Topics in Topology
(Homework 1)
January 5, 2015

- Each question is worth 10 points.
- Due date - January 19, 2015.

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1. Prove that a topological manifold M is connected if and only if it is path-connected.

   Define an equivalence relation on $\mathbb{R}^{n+1} \setminus \{\mathbf{0}\}$ by

   $$x \sim y \iff y = tx, t \in \mathbb{R} - \{0\}.$$

   The n-dimensional, real projective space $\mathbb{RP}^n$ is the quotient of $\mathbb{R}^{n+1}$ by the above equivalence relation.

2. Prove that $\mathbb{RP}^n$ is second countable (Hint: if the quotient map from a second countable space is open then the quotient is also open.)

3. Prove that $\mathbb{RP}^n$ is Hausdorff. (Hint: this is equivalent to saying that the graph of the equivalence relation is closed.)

4. Prove that $\mathbb{RP}^n$ is an n-manifold. (Hint: for each i consider the subset containing elements with ith coordinate nonzero.)

5. Let Y denote the quotient space obtained by identifying the antipodal points of S^{n+1}. Prove that the map $f : \mathbb{R}^{n+1} \setminus \{\mathbf{0}\} \to S^n$ given by $x \mapsto x/\|x\|$ induces a homeomorphism $\overline{f} : \mathbb{RP}^n \to Y$.