Topics in Topology
(Homework 2)
January 19, 2015

- Each question is worth 10 points.
- Due date - February 2, 2015.

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1. Suppose M is an n-manifold which is not R-orientable. Then prove that the map $H_n(M;R) \to H_n(M|x;R)$ is injective for every x, with the image given by
$$\{r \in R \mid 2r = 0\}.$$

2. Let M be a closed orientable n-manifold with a Δ-complex structure. Let $\{\sigma_1 \dots, \sigma_k\}$ be the set of all n-simplices. Then prove that the fundamental class $[M]$ can be represented by (the cycle) $\sum_{i=1}^{k} \sigma_i$ in simplicial homology.

3. For a map $f\colon M \to N$ between connected closed orientable n-manifolds with fundamental classes $[M]$ and $[N]$, define the *degree* of f to be the integer d such that
$$f_*([M]) = d[N].$$
Show that there always exists a degree 1 map from $M \to S^n$.