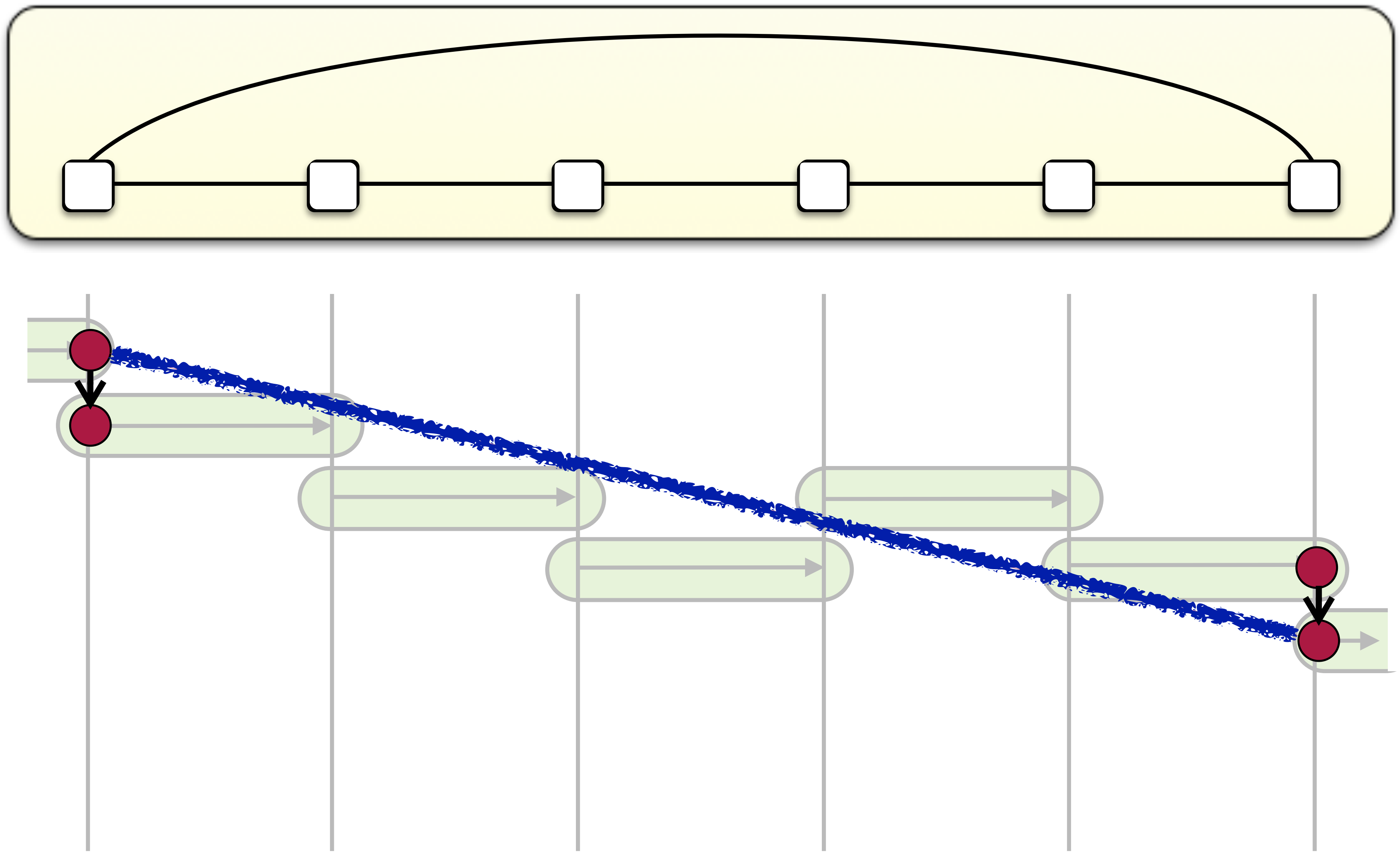

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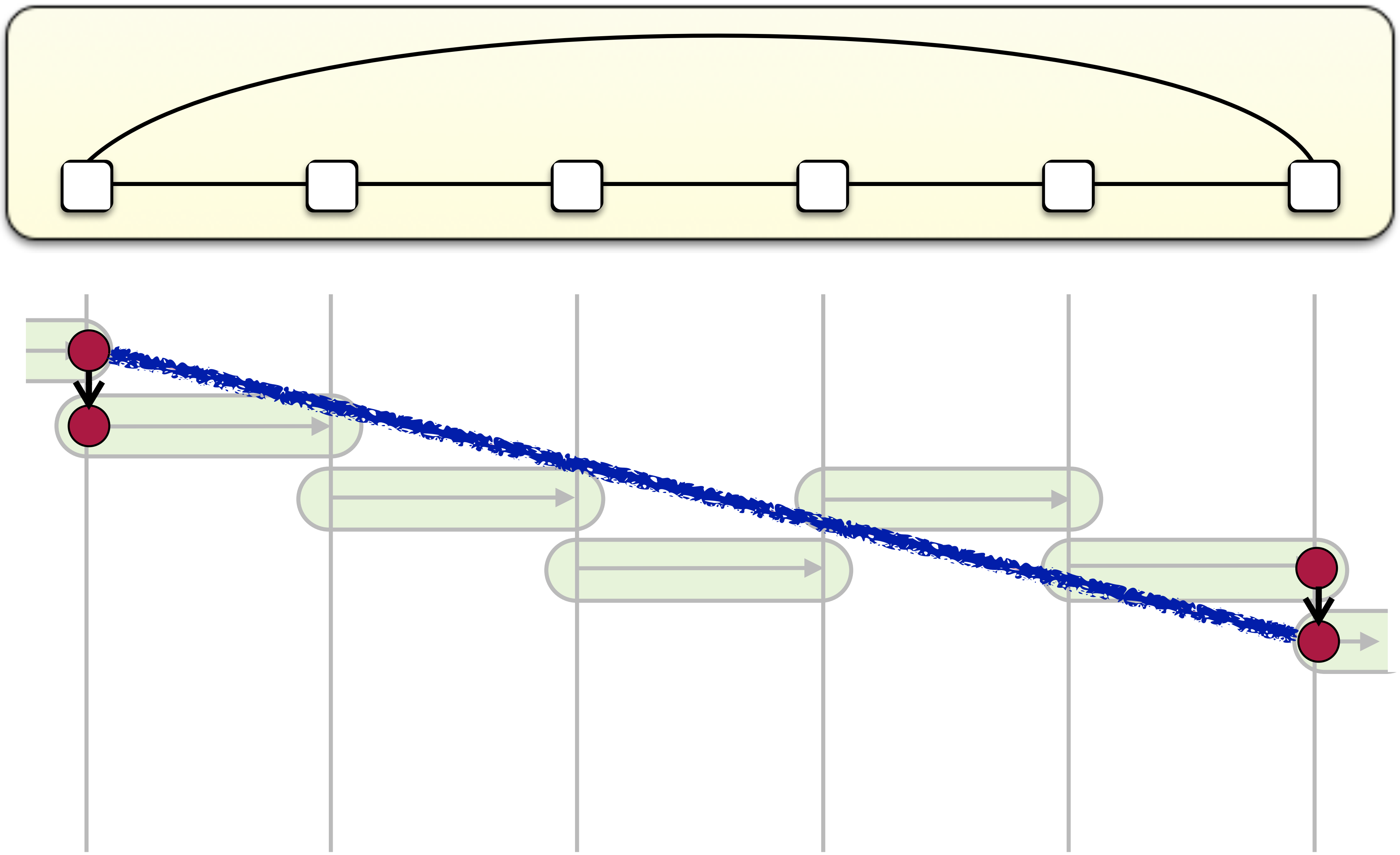

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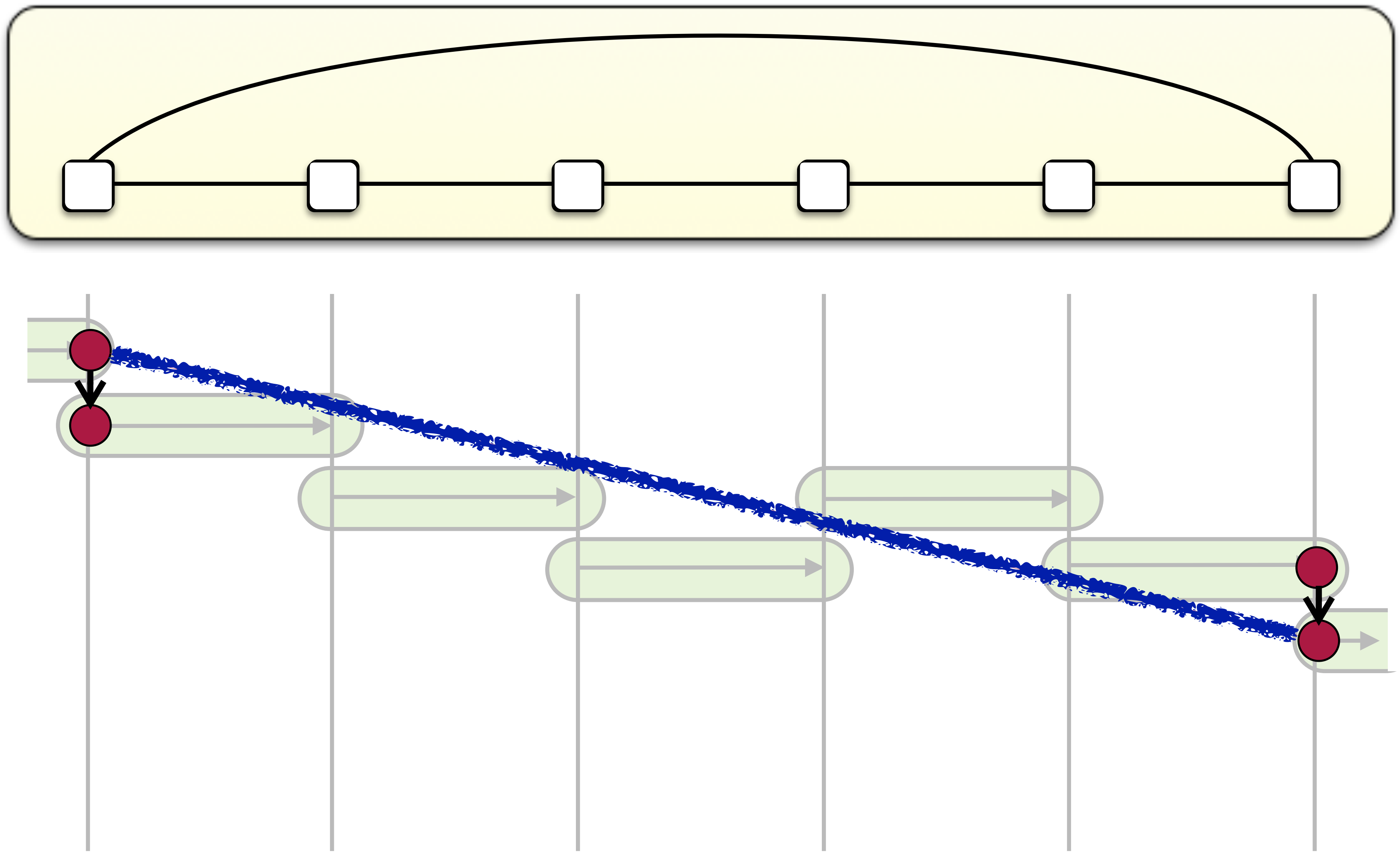

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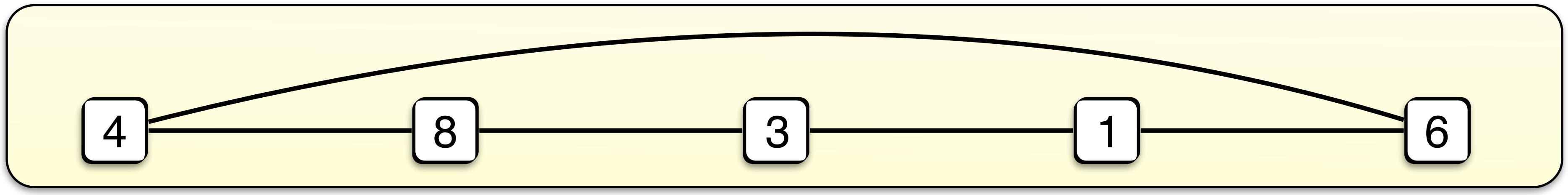

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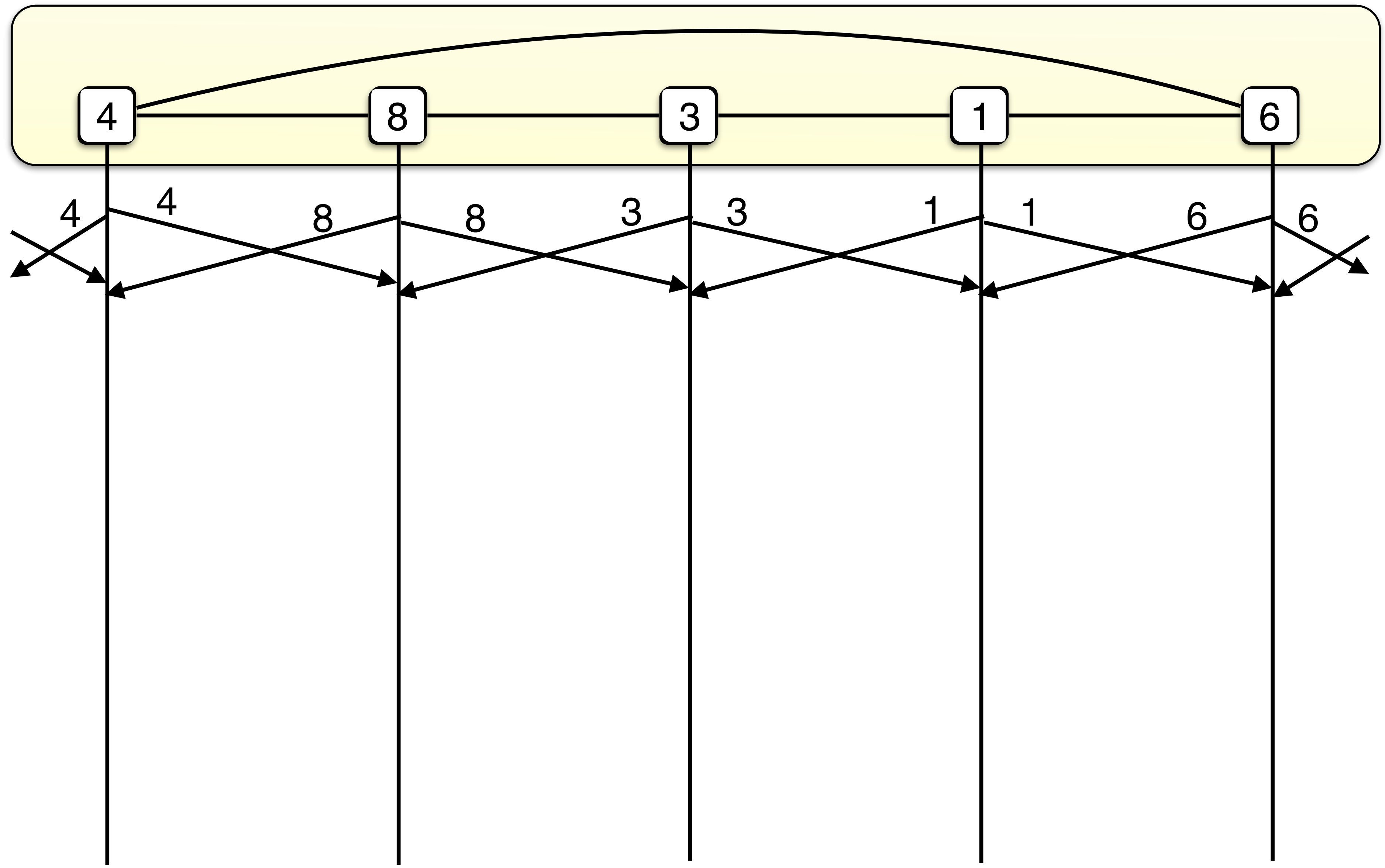

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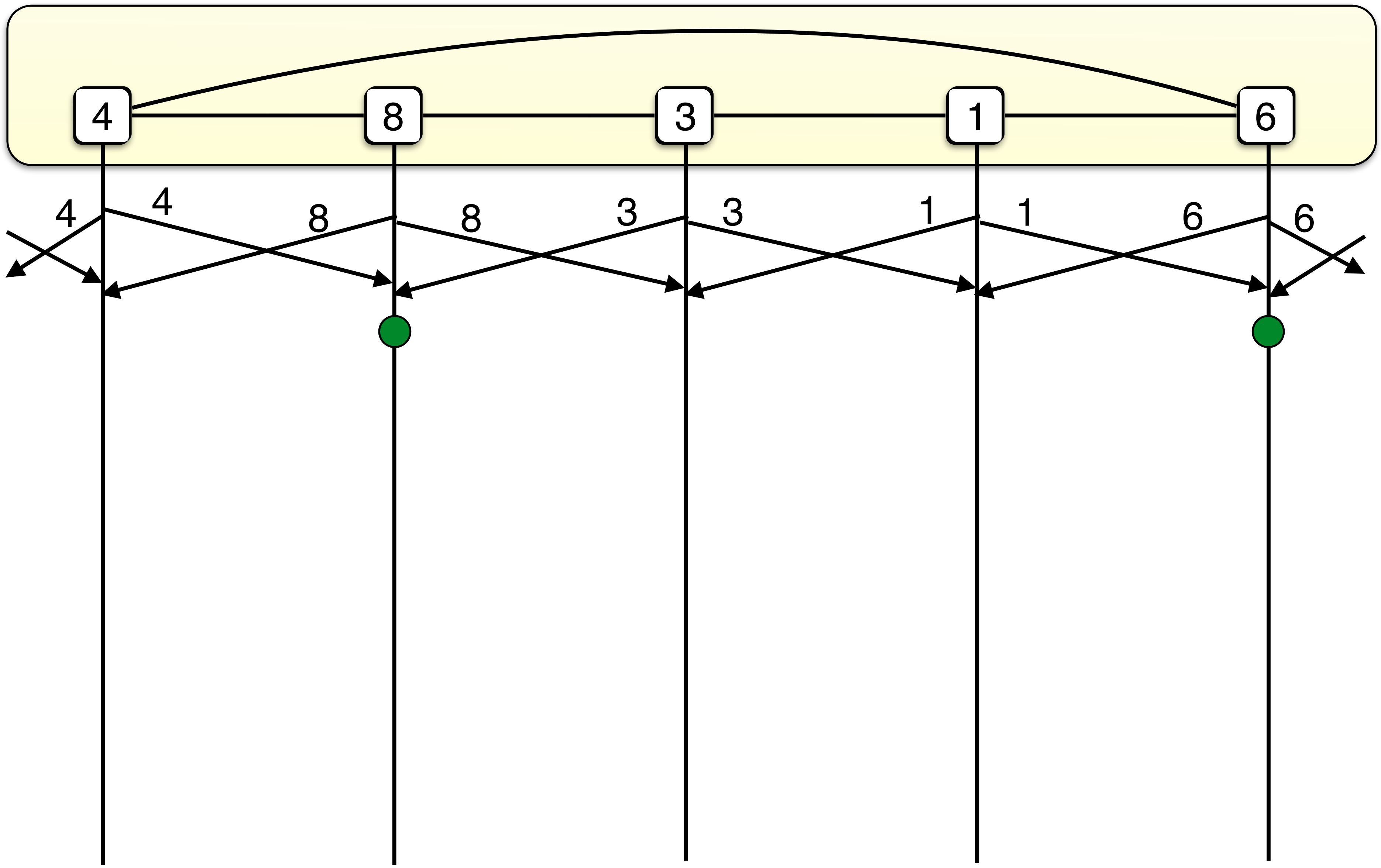

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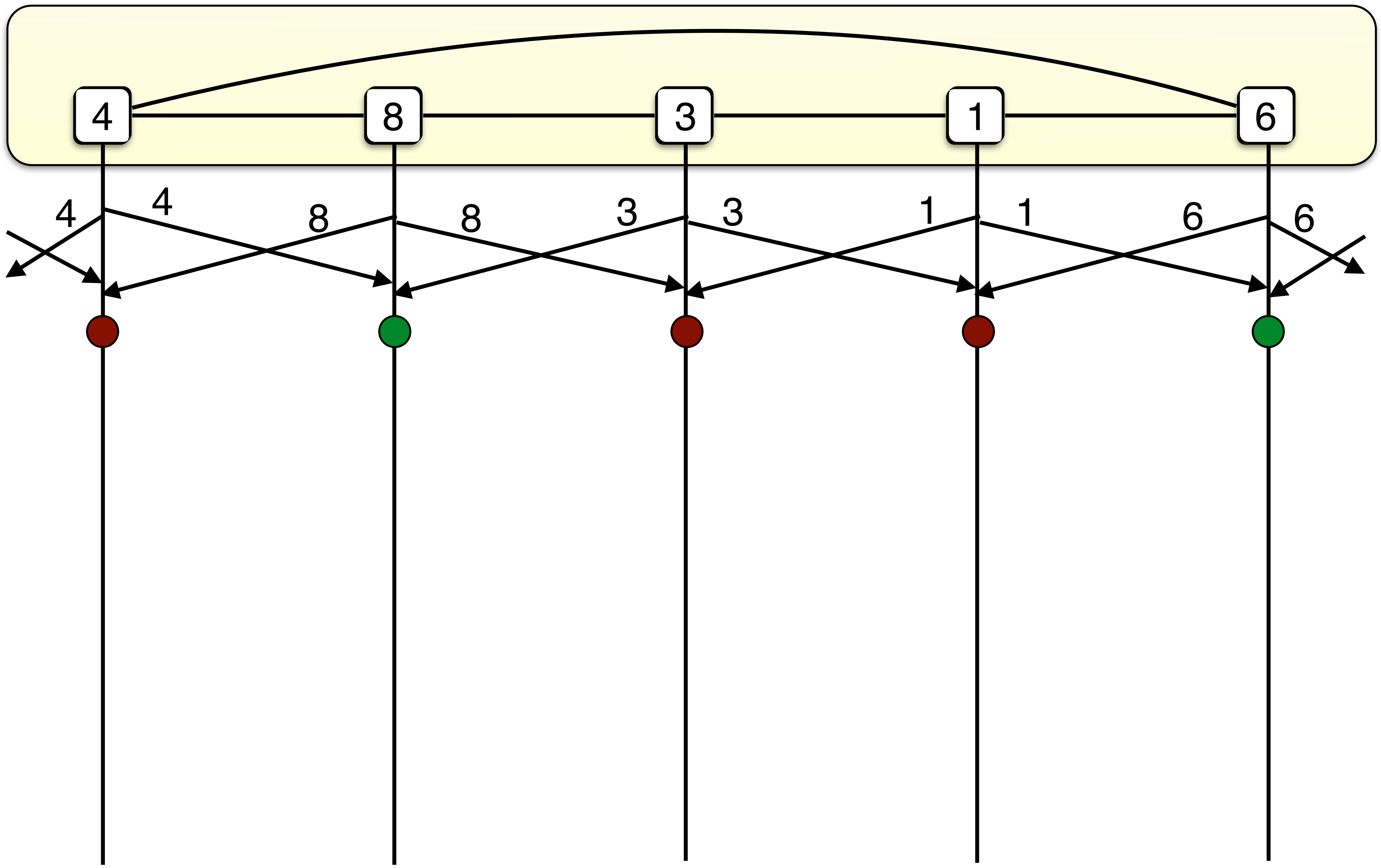

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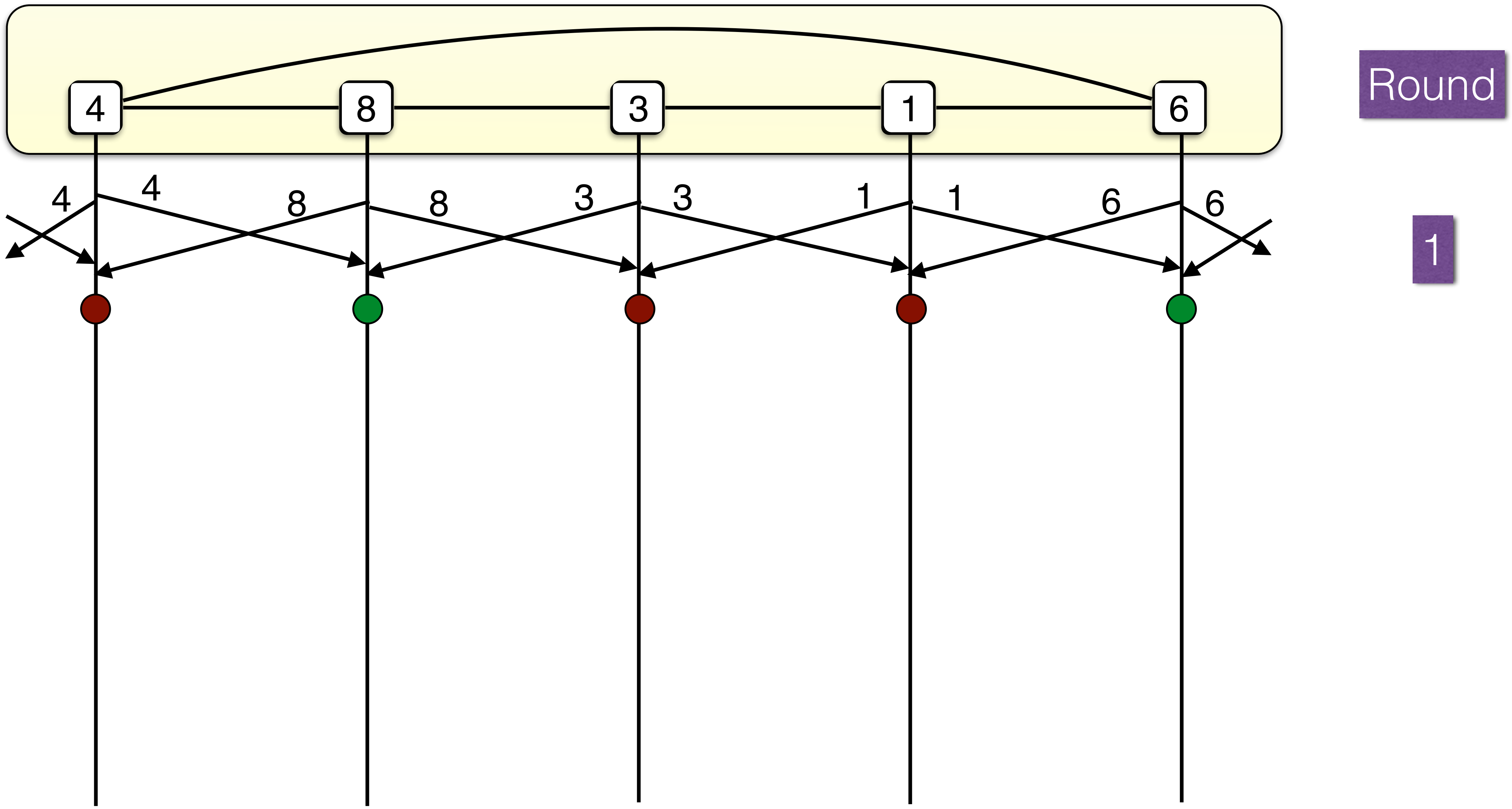

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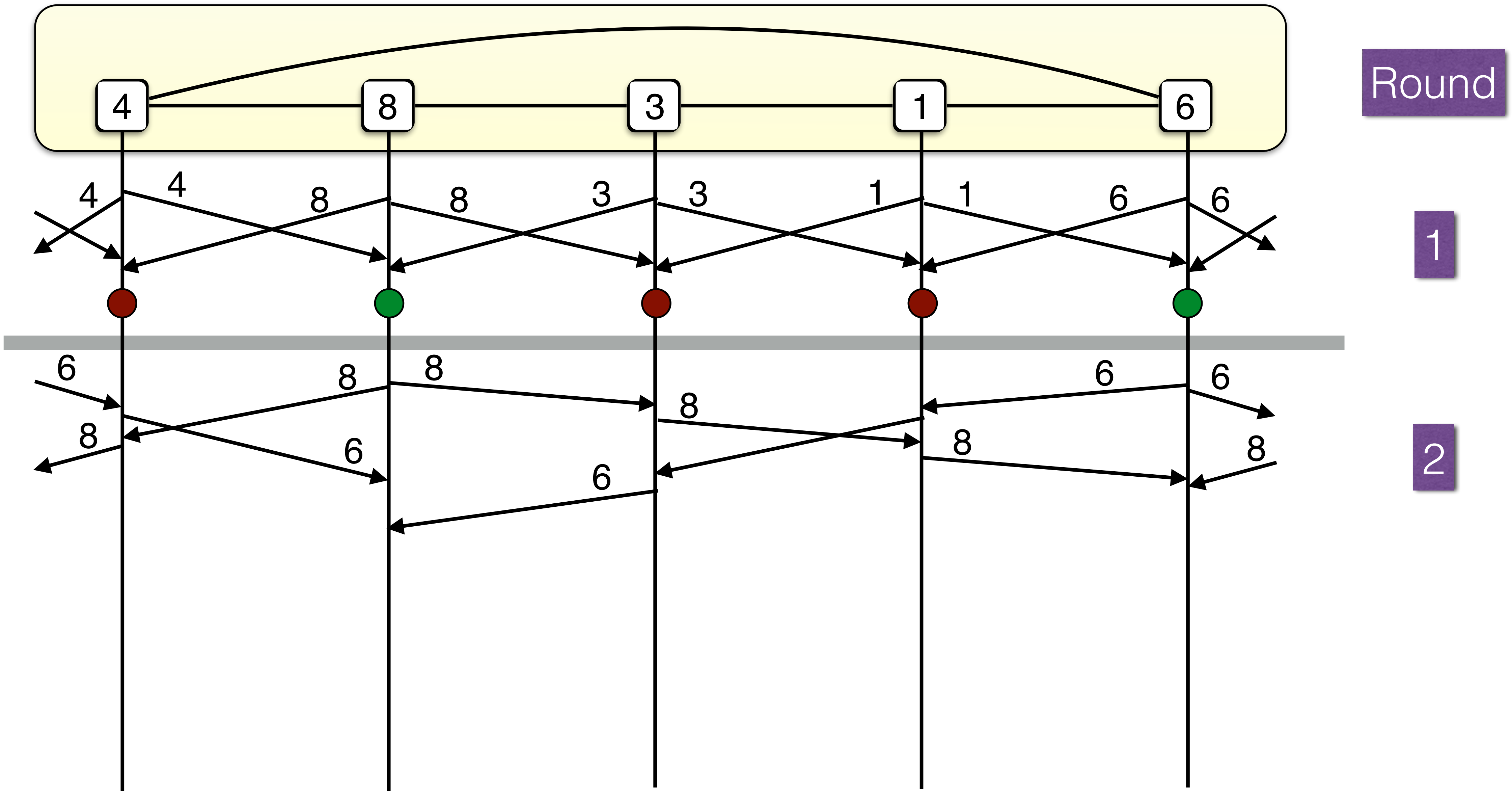

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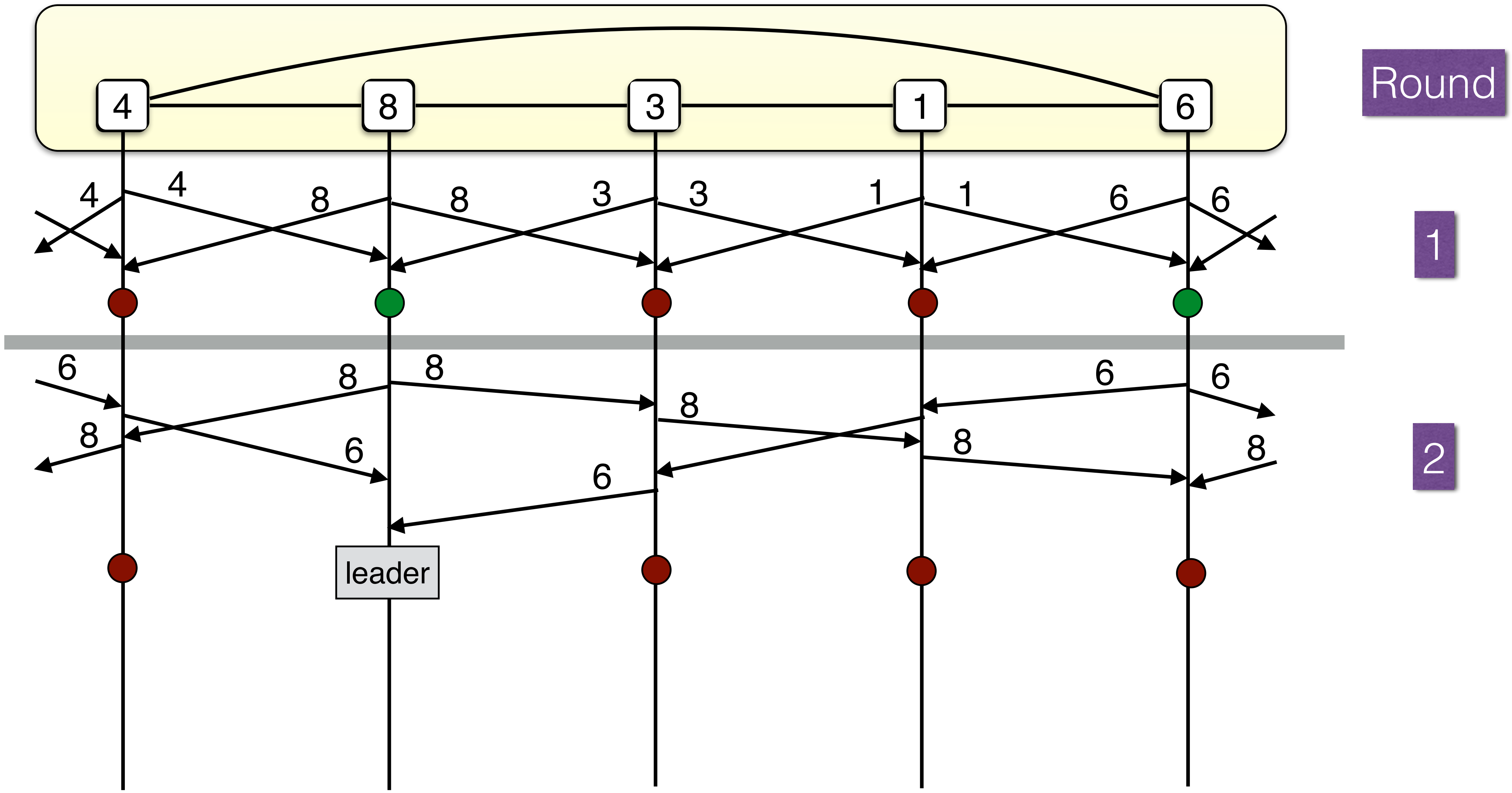

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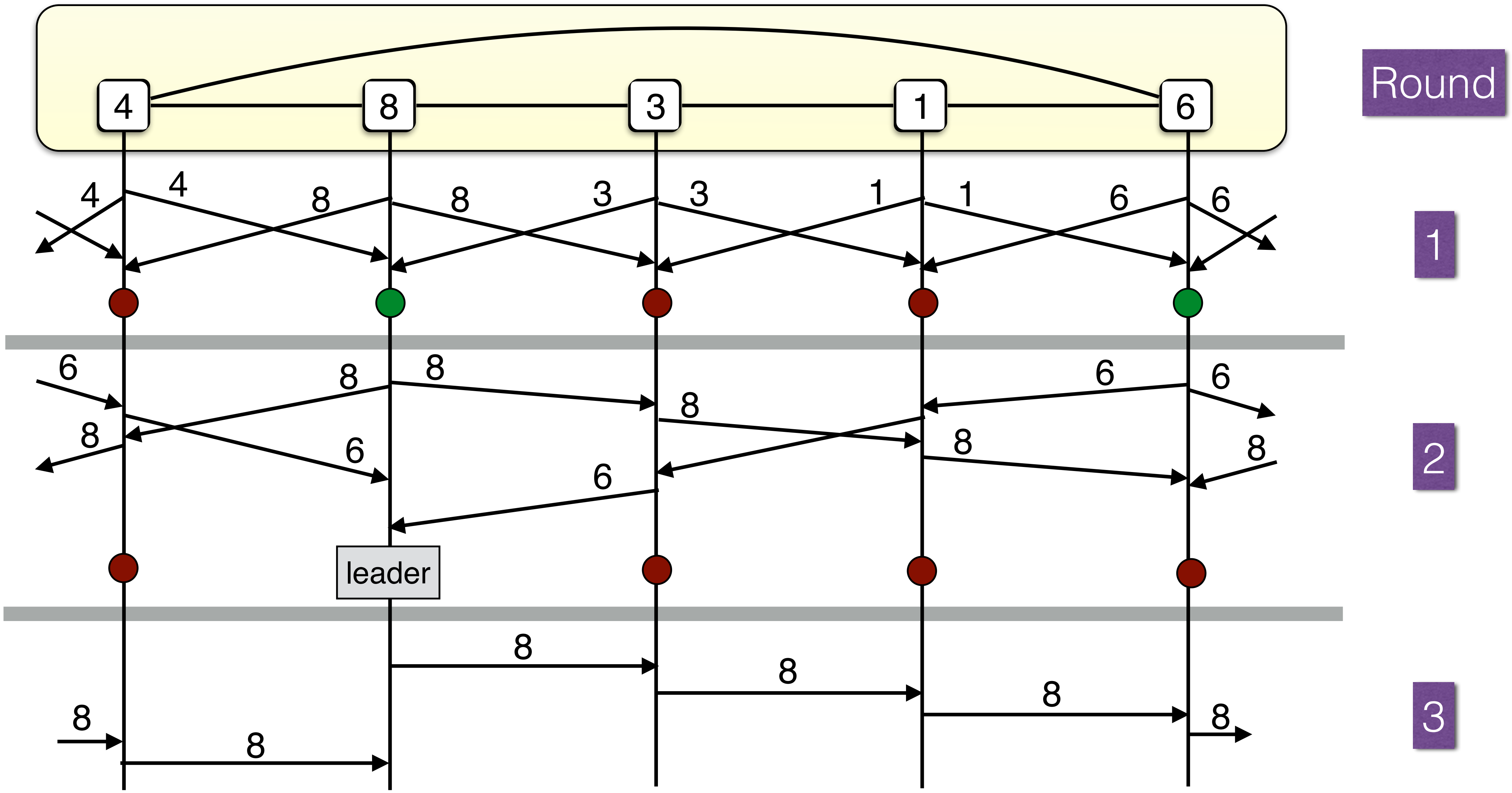

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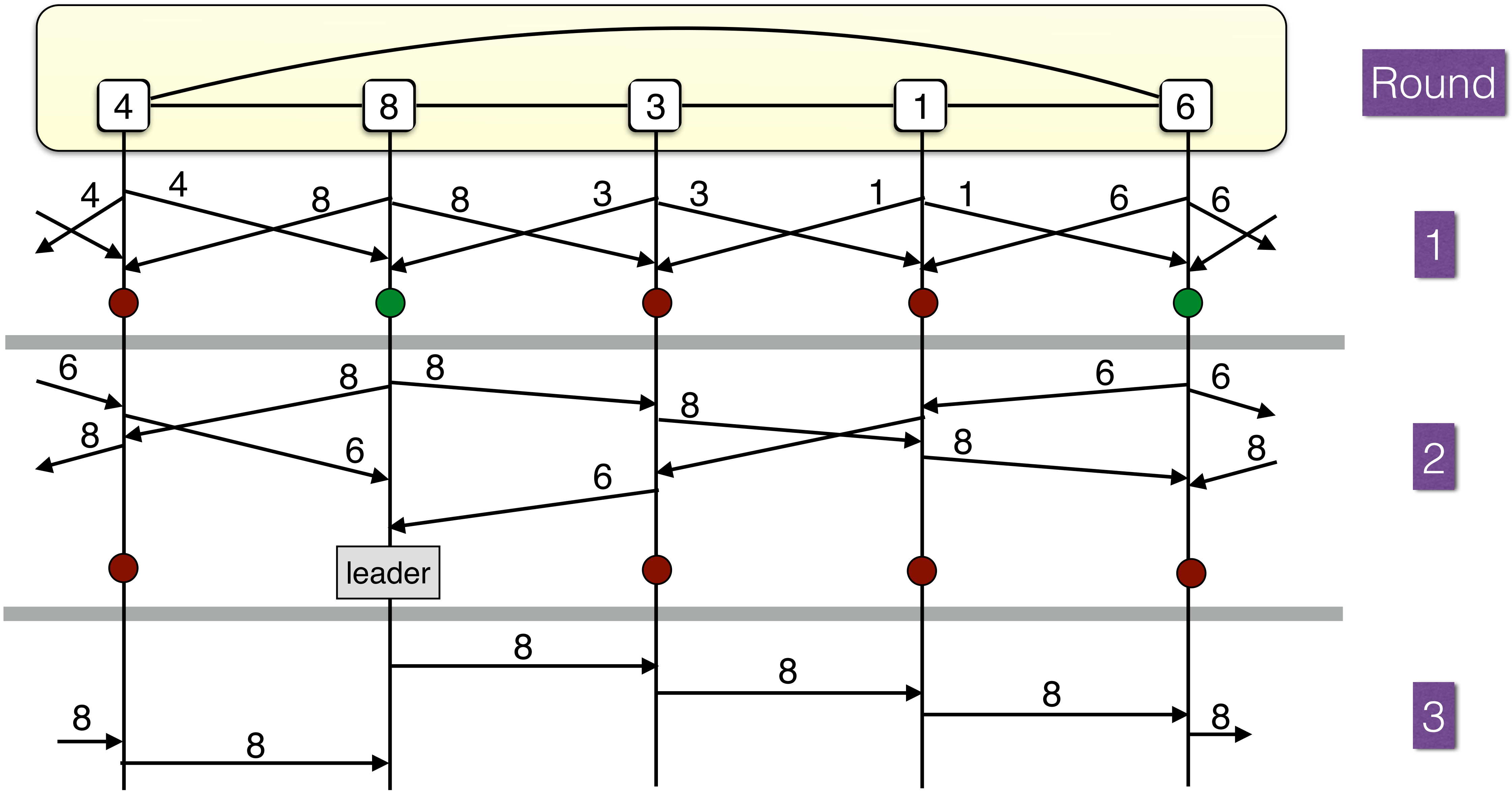

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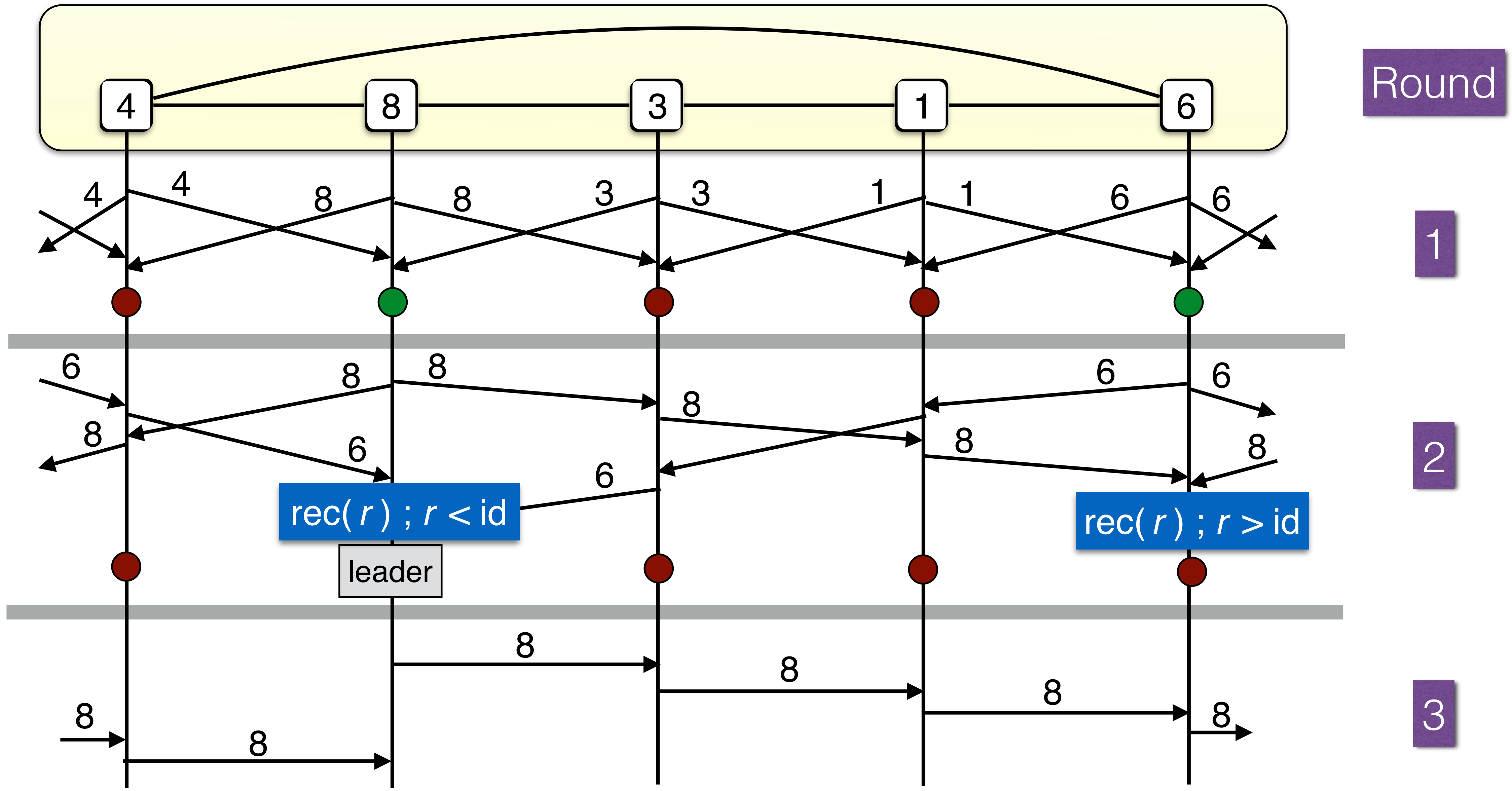

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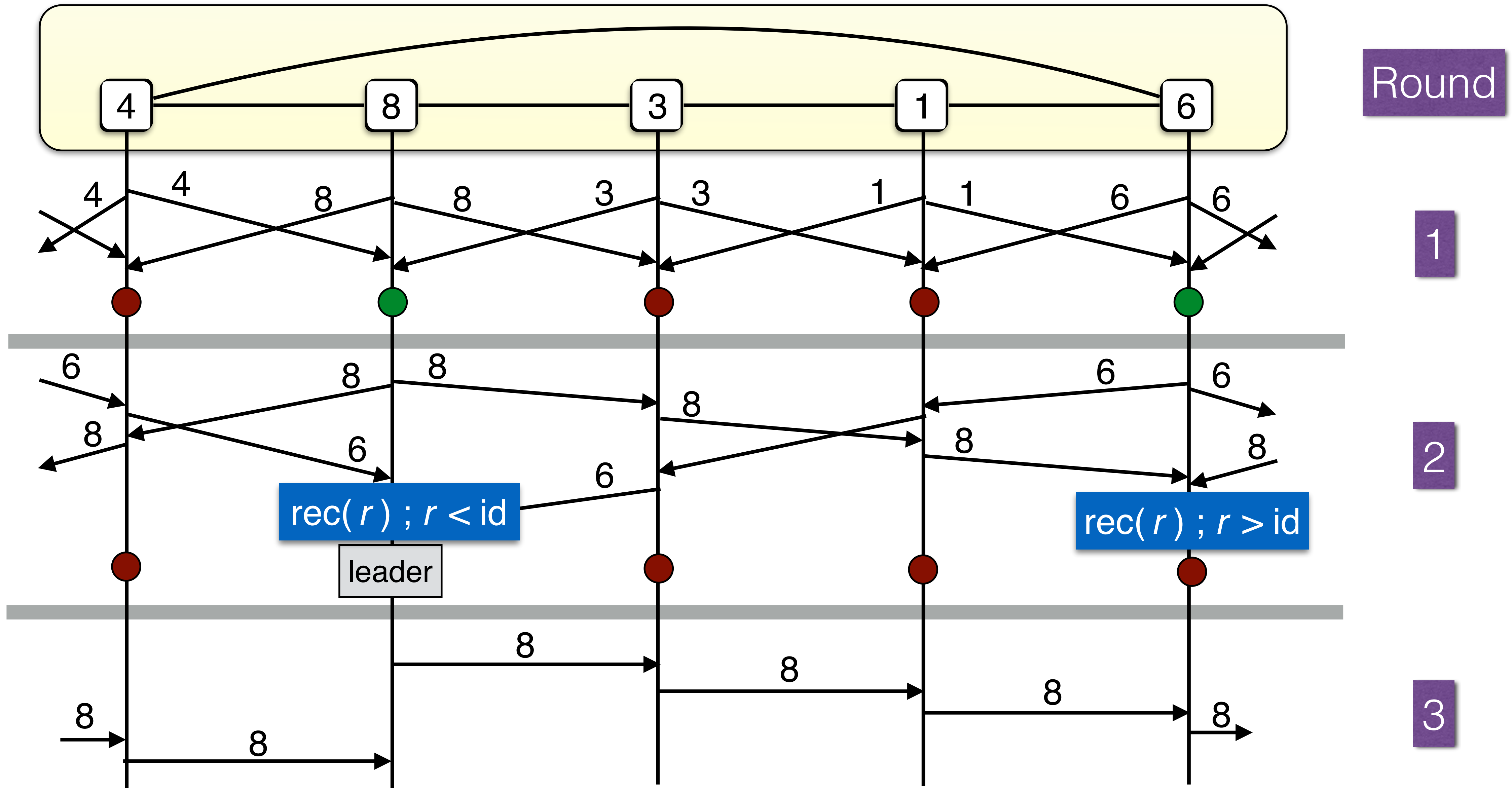

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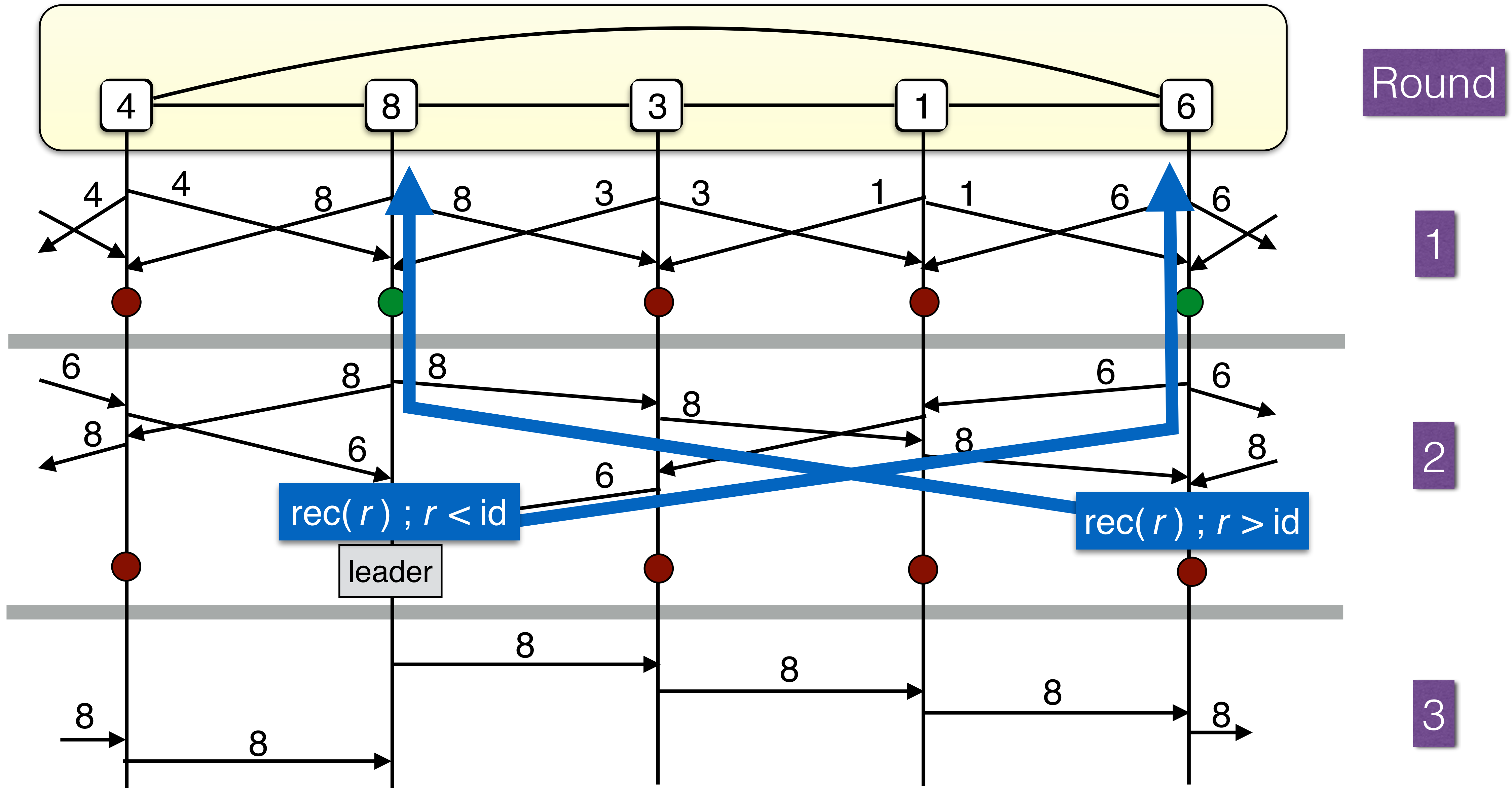

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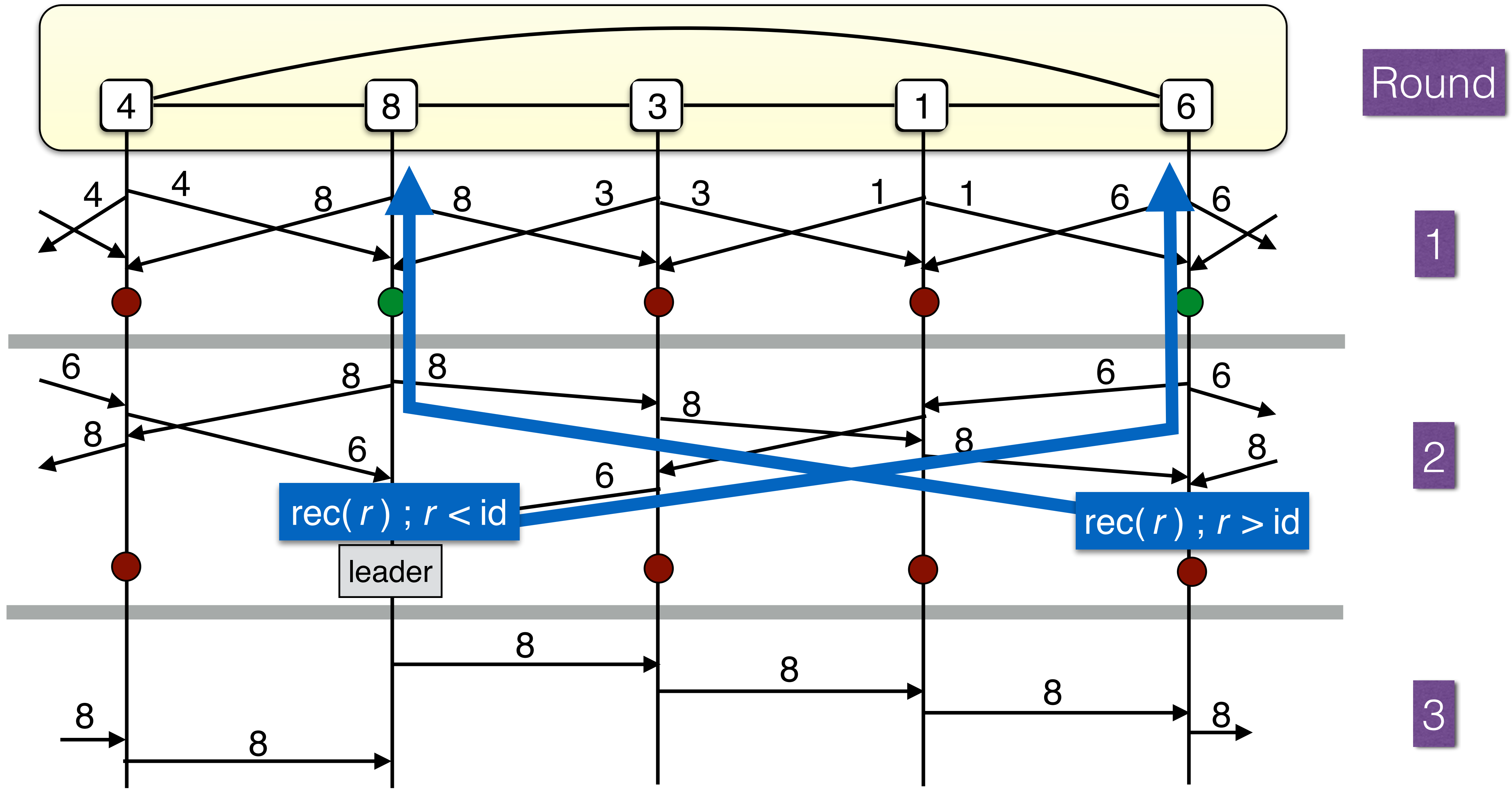

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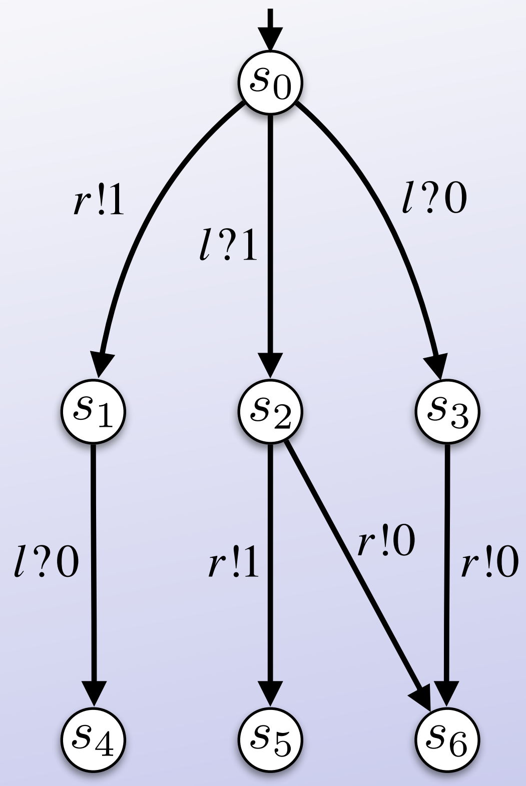
- Number of rounds is sometimes logarithmic in the number of processes.
- MSO can trace back origin of unique process identifiers (pids).
- Underapproximate verification of distributed algorithms that send and compare pids.

Beyond Context Bounds …



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weak PCA

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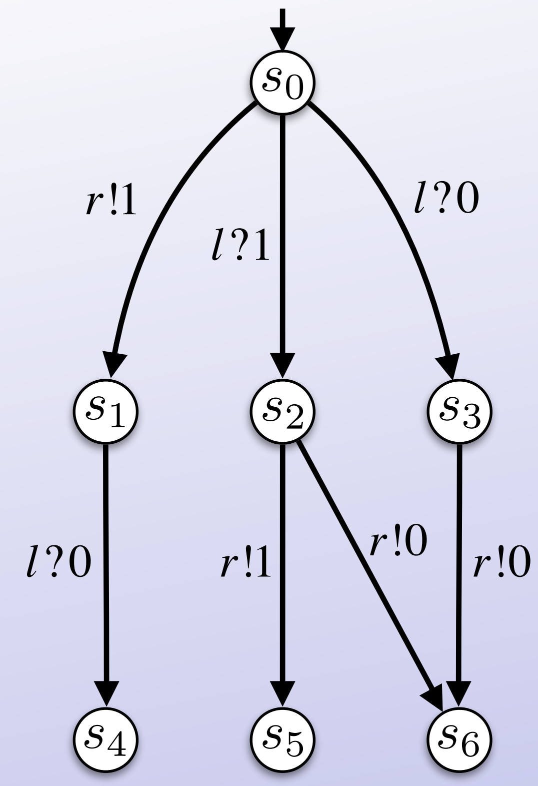

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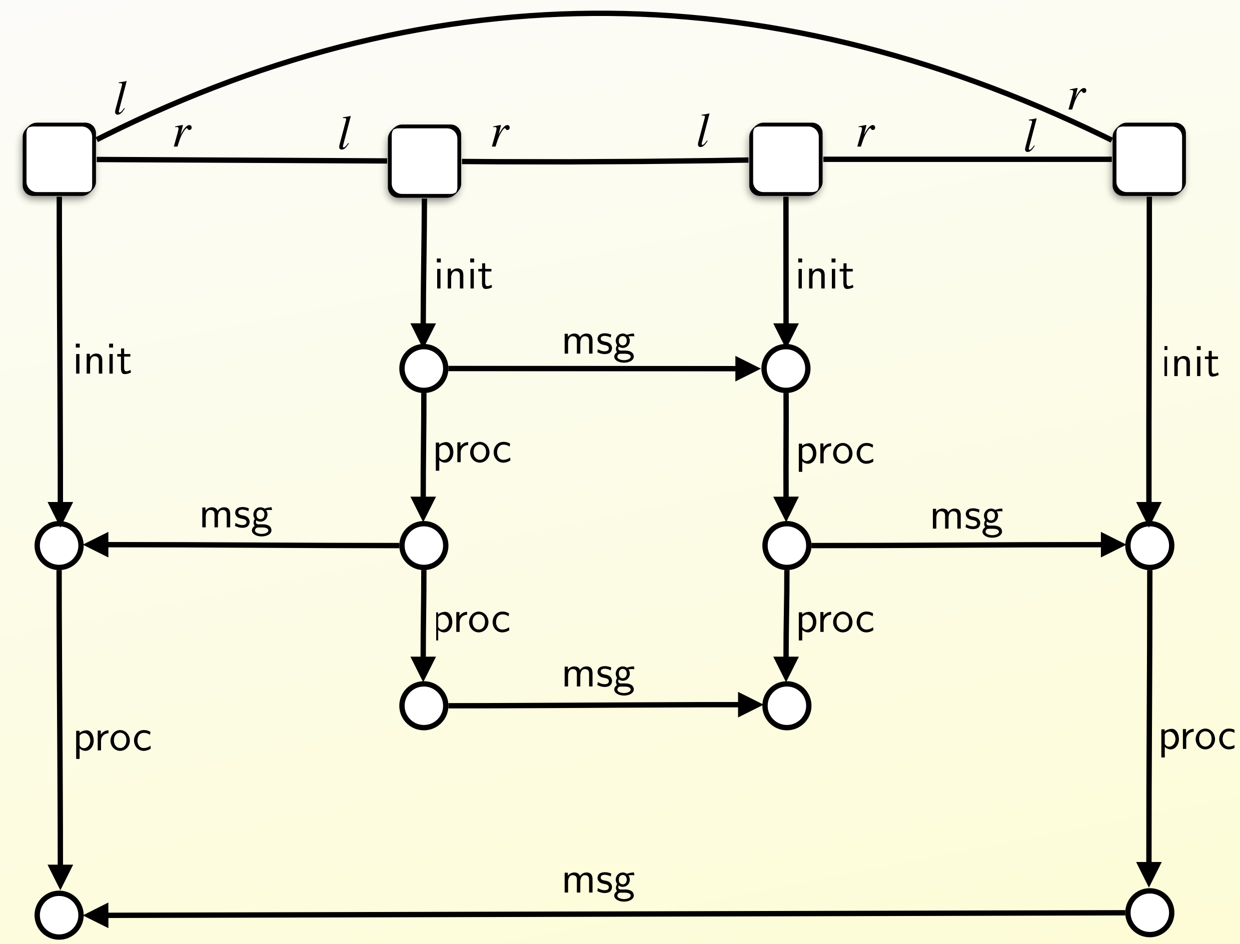

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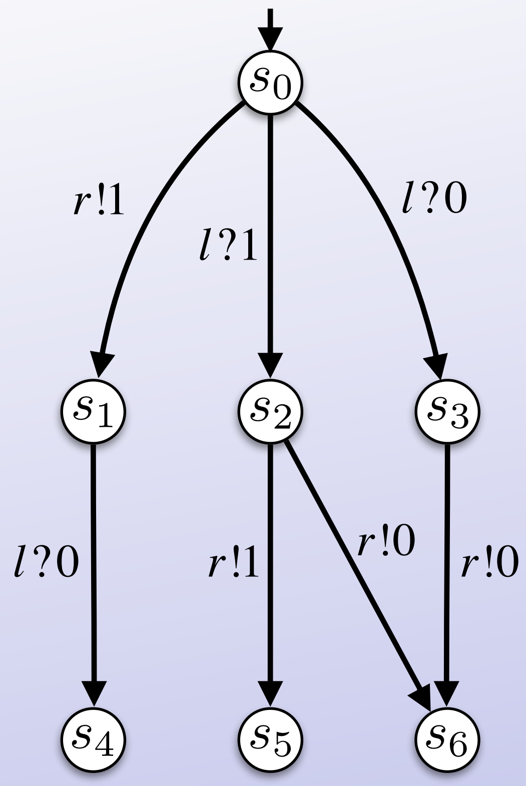

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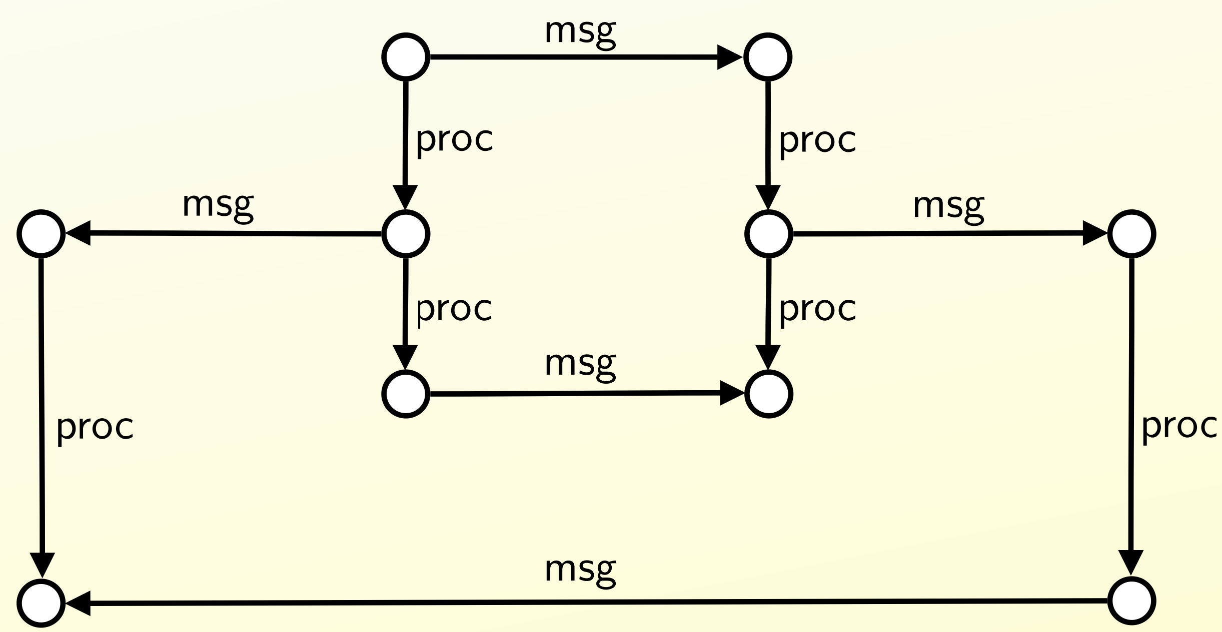

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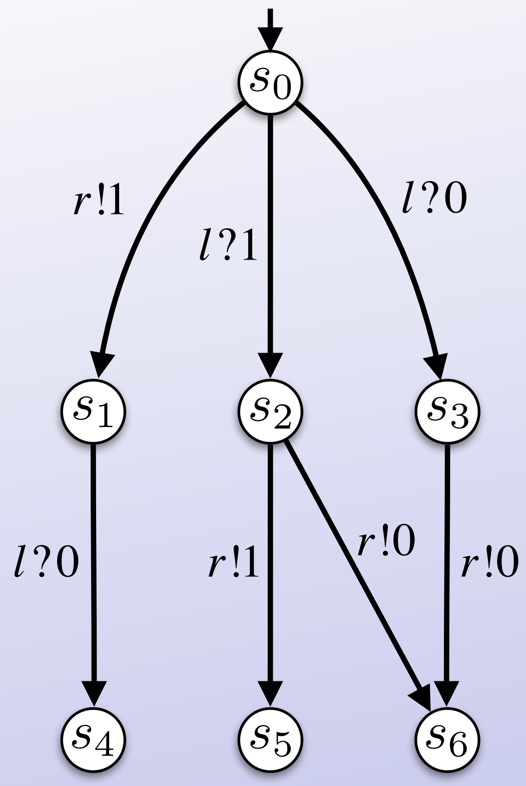

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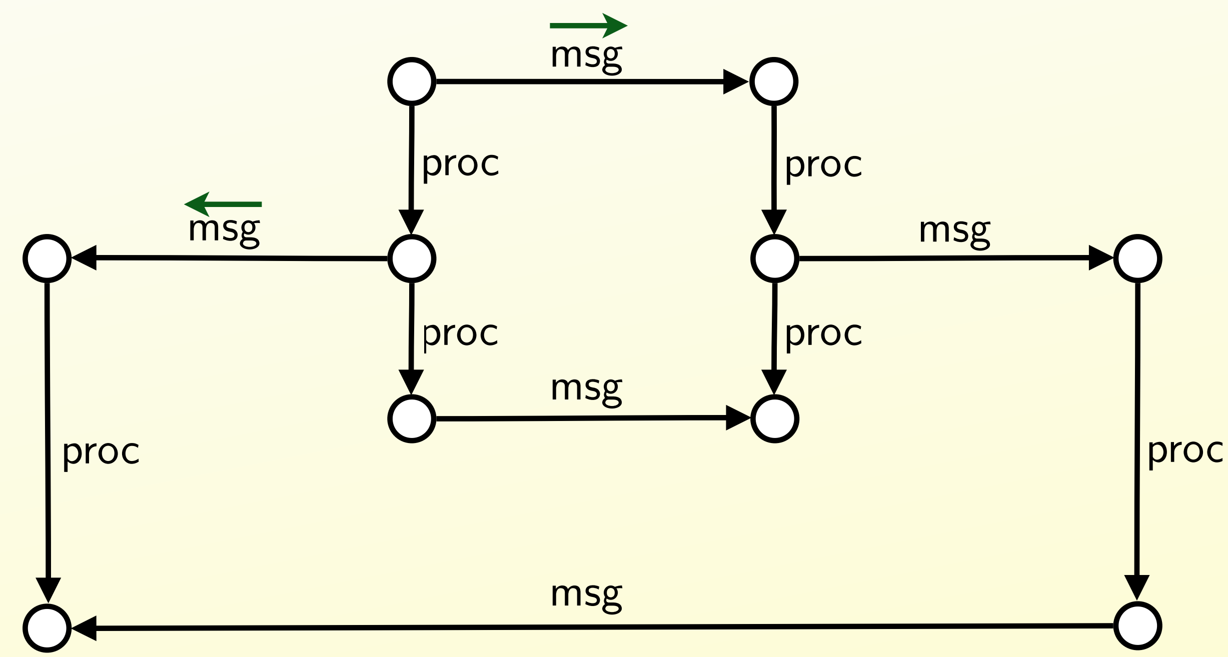

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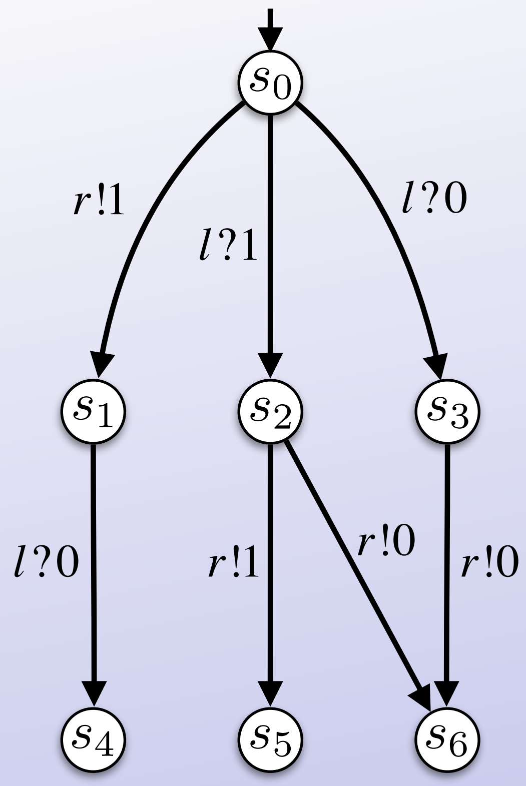

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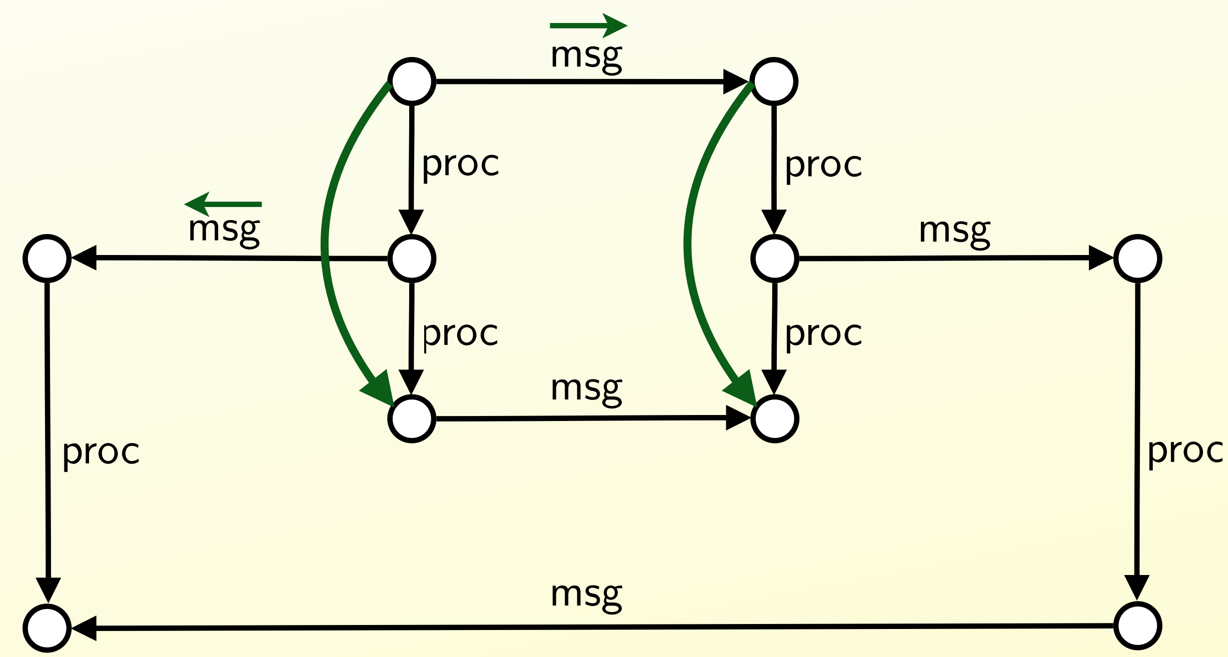

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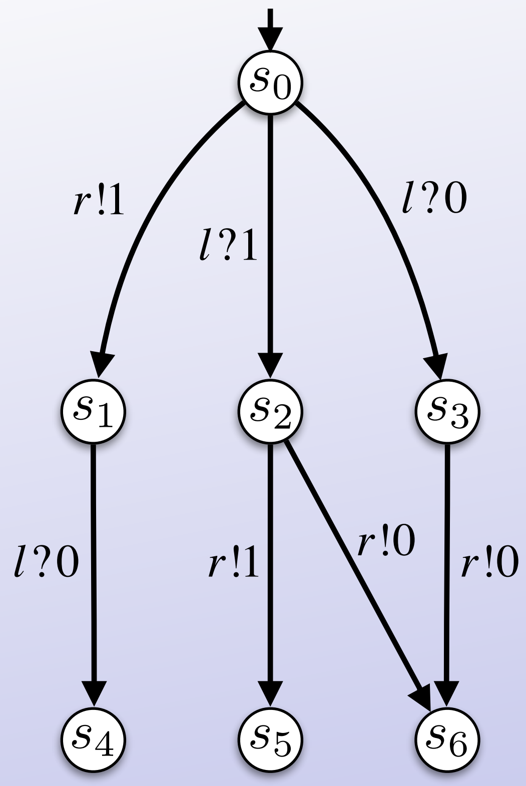

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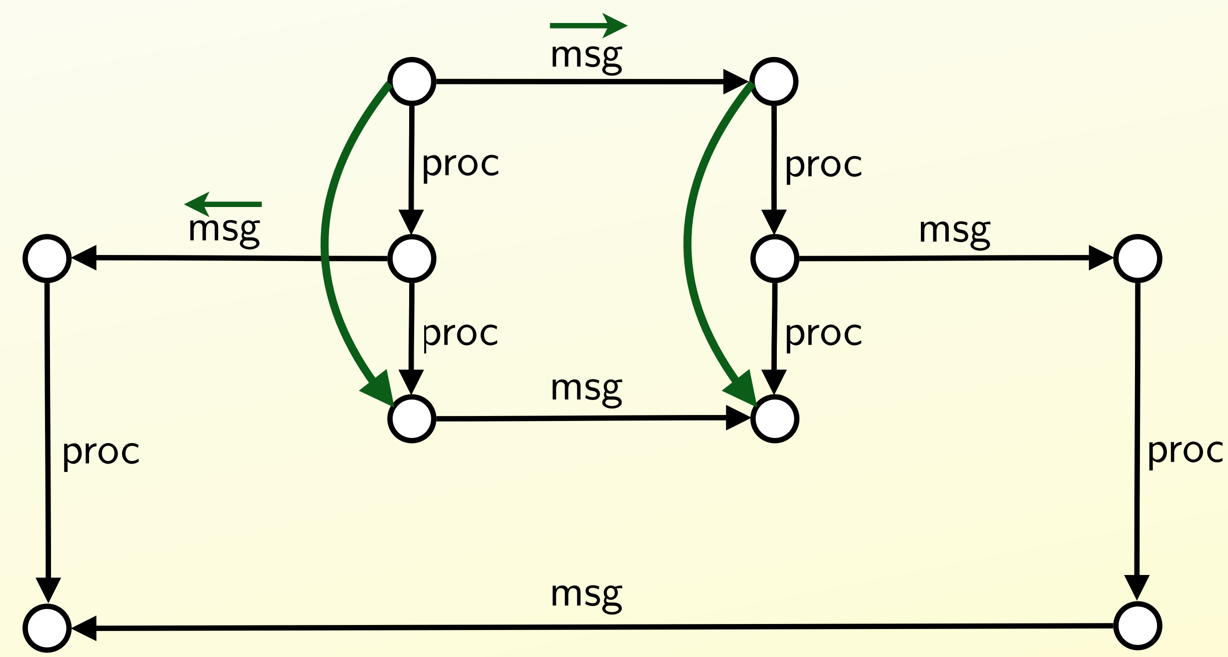

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Let T be any of the following topology classes: rings, grids, binary trees.

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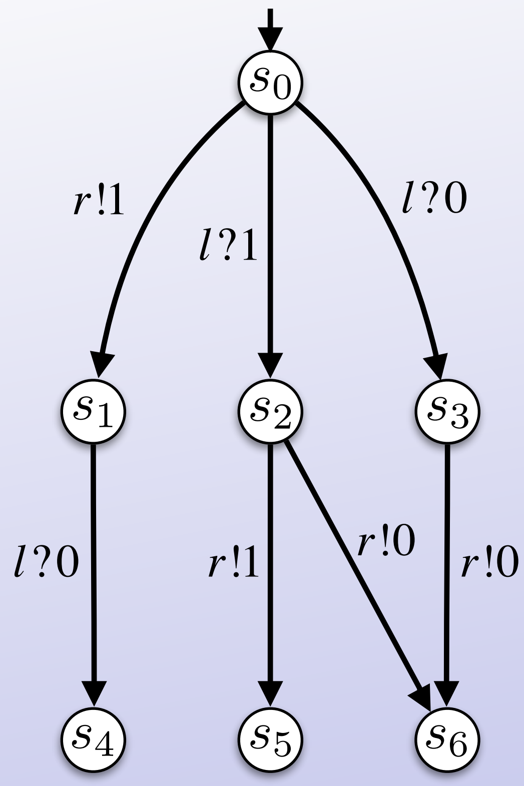

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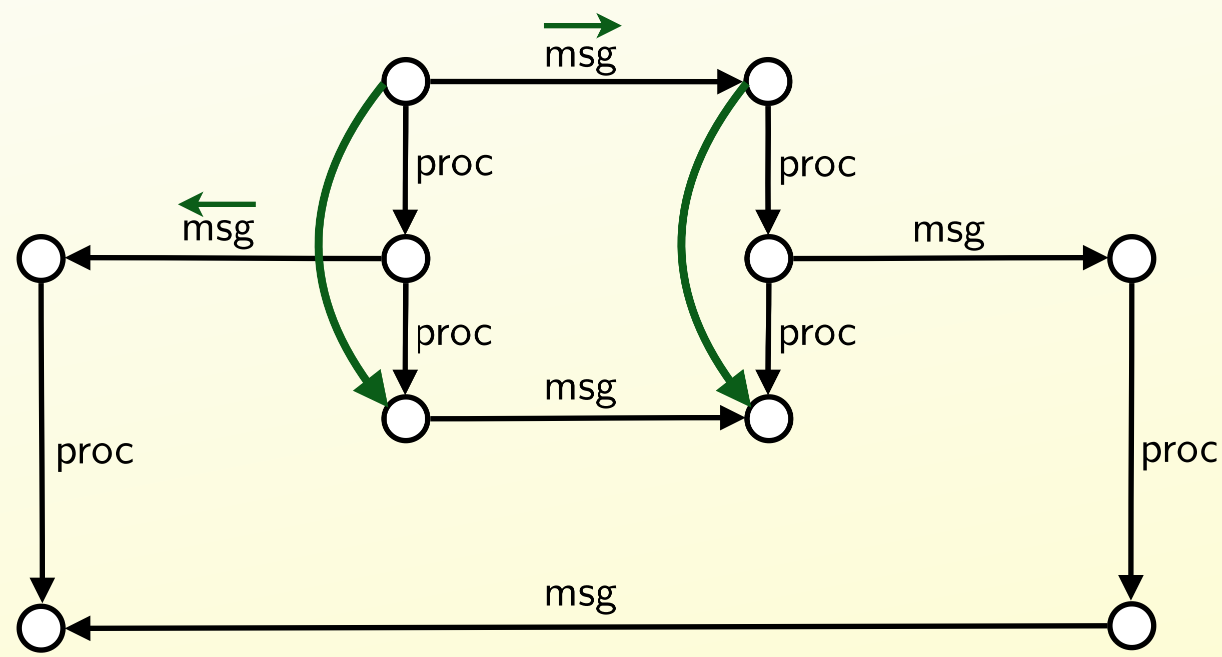

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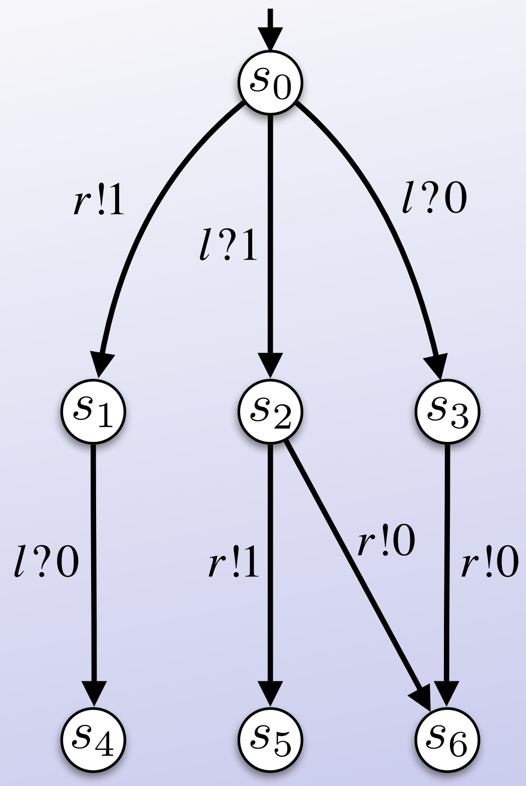

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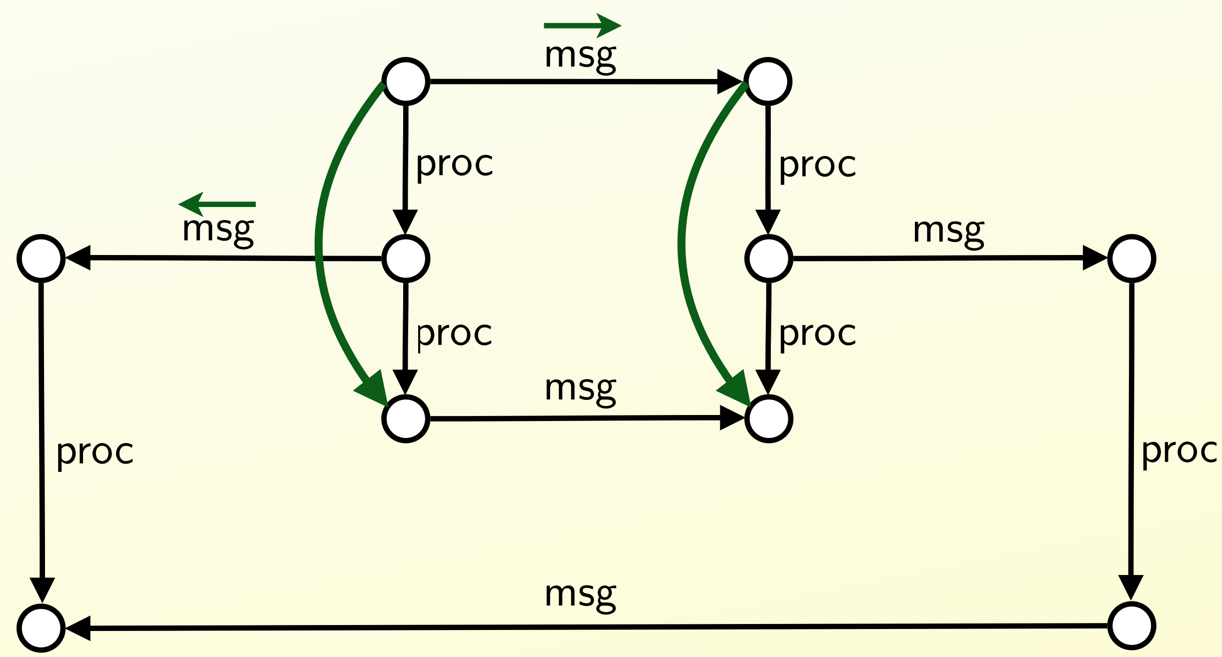

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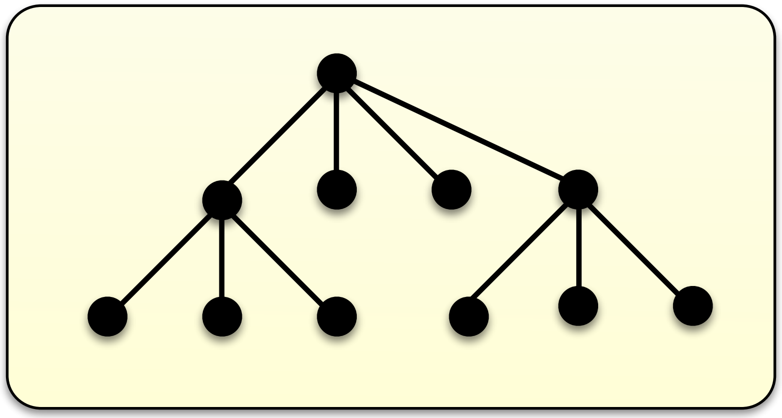
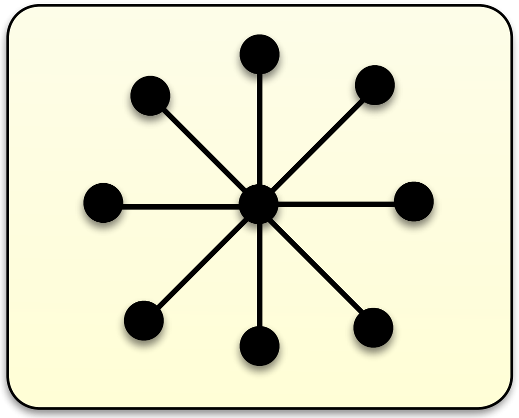
Proof uses [Schwentick-Barthelmann 1999] & [Genest-Kuske-Muscholl 2006].

Other Future Work

- Topologies of unbounded degree (unranked trees, stars, …)





- Temporal logics and efficient model checking

- Split-width for parameterized systems
  [Aiswarya-Gastin-Narayan Kumar 2012]

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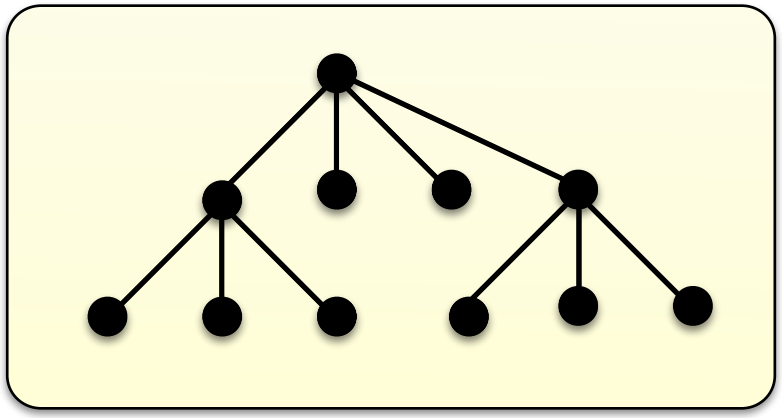

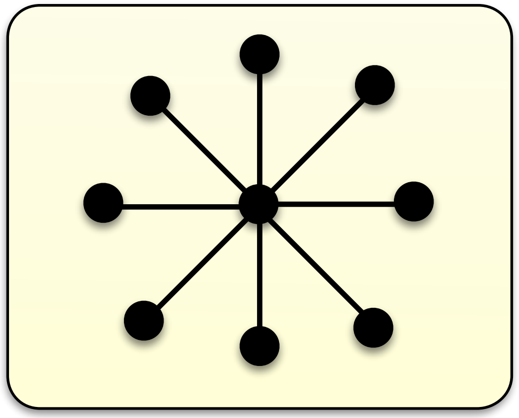

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Thank You!