Introduction to Manifolds

Assignment 6
Due Date: 02/11/2017

Problem 1: In each of the following find the maximal integral curve of the given vector field.

1. $X = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}$ on $\mathbb{R}^2$ with the initial point $p = (0, 0)$.
2. $X = \frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ on $\mathbb{R}^2$ with the initial point $p = (a, b)$.
3. $X = x\frac{\mathrm{d}}{\mathrm{d}x}$ on $\mathbb{R}$ with the initial point any $p \in \mathbb{R}$.
4. $X = x^2\frac{\mathrm{d}}{\mathrm{d}x}$ on $\mathbb{R}$ with the initial point any $p \in \mathbb{R}_{>0}$.

Problem 2: Let f, g be two smooth real valued functions and let X, Y be two smooth vector fields on a manifold M. Prove that

$$[fX, gY] = fg[X, Y] + f(Xg)Y - g(Yf)X.$$

Problem 3: Consider two smooth vector fields X, y on $\mathbb{R}^n$:

$$X = \sum_i a_i \frac{\partial}{\partial x_i} \quad Y = \sum_j b_j \frac{\partial}{\partial x_j}.$$

Find the coefficients of $[X, Y]$ in terms of a_i's and b_j's.