

**Topics in Topology**  
(Homeworks 7 and 8)  
March 25, 2015

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Due dates

- Homework 7 - April 8, 2015.
  - Homework 8 - April 15, 2015
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Homework 7

1. Compute the twisted cohomology  $H^i(S^1; A_\rho)$  where  $A$  is a module over the group ring of  $\mathbb{Z}$  specified by the map  $\rho : \mathbb{Z} \rightarrow \text{Aut}(A)$ . [15 points]
2. Compute the twisted cohomology  $H^i(\mathbb{R}P^n; \mathbb{Z}_-)$ . (Recall that  $\mathbb{Z}_-$  is the non-trivial  $\mathbb{Z}[\mathbb{Z}/2]$ -module.) [15 points]

Homework 8

1. The 2-sphere  $S^2$  can be covered by a four disks, for example, as the union of northern, southern, eastern and western hemispheres. First, find the nerve of this cover and then compute  $H_*(S^2)$  by setting up the Mayer-Vietoris spectral sequence. Explain all the relevant details. [20 points]
2. Show that the map  $\pi : SO(3) \rightarrow S^2$  given by  $A \mapsto Ae_1$  is a fiber bundle. Identify the fiber as well as the structure group. [10 points]