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## Introduction to Manifolds

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### Assignment 8

Due Date: 20/11/2017

**Problem 1:** Let  $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by

$$F(x, y) = (x^2 + y^2, xy) = (u, v).$$

Compute  $F^*(u dv + v du)$ .

**Problem 2:** Let  $\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  be the smooth map

$$\Phi(u, v) = \left( u + v, u - v, v^2, \frac{1}{1 + u^2} \right).$$

Compute  $\Phi^*(\eta)$  where

$$\eta = x dy \wedge dz - e^z dz \wedge dw \in \Omega^2(\mathbb{R}^4).$$

**Problem 3:** Consider the following 1-form on  $\mathbb{R}^2 \setminus \{(0, 0)\}$ :

$$\tau = \frac{-y dx + x dy}{x^2 + y^2}.$$

Let  $\gamma : \mathbb{R} \rightarrow \mathbb{R}^2 \setminus \{(0, 0)\}$  be  $\gamma(t) = (\cos t, \sin t)$ . Compute  $\gamma^*(\tau)$ .

**Problem 4:** Construct a nowhere-vanishing smooth 2-form on the 2-sphere. (use the fact that the 2-sphere is the regular value set of  $x^2 + y^2 + z^2 - 1$ ).