Introduction to Manifolds

Assignment 3
Due Date: 06/09/2018

Problem 1: Let X be a finite-dimensional (abstract) smooth manifold. Show that with respect to the manifold topology on X the coordinate maps $\phi_\alpha : U_\alpha \to \phi_\alpha(U_\alpha)$ are homeomorphisms.

Problem 2: Show that it is not possible to cover the unit d-sphere in $\mathbb{R}^{d+1}$ by a single coordinate chart.

Problem 3: Let $B_a = \{x \in \mathbb{R}^n \mid \|x\|^2 < a^2\}$ be an open ball of radius a in $\mathbb{R}^n$. Show that the map

$$x \mapsto \frac{ax}{\sqrt{a^2 - \|x\|^2}}$$

is a diffeomorphism of B_a onto $\mathbb{R}^n$.

Problem 4: Prove that the union of the two coordinate axes in the Euclidean plane is not a manifold.

Problem 5: Let M and N be two smooth manifolds of dimension m and n respectively. Prove that the cartesian product $M \times N$ is a smooth manifold of dimension $m + n$.