Topics in Topology (Homework 5) February 9, 2015

- Due date February 18, 2015.
- 1. Prove that a closed manifold of odd dimension has Euler characteristic zero. [5 points]
- 2. Let M be a closed connected, non-orientable 3-manifold and let $H_1(M, \mathbb{Z}) = \mathbb{Z}^r \oplus A$ where A is a finite abelian group. Prove that $r \ge 1$ and also compute the torsion in $H_2(M, \mathbb{Z})$. [10 points]
- 3. Show that if a closed orientable manifold M of dimension 2k has $H_{k-1}(M;\mathbb{Z})$ torsion-free, then $H_k(M,\mathbb{Z})$ is also torsion-free. [5 points]